Paper with Solution

GATE

2001 – 2017

PHYSICS-PH

First Edition : January 2018

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CONTENTS

PAPER WITH SOLUTION : 2001 TO 2017

PAPER

PAGES

- **GATE 2001** **01-08**
- **GATE 2002** **09-16**
- **GATE 2003** **17-28**
- **GATE 2004** **29-40**
- **GATE 2005** **41-52**
- **GATE 2006** **53-63**
- **GATE 2007** **64-76**
- **GATE 2008** **77-92**
- **GATE 2009** **93-102**
- **GATE 2010** **103-111**
- **GATE 2011** **112-120**
- **GATE 2012** **121-130**
- **GATE 2013** **131-137**
- **GATE 2014** **138-145**
- **GATE 2015** **146-155**
- **GATE 2016** **156-162**
- **GATE 2017** **163-168**

SOLUTION

- **GATE 2001** **169-195**
- **GATE 2002** **196-214**
- **GATE 2003** **215-235**
- **GATE 2004** **236-256**
- **GATE 2005** **257-276**
- **GATE 2006** **277-297**
- **GATE 2007** **298-317**
- **GATE 2008** **318-339**
- **GATE 2009** **340-356**
- **GATE 2010** **357-369**
- **GATE 2011** **370-380**
- **GATE 2012** **381-394**
- **GATE 2013** **395-409**
- **GATE 2014** **410-424**
- **GATE 2015** **425-441**
- **GATE 2016** **442-456**
- **GATE 2017** **457-470**

PAPER

2001 – 2017

SECTION-A

1. ***This question consists of TWENTY FIVE sub-questions (1.1 to 1.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×1 = 25]**

1.1. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the position vector, then the value of the following integral $\iint_s \vec{r}\cdot\hat{n}dS$ is

(a) V (b) 2V (c) 0 (d) 3V

1.2. For any operator A, $i(A^+ - A)$ is

(a) Hermitian (b) anti-Hermitian (c) unitary (d) orthogonal

1.3. The value of the integral $\int_C z^{10}dz$, where C is the unit circle with the origin as the centre is

(a) 0 (b) $z^{11}/11$ (c) $2\pi i z^{11}/11$ (d) 1/11

1.4. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

(a) The three vectors are orthonormal

(b) The three vectors are linearly independent

(c) The three vectors cannot form a basis in a three-dimensional real vector space

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as a linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

1.5. The Lagrangian for the Kepler problem is given by

$$L = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu}{r}, (\mu > 0),$$

where (r, θ) denote the polar coordinates and the mass of the particle is unity. Then

(a) $p_\theta = 2r^2\dot{\theta}$ (b) $p_r = 2\dot{r}$

(c) the angular momentum of the particle about the centre of attraction is a constant

(d) the total energy of the particle is time dependent

1.6 Which of the following equations is relativistically invariant? (α, β, γ and δ are constants of suitable dimensions)

(a) $\frac{\partial\phi(x,t)}{\partial t} = \alpha\frac{\partial^2\phi(x,t)}{\partial x^2}$ (b) $\frac{\partial^2\phi(x,t)}{\partial t^2} = \beta\frac{\partial^2\phi(x,t)}{\partial x^2}$

(c) $\frac{\partial^2\phi(x,t)}{\partial t^2} = \gamma\frac{\partial\phi(x,t)}{\partial x}$ (d) $\frac{\partial\phi(x,t)}{\partial t} = \delta\frac{\partial^3\phi(x,t)}{\partial x^3}$

1.7. The Lagrangian for a three particles system is given by:

$$L = \frac{1}{2}\left(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2\right) - a^2(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3),$$

where a is real. Then one of the normal coordinates has a frequency ω given by

(a) $\omega^2 = a^2$ (b) $\omega^2 = a^2/2$ (c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}a^2$

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1.8. Two point charges Q_1 = 1nC and Q_2 = 2nC are kept in free space such that the distance between them is 0.1 m.

(a) The force on Q_2 is along the direction from Q_2 to Q_1

(b) The force on Q_2 is the same in magnitude as that on Q_1

(c) The force on Q_1 is attractive

(d) A point charge Q_3 = –3nC, placed at the midpoint between Q_1 and Q_2, experiences no net force

1.9. A current *I* flows in the anticlockwise direction through a square loop of side *a* lying in the *xoy* plane with its centre at the origin. The magnetic induction at the centre of the square loop is

(a) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_x$ (b) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_z$ (c) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_z$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_x$

1.10. A thin conducting wire is bent into a circular loop of radius *r* and placed in a time dependent magnetic field of magnetic induction.

$\vec{B}(t) = B_0 e^{-\alpha t}\hat{e}_z, (B_0 > 0 \text{ and } \alpha > 0),$

such that, the plane of the loop is perpendicular to $\vec{B}(t)$. Then the induced emf in the loop is

(a) $\pi r^2 \alpha B_0 e^{-\alpha t}$ (b) $\pi r^2 B_0 e^{-\alpha t}$ (c) $-\pi r^2 \alpha B_0 e^{-\alpha t}$ (d) $-\pi r^2 B_0 e^{-\alpha t}$

1.11. Consider an electric field $\vec{E}$ existing in the interface between a conductor and free space. Then the electric field $\vec{E}$ is

(a) external to the conductor and normal to the conductor's surface

(b) internal to the conductor and normal to the conductor's surface

(c) external to the conductor and tangential to the conductor's surface

(d) both external and internal to the conductor and normal to the conductor's surface

1.12. A spinless particle moves in a central potential V(r)

(a) The kinetic energy and the potential energy of the particle cannot simultaneously have sharp values

(b) The total energy and the potential energy of the particle can simultaneously have sharp values

(c) The total energy and the square of the orbital angular momentum about the origin cannot simultaneously have sharp values.

(d) The total energy of the particle can have only discrete eigenvalues

1.13. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$.

(a) $\varphi(x) = A\tan x, A > 0$ (b) $\varphi(x) = B\cos x, B$ real

(c) $\varphi(x) = C\exp(-D/x^2), C > 0, D < 0$ (d) $\varphi(x) = E\,x\exp(-Fx^2), E, F > 0$

1.14. A quantum harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda(a^t a)^2$ acts on the particle, where λ is a constant of suitable dimensions and *a* and $a^\dagger$ are lowering and raising operators respectively. Then the first order energy shift is given by

(a) λn (b) $\lambda^2 n$ (c) λn^2 (d) $(\lambda n)^2$

1.15. Two particles are said to be distinguishable when

(a) the average distance between them is large compared to their de Broglie wavelengths

(b) the average distance between them is small compared to their de Broglie wavelengths

(c) they have overlapping wavepackets

(d) their total wave function is symmetric under particle exchange

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1.16. For an energy state E of a photon gas, the density of states is proportional to
(a) $\sqrt{E}$ (b) E (c) $E^{3/2}$ (d) E^2

1.17. X-rays were produced using Cobalt (Z = 27) as target. It was observed that the X-ray spectrum contained a strong K_α line of wavelength 0.1785 nm and a weak K_α line of wavelength 0.1930 nm. Then, the weak K_α line is due to an impurity whose atomic number is
(a) 25 (b) 26 (c) 28 (d) 30

1.18. A sample of Silicon of thickness $200\mu m$ is doped with 10^{23} Phosphorous atoms per m^3. If the sample is kept in a magnetic field of 0.2 Wb/m^2 and a current of 1 mA is passed through the sample, the Hall voltage produced is
(a) $62.5\mu V$ (b) $-6.25\mu V$ (c) $+6.25\mu V$ (d) $-62.5\mu V$

1.19. The probability that a state which is 0.2 eV above the Fermi energy in a metal atom at 700K is
(a) 96.2% (b) 62.3% (c) 3.5% (d) 37.7%

1.20. The distance between the adjacent atomic planes in $CaCO_3$ is 0.3 nM. The smallest angle of Bragg scattering for 0.03 nm X-ray is
(a) 2.9º (b) 1.5º (c) 0.29º (d) 5.8º

1.21. Infrared absorption can be observed in which of the following molecules?
(a) N_2 (b) O_2 (c) HCl (d) C_2

1.22. The cross-sections of the reactions $p + \Pi^- \rightarrow \Sigma^- + K^+$ and $p^- + \Pi^+ \rightarrow \bar{\Sigma}^- + K^-$ at a given energy are the same due to
(a) baryon number conservation (b) time-reversal invariance
(c) charge conjugation (d) parity conservation

1.23. RAM and ROM are
(a) charge coupled devices used in computers (b) computer memories
(c) logic gates (d) binary counters used in computers

1.24. In an *n-p-n* transistor, the leakage current consists of
(a) electrons moving from the base to the emitter
(b) electrons moving from the collector to the base
(c) electrons moving from the collector to the emitter
(d) electrons moving from the base to the collector

1.25. A piece of semiconducting material is introduced into a circuit. If the temperature of the material is raised, the circuit current will
(a) increase (b) remain the same (c) decrease (d) cease to flow

2. ***This question consists of TWENTY FIVE sub-questions (2.1 to 2.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×2 = 50]**

2.1. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals
(a) 1 (b) 3 (c) 0 (d) –3

2.2. The value of the residue of $\frac{\sin z}{z^6}$ is
(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$ (c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

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2.3. If $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$, then $\hat{F}^2[f(x)]$ is equal to

(a) $f(x)$ (b) $-f(x)$ (c) $f(-x)$ (d) $[f(x) + f(-x)]/2$

2.4. A particle of mass *m* is constrained to move on the plane curve $xy = C (C > 0)$ under gravity (*y* axis vertical). The Lagrangian of the particle is given by

(a) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C^2}{x^4}\right)+\frac{mgC}{x}$ (b) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C^2}{x^4}\right)-\frac{mgC}{x}$

(c) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C}{x^2}\right)+\frac{mgC}{x}$ (d) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C}{x^2}\right)-\frac{mgC}{x}$

2.5. If poisson bracket $[q, f(p)] = \alpha f(p)$, where α is a scalar then $f(p)$ is equal to

(a) $e^{\alpha p}$ (b) $e^{-\alpha p}$ (c) $\alpha e^{-\alpha p}$ (d) αe^{-p}

2.6. *x* and *p* are two operators which satisfy $[x, p] = i$. The operators X and P are defined as $X = x\cos\phi + p\sin\phi$ and $Y = -x\sin\phi + p\cos\phi$, for ϕ real. Then $[X, Y]$ equals

(a) 1 (b) –1 (c) *i* (d) –*i*

2.7. A quantum particle of mass *m* is confined to a square region in *xoy*-plane whose vertices are given by (0, 0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for *l*, *m*, *n* positive integers)?

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$ (b) $\frac{2}{L}\cos\left(\frac{l\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$ (d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{l\pi y}{L}\right)$

2.8. Let $\vec{L} = (L_x, L_y, L_z)$ denote the orbital angular momentum operators of a particle and let $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$. The particle is in an eigenstate of L^2 and L_z eigenvalues $\hbar^2 \ell(\ell+1)$ and $\hbar\ell$ respectively. The expectation value of $L_+ L_-$ in this state is

(a) $\ell\hbar^2$ (b) $2\ell\hbar^2$ (c) 0 (d) $\ell\hbar$

2.9. A normalized state of a particle moving in a potential V(*x*) is given by $\psi(x,t) = \sum_{n=0}^{\infty} C_n \phi_n(x) e^{-E_n t/h}$ where $\phi_n(x)$'s are the normalized eigenfunctions of the particle corresponding to the energies E_n's. Then

(a) $\sum_{n=0} |C_n|^2 = 1$

(b) The average energy of the particle in the state $\psi(x,t)$ is $\sum_{n=\infty}^{\infty} |C_n| E_n$

(c) $\psi(x,t)$ is an eigenfunction of the Hamiltonian of the particle

(d) $\psi(x,t)$ is an eigenfunction of the momentum operator

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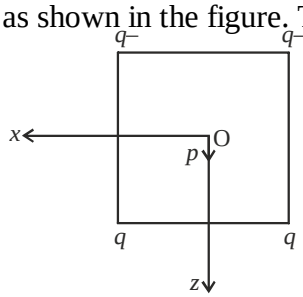
2.10. A coaxial cable of uniform cross-section contains an insulating material of dielectric constant 3.5. The radius of the central wire is 0.01 m and that of the sheath is 0.02 m. The capacitance per kilometer of a cable is

(a) 280.5 nF (b) 28.05 nF (c) 56.10 nF (d) 2.805 nF

2.11. The *xoy* plane carries a uniform surface current of density $\vec{K} = 50\hat{e}_z$ A/m. The magnetic field at the point $z = -0.5\text{m}$ is

(a) 10×10^{-6} Wb (b) 1×10^{-6} Wb (c) $\pi\times10^{-6}$ Wb (d) $10\pi\times10^{-6}$ Wb

2.12. Four point charges are placed at the corners of a square whose center is at the origin of a Cartesian coordinate system. A point dipole $\vec{p}$ is placed at the centre of the square as shown in the figure. Then,



(a) there is no force acting on the dipole

(b) there is no torque about the centre of O on the dipole

(c) the dipole has minimum energy if it is in $\hat{e}_x$ direction

(d) the force on the dipole is increased if the medium is replaced by another medium with larger dielectric constant

2.13. The electric field $E(r,t)$ at a point *r* at time *t* in a metal due to the passage of electrons can be described by the equation

$$\nabla^2\vec{E}(\vec{r},t) = \frac{1}{c^2}\left[\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2} + \omega'^2\vec{E}(\vec{r},t)\right]$$

where ω' is a characteristic associated with the metal and *c* is the speed of light in vacuum. The dispersion relation corresponding to the plane wave solutions of the form $\exp\left[i(\vec{k}\cdot\vec{r} - \omega t)\right]$ is given by

(a) $\omega^2 = c^2k^2 - \omega'^2$ (b) $\omega^2 = c^2k^2 + \omega'^2$ (c) $\omega = ck - \omega'$ (d) $\omega = ck + \omega'$

2.14. A copper wire of uniform cross-sectional area 1.0×10^{-6} m^2 carries a current of 1A. Assuming that each copper atom contributes one electron to the electron gas, the drift velocity of the free electrons (density of copper is 8.94×10^3 kg/m^3 and its atomic mass is 1.05×10^{-25} kg) is

(a) 7.4×10^{-4} m/s (b) 74×10^{-4} m/s (c) 74×10^{-3} m/s (d) 7.4×10^{-5} m/s

2.15. The number of hyperfine components observed in the electronic transition $^2p_{1/2} \rightarrow {}^2S_{1/2}$ of an atom with nuclear spin $\frac{1}{2}$ is

(a) 3 (b) 4 (c) 6 (d) 5

2.16. Which of the following functions describes the nature of interaction potential V(r) between two quarks inside a nucleon? (*r* is the distance between the quarks and *a* and *b* positive constants of suitable dimensions)

(a) $V(r) = \frac{a}{r} + br$ (b) $V(r) = -\frac{a}{r} + br$ (c) $V(r) = \frac{a}{r} - br$ (d) $V(r) = -\frac{a}{r} - br$

2.17. Which of the following reactions violates lepton number conservation?

(a) $e^+ + e^- \rightarrow \upsilon + \overline{\upsilon}$ (b) $e^- + p \rightarrow \upsilon + n$

(c) $e^+ + n \rightarrow p + \upsilon$ (d) $\mu^- \rightarrow e^- + \upsilon + \overline{\upsilon}$

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2.18. The Lande *g*-factor for the $^3P_1$ level of an atom is

(a) 1/2 (b) 3/2 (c) 5/2 (d) 7/2

2.19. The pure rotational levels of a molecule in the far-infrared region follows the formula F(J) = BJ (J+1), where F(J) is the energy of the rotational level with quantum number J and B is the rotational constant. The lowest rotational energy gap in rotational Raman spectrum is

(a) 2B (b) 4B (c) 6B (d) 8B

2.20. The total number of Zeeman components observed in an electronic transition $^2D_{5/2} \rightarrow {}^2P_{3/2}$ of an atom in a weak field is

(a) 4 (b) 6 (c) 12 (d) 10

2.21. A resistance of 600Ω is parallel to an inductance of reactance $600\,(\Omega)$ applied voltage, then the total impedance of the circuit is

(a) 628Ω (b) 268Ω (c) $424\,\Omega$ (d) 300Ω

2.22. An *n*-channel silicon (dielectric constant = 12) FET with a channel width $a = 2 \times 10^{-6}$ m is doped with 10^{21} electrons / m^3. The pinch-off voltage is

(a) 0.86V (b) 0.68V (c) 8.6V (d) 6.8V

2.23. The solution of the system of differential equations

$$\frac{dy}{dx} = y - z \text{ and } \frac{dz}{dx} = -4y + z$$

is given by (for A and B are arbitrary constants)

(a) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} + 2Be^{-x}$ (b) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} + 2Be^{-x}$

(c) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} - 2Be^{-x}$ (d) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} - 2Be^{-x}$

2.24. If $u(x, y, z, t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions, is a solution of the wave function

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \text{ then } \beta \text{ is}$$

(a) $\left(1 - \frac{v}{c}\right)^{1/2}$ (b) $\left(1 - \frac{v}{c}\right)$ (c) $\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1 - \frac{v^2}{c^2}\right)$

2.25. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by

(a) $\frac{Ik_BT}{\hbar^2}$ (b) $\frac{2Ik_BT}{\hbar^2}$ (c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$

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SECTION-B

***This section consists of TWENTY* questions of *FIVE* marks each. *ANY FIFTEEN* out of these questions have to answered on the Answer Book provided. [75 Marks]**

3. Given $\vec{A} = y^2\hat{e}_x + 2yx\hat{e}_y + (xye^z - \sin x)\hat{e}_z$, calculate the value of $\iint_S \left(\nabla \times \vec{A}\right)\cdot \hat{n}\, ds$ over the part of the sphere $x^2 + y^2 + z^2 = 1$ above the *xoy* plane.

4. Find the matrix of the linear transformation T on $V_3(R)$ (i.e., three dimensional real vector space) defined as $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b \\ b+c \\ c+a \end{pmatrix}$, with respect to the basis $B = \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$, where $\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

   Also calculate the matrix representation of T^{-1}.

5. Find the general solution of $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0,$ using the Frobenius power series method.

6. Consider the Lagrangian $L = \frac{q\ln(\dot{q})^2}{2} - \lambda q + qf(q),$ where $f(q)$ is an arbitrary function and $\lambda > 0$.

   (a) Write down the Euler-Lagrange equation of motion

   (b) Does the Lagrangian transform covariantly under the transformation $q \to \alpha q$ for α a real constant?

   (c) Calculate the Hamiltonian of the system

   (d) Is total energy a constant of motion?

7. A square lamina OABC of side *l* and negligible thickness is lying in the XOY plane of a Cartesian coordinate system such that O is at the origin and the sides OA and OC are along the positive X and Y directions respectively. Calculate the moment of inertia tensor and the directions of the three principal moments. The mass of the lamina is *m*.

8. A particle of mass *m* is subjected to a potential $V(x) = k|x|, k > 0$

   (a) If H is the Hamiltonian of the particle, calculate $[H, V(x)]$

   (b) Use the uncertainty principle in the form $\Delta x\, \Delta p \sim \frac{\hbar}{2}$ to estimate the ground state energy of the particle.

9. A particle of mass *m* in the one-dimensional energy well

$$V(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & \text{elsewhere,} \end{cases}$$

   is in a state whose coordinate wave function is given by $\phi(x) = Cx(L - x),$ where *c* is the normalization constant.

   (a) Determine the expectation value of the energy in the state $\phi(x)$

   (b) Calculate the probability that on measurement of energy, the particle will be found in its ground state.

$$\left[\text{Standard Integrals}: \int_0^L dx\, x\sin(\pi x/L) = L^2/\pi, \int_0^L dx\, x^2\sin(\pi x/L) = \frac{L^3}{\pi} - \frac{4L^3}{\pi^3}\right]$$

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10. Consider the harmonic oscillator in the form $H = (p^2 + x^2)/2$ (we have set $m = 1$, $\omega = 1$ and $\hbar = 1$). The harmonic oscillator is in its *n*th energy eigenstate and subjected to a time-independent perturbation $\lambda(xp + px)$, for λ real. Calculate the first-order energy shift and first-order correction to the wave function.

11. An ideal electron gas is confined to an area A in a two-dimensional plane at temperature T. Calculate
    (a) the density of states
    (b) N, the number of electrons
    (c) E_F, the Fermi energy as a function of N

12. Write down the partition function of a particle of mass *m* whose potential energy is given by $V(x, y, z) = ax^2 + b(y^2 + z^2)^{1/2}$, where *a* and *b* are positive constants of suitable dimensions. Also calculate the average energy of the particle.

$$\left[\text{Standard Integral}: \int_{-\infty}^{\infty} dx\, e^{-x^2/2\beta^2} = \sqrt{2\pi}\,\beta\right]$$

13. Given that the molecular weight of KCl is 74.6 and its density is 1.99×10^3 kg/m^3, calculate the following:
    (a) the distance between the atomic planes
    (b) the lattice constant

14. The reaction ${}^3H(p,n)^2He$ has a Q value of –0.764 MeV. Calculate the threshold energy of incident protons for which neutrons are emitted in the forward direction.

15. A circular conducting loop C_1 of radius 2m is located in the XOY plane such that its centre is at (0, 0, 0). Another circular conducting loop C_2 of radius 2m is located at (0, 0, 4) such that the plane of C_2 is parallel to the XOY plane. A current of 5A is flowing in each of these loops such that the positive Z-axis lies to the left of the directions of the currents. Find the magnetic induction $\vec{B}$ produced at (0, 0, 0), neglecting the mutual induction of the loops.

16. Draw the electrical circuits for each of the following ********** source (battery), a detector (lamp), and switch (es).
    (a) AND (b) OR (c) NOT (d) NAND (e) NOR

17. The pinch-down voltage of a *p*-channel junction FET is V_P = 5V and the drain-to-source saturation current I_{DSS} = –40mA. The value of drain-source voltage V_{DS} is such that the transistor is operating in the saturated region. The drain current is given as I_D = –15 mA. Find the gate-source voltage V_{GS}.

18. A narrow beam of electrons, accelerated under a potential difference, incident on a crystal whose grating space is 0.3 nm. If the first diffraction ring is produced at an angle 5.8º from the incident beam, find the momentum of the electrons and the potential difference applied.

19. The region $z > 0$ of a Cartesian coordinate system contains a linear isotropic dielectric of dielectric constant 2.0. The region $z < 0$ is the free space. A free space charge density of 5nC/m^2 is at the interface $z = 0$. If the displacement vector in the dielectric is $\vec{D}_2 = 3\hat{e}_x + 4\hat{e}_y + 6\hat{e}_z$ nC/m^2, find the corresponding displacement vector $\vec{D}_1$ in the free space.

20. The series limit of the Balmer series for hydrogen atom is given as 360 nm. Calculate the atomic number of the element that gives the lowest *x*-ray wavelength at 0.1 nm of the K-series.

21. The first few electronic energy states for neutral copper atom (Z = 29) are given as $E_1 < E_2 < E_3$, where E_1 being the ground electronic state. The states E_2 and E_3 are doubly degenerate due to spin splitting. Write the electron configuration of the states and arrange the spectral terms of the split levels following Hund's rules. Explain why $E_2 < E_3$.

22. The rotational lines of the CN band system at 3883.4Å is represented by a formula $\bar{\nu}$ = (25798 + 3.850m + 0.068 m^2) cm^{-1}, where *m* is a running number. Calculate the values of the rotational constants B'_v and B''_v, the location of the band head and the degradation of the band.

CAREER ENDEAVOUR

SECTION-A

1. ***This question consists of TWENTY FIVE sub-questions (1.1 to 1.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×1 = 25]**

1.1. If two matrices A and B can be diagonalized simultaneously, which of the following is true?

(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$

1.2. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

1.3. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true?

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

1.4. The homogeneity of time leads to the law of conservation of

(a) linear momentum (b) angular momentum

(c) energy (d) parity

1.5. Hamilton canonical equations of motion for a conservative system are

(a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (b) $\frac{dp_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

CAREER ENDEAVOUR

1.6. If R_1 is the value of the Rydberg constant assuming mass of the nucleus to be infinitely large compared to that of an electron, and if R_2 is the Rydberg constant taking nuclear mass to be 7500 times the mass of the electron, then the ratio R_2/R_1 is

(a) a little less than unity (b) a little more than unity

(c) infinitely small (d) infinitely large

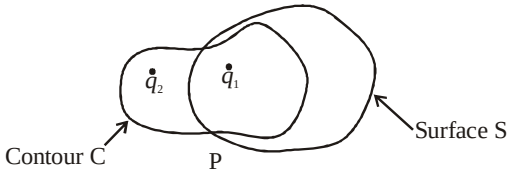
1.7. Consider an infinitely long straight cylindrical conductor of radius R with its axis along the z-direction, which carries a current of 1A uniformly distributed over its cross section. Which of the following statements is correct?

(a) $\vec{\nabla} \times \vec{B} = 0$ everywhere (b) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ everywhere,

(c) $\vec{\nabla} \times \vec{B} = 0$ for $r > R$, (d) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ for $r > R$

where r is the radial distance from the axis of the cylinder.

1.8. Consider a set of two stationary point charges q_1 and q_2 as shown in the figure. Which of the following statements is correct?



(a) The electric field at P is independent of q_2

(b) The electric flux crossing the closed surface S is independent of q_2

(c) The line integral of the electric field $\vec{E}$ over the closed contour C depends on q_1 and q_2.

(d) $\vec{\nabla}\cdot\vec{E}=0$ everywhere

1.9. If the wave function of a particle trapped in space between $x=0$ and $x=L$ is given by $\psi(x)=A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant, for which value(s) of *x* will the probability of finding the particle be the maximum?

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) $\frac{L}{6}$ and $\frac{L}{3}$ (d) $\frac{L}{4}$ and $\frac{3L}{4}$

1.10. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $\left|+\hat{z}\uparrow\right\rangle$ state. Which of the following statements is true?

(a) The particle has a definite value of the *y*-component of the spin angular momentum

(b) The particle has a definite value of the square of the spin angular momentum

(c) The particle has a definite value of the *x*-components of spin angular momentum

(d) The particle has definite values of *x*-and *y*-components of spin angular momentum

1.11. If σ is the total cross-section and $f(\theta),\theta$ being the angle of scattering, is the scattering amplitude for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that *k* is the magnitude of the wave vector along the $\hat{z}$ direction.

(a) $\sigma=|f(\theta)|^2$ (b) $\sigma=\frac{4\pi}{k}|f(\theta=0)|^2$

(c) $\sigma=\frac{4\pi}{k}\times$ Imaginary part of $[f(\theta=0)]$ (d) $\sigma=\frac{4\pi}{k}|f(\theta)|^2$

CAREER ENDEAVOUR

1.12. In a classical micro-canonical ensemble for a system of N non-interacting particles, the fundamental volume in phase space which is regarded as "equivalent to one micro-state" is

(a) h^{3N} (b) h^{2N} (c) h^N (d) h

where *h* is the Planck's constant

1.13. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place?

(a) $T<\frac{h^2}{2\pi mk_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{3/2}$ (b) $T<\frac{h^2}{2\pi mk_B}\left\{\frac{V}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{3/2}$

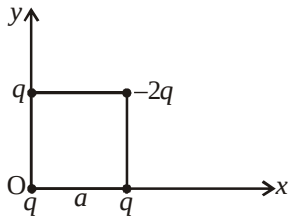
(c) $T<\frac{h^2}{2\pi mk_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$ (d) $T<\frac{h^2}{2\pi mk_B}\left\{\frac{V}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

where *m* is the mass of each particle of the system, k_B is the Boltzmann constant, *h* is the Planck's constant and ς is the well known Zeta function.

1.14. A large circular coil of N turns and radius R carries a time varying current $I = I_0 \sin(\omega t)$. A small circular coil of *n* turns and radius $r(r << R)$ is placed at the center of the large coil such that the coils are concentric and coplanar. The induced emf in the small coil

(a) leads the current in the large coil by $\pi/2$
(b) lags the current in the large coil by π
(c) is the phase with the current in the large coil
(d) lags the current in the large coil by $\pi/2$

1.15. Four charges are placed at the four corners of a square of side *a* as shown in the figure. The electric dipole moment of this configuration is



(a) $\vec{p} = qa\hat{i} + qa\hat{j}$

(b) $\vec{p} = -qa\hat{i} + qa\hat{j}$

(c) $\vec{p} = -qa\hat{i} - qa\hat{j}$

(d) $\vec{p} = qa\hat{i} - qa\hat{j}$

1.16. Which of the following statements is true?

(a) In a micro-canonical ensemble the total number of particles N and the energy E are constants while in a canonical ensemble N and temperature T are constants

(b) In a micro-canonical ensemble the total number of particles N is a constant but the energy E is variable while in a canonical ensemble N and T are constants

(c) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N and T both vary

(d) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N is a constant but T varies

1.17. In a one-dimensional Kronig Penny model, the total number of possible wave functions is equal to

(a) twice the number of unit cells
(b) number of unit cells
(c) half the number of unit cells
(d) independent of the number of unit cells

1.18. The potential in a divalent solid at a particular temperature is represented by a one-dimensional periodic model. The solid should behave electrically as

(a) a semiconductor
(b) a conductor
(c) an insulator
(d) a superconductor

CAREER ENDEAVOUR

1.19. In a cubic system with cell edge *a*, two phonons with wave vectors $\vec{q}_1$ and $\vec{q}_2$ collide and produce a third phonon with a wave vector $\vec{q}_3$ such that

$$\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R},$$

where $\vec{R}$ is a lattice vector. Such a collision process will lead to

(a) finite thermal resistance
(b) zero thermal resistance
(c) an infinite thermal resistance
(d) a finite thermal resistance for certain $|\vec{R}|$ only

1.20. The baryon number of proton, the lepton number of proton, the baryon number of electron and the lepton number of electron are respectively

(a) zero, zero, one and zero
(b) one, one, zero and one
(c) one, zero, zero and one
(d) zero, one, one and zero

1.21. Typical energies released in a nuclear fission and a nuclear fusion reaction are respectively

(a) 50 MeV and 1000 MeV
(b) 200 MeV and 1000 MeV
(c) 1000 MeV and 50 MeV
(d) 200 MeV and 10 MeV

1.22. Nuclear forces are

(a) spin dependent and have no non-central part (b) spin dependent and have a non-central part

(c) spin independent and have no non-central part (d) spin independent and have a non-central part

1.23. The nuclear spins of $_6C^{14}$ and $_{12}Mg^{25}$ nuclei are respectively

(a) zero and half-integer (b) half-integer and zero

(c) an integer and half-integer (d) both half-integers

1.24. The asymmetry terms in the Weizsacker semi-empirical mass formula is because of

(a) non-spherical shape of the nucleus

(b) non-zero spin of nucleus

(c) unequal number of protons and neutrons inside the nucleus

(d) odd number of protons inside the nucleus

1.25. Which of the following options is true for a two input XOR gate?

	Input		**Output**
	A	**B**	
(a)	0	1	1
(b)	1	0	0
(c)	0	0	1
(d)	1	1	1

2. ***This question consists of TWENTY FIVE sub-questions (2.1 to 2.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×2 = 50]**

2.1. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$, where a and b $(a \neq b)$ are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors?

(a) $-b\hat{i} + a\hat{j}$ (b) $-a\hat{i} + b\hat{j}$ (c) $-a\hat{i} - b\hat{j}$ (d) $-b\hat{i} - a\hat{j}$

2.2. Fourier transform of which of the following functions does not exist?

(a) $e^{-|x|}$ (b) xe^{-x^2} (c) e^{x^2} (d) e^{-x^2}

2.3. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1, 1, 7) is

(a) $(-6\hat{i} + 4\hat{j} + \hat{k})/\sqrt{53}$ (b) $(4\hat{i} + 6\hat{j} - \hat{k})/\sqrt{53}$

(c) $(6\hat{i} + 4\hat{j} - \hat{k})/\sqrt{53}$ (d) $(4\hat{i} + 6\hat{j} + \hat{k})/\sqrt{53}$

2.4. The solution of the differential equation

$$(1+x)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx} - y(x) = 0 \text{ is}$$

(a) $Ax^2 + B$ (b) $Ax + Be^{-x}$ (c) $Ax + Be^{x}$ (d) $Ax + Bx^2$

where A and B are constants

2.5. A particle of mass M moving in a straight line with speed v collides with a stationary particle of the same mass. In the center of mass coordinate system, the first particle is deflected by 90°.The speed of the second particle, after collision, in the laboratory system will be

(a) $v/\sqrt{2}$ (b) $\sqrt{2}\,v$ (c) v (d) $v/2$

CAREER ENDEAVOUR

2.6. The scalar potential corresponding to the force field $\vec{F} = \hat{i}(y+z)$

(a) is $y^2/2$ (b) is 1 (c) is zero (d) does not exist

2.7. Two particles of equal mass are connected by an inextensible string of length L. One of the masses is constrained to move on the surface of a horizontal table. The string passes through a small hole in the table and the other mass is hanging below the table. The only constraint is that the first mass moves on the surface of the table. The number of degrees of freedom of the masses-string system is

(a) five (b) four (c) two (d) one

2.8. An electron is accelerated from rest by 10.2 million volts. The percent increase in its mass is

(a) 20,000 (b) 2,000 (c) 200 (d) 20

2.9. An infinitely long closely wound solenoid carries a sinusoidally varying current. The induced electric field is

(a) zero everywhere

(b) non-zero inside and zero outside the solenoid

(c) non-zero inside as well as outside the solenoid

(d) zero inside and non-zero outside the solenoid

2.10. A laser beam of wavelength 600 nm with a circular cross section having a radius of 10mm falls normally on a lens of radius 20 mm and focal length 10 cm. The radius of the focussed spot is approximately

(a) $0.3\,\mu m$ (b) $0.6\,\mu m$ (c) $3.0\,\mu m$ (d) $6.0\,\mu m$

2.11. A left circularly polarized light beam of wavelength 600 nm is incident on a crystal of thickness *d* and propagates perpendicular to its optic axis. The ordinary and extraordinary refractive indices of the crystal are $n_0 = 1.54$ and $n_e = 1.55$ respectively. The emergent light will be right circularly polarized if *d* is

(a) $120\,\mu m$ (b) $60\,\mu m$ (c) $30\,\mu m$ (d) $15\,\mu m$

2.12. In a two beam interference pattern, the maximum and minimum intensity values are found to be $25I_0$ and $9I_0$ respectively, where I_0 is a constant. The intensities of the two interfering beams are

(a) $16I_0$ and I_0 (b) $5I_0$ and $3I_0$ (c) $17I_0$ and $8I_0$ (d) $8I_0$ and $2I_0$

2.13. An electron propagating along the *x*-axis passes through a slit of width $\Delta y = 1\text{nm}$. The uncertainty in the *y*-component of its velocity after passing through the slit is

(a) 7.322×10^5 m/s (b) 1.166×10^5 m/s

(c) 3.436×10^5 m/s (d) 2.326×10^4 m/s

2.14. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $[\hat{A}, \hat{B}]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $\left[[\hat{A}, \hat{B}], [\hat{B}, \hat{A}]\right]$ is equal to

(a) $\hat{A}\hat{B}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{B}\hat{A}$ (b) $\hat{A}(\hat{A}\hat{B} - \hat{B}\hat{A}) - \hat{B}(\hat{B}\hat{A} - \hat{A}\hat{B})$

(c) zero (d) $\left([\hat{A}, \hat{B}]\right)^2$

2.15. An electron is in a state with spin wave function $\phi_s = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$ in the S_z representation. What is the probability of finding the z-component of its spin along the $-\hat{z}$ direction?

(a) 0.75 (b) 0.50 (c) 0.35 (d) 0.25

CAREER ENDEAVOUR

2.16. If the wavelength of the first line of the Balmer series in the hydrogen spectrum is λ, then the wavelength of the first line of the Lyman series is

(a) $(27/5)\lambda$ (b) $(5/27)\lambda$ (c) $(32/27)\lambda$ (d) $(27/32)\lambda$

2.17. A vector $\vec{A} = (5x + 2y)\hat{i} + (3y - z)\hat{j} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value

(a) 4 (b) –4 (c) 8 (d) –8

2.18. The mean free path of the particles of a gas at a temperature T_0 and pressure p_0 has a value λ_0. If the pressure is increased to $1.5p_0$ and the temperature is reduced to 0.75 T_0, the mean free path

(a) remains unchanged (b) is reduced to half

(c) is doubled (d) is equal to $1.125\,\lambda_0$

2.19. Which of the following relations between the particle number density n and temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?

(a) $n/T^{1/2} << 1$ (b) $n/T^{3/2} << 1$

(c) $n/T^{3/2} >> 1$ (d) $n/T^{1/2}$ and $n/T^{3/2}$ can have any value

2.20. For a prefect free-electron gas in a metal, the magnitudes of phase velocity (v_p) and group velocity (v_g) are such that

(a) $v_p = v_g$ (b) $v_p = \frac{1}{2}v_g$

(c) $v_p = \sqrt{2}v_g$ (d) $v_p = 2v_g$

2.21. A metal has free-electron density $n = 10^{29}$ m^{-3}. Which of the following wavelengths will excite plasma oscillations?

(a) $0.033\,\mu$m (b) $0.330\,\mu$m (c) $3.300\,\mu$m (d) $33.000\,\mu$m

2.22. For an NaCl crystal, the cell-edge a = 0.563 nm. The smallest angle at which Bragg reflection can occur corresponds to a set of planes whose incides are

(a) 100 (b) 110 (c) 111 (d) 200

2.23. Silicon has diamond structure with unit-cell-edge a = 0.563 nm. The interatomic separation is

(a) 0.122 nm (b) 0.234 nm (c) 0.383 nm (d) 0.542 nm

2.24. The spin and parity of $_4Be^9$ nucleus, as predicted by the shell model, are respectively.

(a) 3/2 and odd (b) 1/2 and odd (c) 3/2 and even (d) 1/2 and even

2.25. The feedback ratio of an amplifier, which on application of a negative feedback, changes the voltage gain from –250 to –100 is

(a) –0.250 (b) –0.025 (c) –0.060 (d) –0.006

CAREER ENDEAVOUR

SECTION-B

***This section consists of TWENTY* questions of *FIVE* marks each. *ANY FIFTEEN* out of these questions have to answered on the Answer Book provided. [75 Marks]**

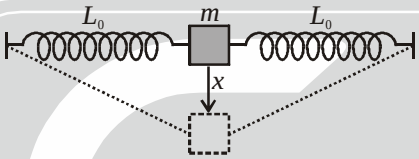
3. Given the differential equation $\frac{d^2y(x)}{dx^2}+2\frac{dy(x)}{dx}+5y(x)=0$ find its solution that satisfies the initial conditions

$y=0$ and $x=0$ and $\frac{dy}{dx}=1$ at $x=0$

4. Find the matrix that diagonalizes the matrix $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

5. Using the residue theorem, compute the integral $I=\int_0^\infty \frac{dx}{(1+x)^4}$.

6. A particle of mass M is attached to two identical springs of unstretched length L_0 and spring constant *k*. The entire system is placed on a horizontal frictionless table as shown in the figure. The mass is slightly puled along the surface of the table and perpendicular to the lengths of the springs and then let go. Using the Lagrangian equation (s) of motion, show whether the mass will execute simple harmonic motion. If so, find the time period.



7. A uniform thin circular disc of mass M and radius R lies in the X-Y plane with its centre at the origin. Find the moments of inertia tensor. What are the values of the principal moments of inertia? Find the principal axes.

8. Two events, 10^{-7} s apart in time, take place at two points 50 m apart on the X-axis. Find the speed of an observer moving along the X-axis who observes the two events simultaneously. What will be the spatial separation between these two events as seen by this observer?

9. Consider a parallel plate air filled capacitor with a plate area of 10 cm^2 separated by a distance of 2 mm. The potential difference across the plates varies as

V = 360 sin ($2\pi \times 10^6$ *t*) volts,

where *t* is measured in seconds. Neglecting fringe effects, calculate the displacement current flowing through the capacitor.

10. The potential of a spherically symmetric charge distribution is given by

$$V(r)=\frac{a}{3}\left(4-\frac{r^2}{R^2}\right);\ \text{for}\ r\le R$$

$$=\frac{aR}{r};\ \text{for}\ r>R,$$

a and *R* being constants. Find the corresponding charge distribution.

11. Consider a plane electromagnetic wave propagating in free space and having an electric field distribution given by

$$\vec{E}=E_0\left(\frac{\sqrt{3}}{2}\hat{j}-\frac{1}{2}\hat{i}\right)\exp\left[i\left(\omega t-\frac{\sqrt{3}}{2}\alpha x-\frac{1}{2}\alpha y\right)\right],$$

where E_0, ω and a are constants. Calculate the corresponding magnetic field $\vec{B}$.

CAREER ENDEAVOUR

12. A particle in the ground state of an infinitely deep one dimensional potential well of width a is subject to a perturbation of the form

$$V = V_0 \cos^2\left(\frac{\pi x}{a}\right)$$

where V_0 is a constant. Find the shift in energy of the particle in the lowest order perturbation theory.

13. A quantum particle is in a state which is the superposition of the eigenstates of the momentum operator $p_x = -i\hbar\frac{\partial}{\partial x}$. If the probability of finding the momentum $\hbar k$ of the particle is 90%, compute its wave function.

14. The wave function of a free particle is given by $\psi(\vec{r}) = Ce^{-(x^2+y^2+z^2)}$, where C is a constant. Compute the momentum space probability density, normalize it to 1 and hence find the value of C.

15. Carbon monoxide has a bond length of 0.1132 nm. What will be the frequency of rotation of the molecule for its lowest excited state?

16. 1 Kg of water at a temperature of 353 K is mixed adiabatically with an equal mass of water at 293 K. Find the change in entropy of the universe assuming the specific heat of water to be constant equal to 238 $J.kg^{-1}.\ K^{-1}$.

17. A conductor having a free electron gas is maintained at a very low temperature ($T \to 0K$). Find the average energy per electron in terms of the electrons density and the electron mass.

18. A small concentration of minority carries is injected into a homogeneous semiconductor held at 300K. An electric field of 30 V/cm is applied across the width of the crystal. As a result, the minority carriers move a distance of 1.5 cm in a time of 300 μs. What is the diffusion coefficient of the minority carriers in the semiconductor?

19. A phonon with wave vector $\vec{q}$ gets absorbed on collision with an electron of wave vector $\vec{k}$. The electron is considered free and its energy is much larger than that of the phonon. If the electron is scattered at an angle α, show that $\alpha = 2\sin^{-1}(q/2k)$.

20. In spherical coordinates, the wave function describing a state of a system is

$$\psi(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}e^{-4/2a_0}\sin\theta e^{-i\phi}$$

where a_0 is a constant. Find the parity of the system in this state.

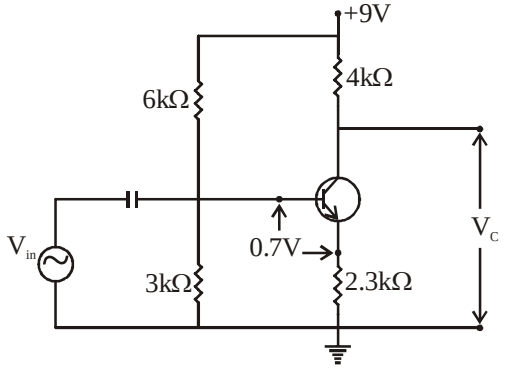
21. Calculate the minimum kinetic energy that the neutron should have in order to induce the reaction

$O^{16}(n^1,\ He^4)C^{13}$

in which C^{13} is left in an excited state of energy 1.79 MeV. Given:

Mass of O^{16} = 16.000000 amu
Mass of n^1 = 1.008986 amu
Mass of He^4 = 4.003874 amu
Mass of C^{13} = 13.007490 amu

22. Calculate the dc collector voltage (V_c) with respect to ground in the amplifier circuit shown in the figure. The current gain β for the transistor is 200.



CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.30 : Carry ONE mark each.

1. The two vectors $P = i, q = (i + j)/\sqrt{2}$ are
   (a) related by a rotation
   (b) related by a reflection through the *xy*-plane
   (c) related by an inversion
   (d) not linearly independent

2. A 3 × 3 matrix has eigenvalues 0, $2+i$ and $2-i$. Which one of the following statements is correct?
   (a) The matrix is Hermitian
   (b) The matrix is unitary
   (c) The inverse of the matrix exists
   (d) The determinant of the matrix is zero

3. The value of the integral $\int_C dz/z^2$, where *z* is a complex variable and C is the unit circle with the origin as its centre, is
   (a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$

4. A particle with an initial velocity $v_0\hat{i}$ enters a region with an electric field $E_0\hat{j}$ and a magnetic field $B_0\hat{j}$. The trajectory of the particle will
   (a) be an ellipse
   (b) be a cycloid
   (c) be a helix with constant pitch
   (d) not be confined to any plane

5. An object of mass *m* rests on a surface with coefficient of static friction μ. Which of the following is NOT correct?
   (a) The force of friction is exactly μmg
   (b) The maximum force of friction is μmg
   (c) The force of friction is along the surface
   (d) The force of friction opposes any effort to move the object

6. The Lagrangian of a particle of mass *m* moving in a plane is given by L = (1/2) $m[v_x^2 + v_y^2] + a(xv_y - yv_x)$, where v_x and v_y are velocity components and *a* is *a* constant. The canonical momenta of the particle are given by
   (a) $p_x = mv_x$ and $p_y = mv_y$
   (b) $p_x = mv_x + ay$ and $p_y = mv_y + ax$
   (c) $p_x = mv_x - ay$ and $p_y = mv_y + ax$
   (d) $p_x = mv_x - ay$ and $p_y = mv_y - ax$

7. Two events are separated by a distance of 6×10^5 km and the first event occurs 1sec before the second event. The interval between the two events
   (a) is time-like
   (b) is light-like (null)
   (c) is space-like
   (d) cannot be determined from the information given

8. An electric charge, +Q is placed on the surface of a solid, conducting sphere of radius *a*. The distance measured from the centre of the sphere is denoted as *r*. Then
   (a) the charge gets distributed uniformly through the volume of the sphere
   (b) the electrostatic potential has the same value for $r < a$
   (c) an equal and opposite charge gets induced in the bottom half of the sphere
   (d) the electric field is given by $1/(4\pi\varepsilon_0 r^2)$ for $r < a$

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9. An electric field applied along the length of a long cylinder produces a polarization P. The depolarization field produced in this configuration is

(a) $4\pi P/3$ (b) $-4\pi P/3$ (c) $2\pi P$ (d) 0

10. Which one of the following Maxwell's equations implies the absence of magnetic monopoles?

(a) $\nabla \cdot E = \pi/\varepsilon_0$ (b) $\nabla \cdot B = 0$

(c) $\nabla \times E = -\partial B/\partial t$ (d) $\nabla \times B = (1/c^2)\partial B/\partial t + \mu_0 J$

11. An electromagnetic wave is propagating in free space in the *z*-direction. If the electric field is given by $E = \cos(\omega t - kz)i,$ where $\omega t = ck,$ then the magnetic field is given by

(a) $B = (1/c)\cos(\omega t - kz)\, j$ (b) $B = (1/c)\sin(\omega t - kz)\, j$

(c) $B = (1/c)\cos(\omega t - kz)i$ (d) $B = (1/c)\cos(\omega t - kz)\, j$

12. Given a wave with the dispersion relation $\omega = ck + m$ for $k > 0$ and $m > 0,$ which one of the following is true?

(a) The group velocity is greater than the phase velocity
(b) The group velocity is less than the phase velocity
(c) The group velocity and the phase velocity are equal
(d) There is no definite relation between the group velocity and the phase velocity

13. Which of the following is a valid normalized wave function for a particle in a one dimensional infinite potential well of width L centered at $x = 0$?

(a) $(2/L)[\cos(2\pi x/L) + \sin(2\pi x/L)]$ (b) $(2/L)^{1/2}\sin[n\pi x/L]$ for odd n

(c) $(2/L)^{1/2}\cos[n\pi x/L]$ for odd n (d) $(2/L)\cos(\pi x/L)$

14. The commutator $[x, P^2],$ where *x* and *P* are position and momentum operators respectively, is

(a) $2i\hbar P$ (b) $-i\hbar P$ (c) $2i\hbar xP$ (d) $-2i\hbar xP$

15. A spin half particle is in the state $S_z = \hbar/2$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by

(a) $0, 0, \hbar^2/4, \hbar^2/4$ (b) $0, \hbar^2/4, \hbar^2/4, 0$

(c) $0, \hbar^2/4, 0, \hbar^2/4$ (d) $\hbar^2/4, \hbar^2/4, 0, 0$

16. The spectral term for the atom with 70% filled subshell and only S = 3/2 is

(a) $^3P_0$ (b) $^4F_{9/2}$ (c) $^3F_{1/2}$ (d) $^4P_{1/2}$

17. The hyperfine splitting of the spectral lines of an atom is due to

(a) the coupling between the spins of two or more electrons
(b) the coupling between the spins and the orbital angular momenta of the electrons
(c) the coupling between the electron spins and the nuclear spin
(d) the effect of external electromagnetic fields

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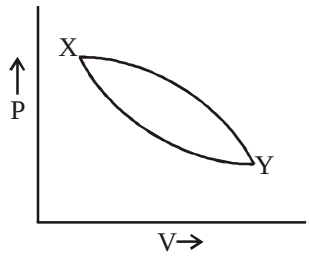
18. A piston containing an ideal gas is originally in the state X (see figure). The gas is taken through a thermal cycle $X \rightarrow Y \rightarrow X$ as shown. The work done by the gas is positive if the direction of the thermal cycle is



(a) clockwise
(b) counter-clockwise
(c) neither clockwise nor counter-clockwise
(d) clockwise from $X \rightarrow Y$ and counter-clockwise from $Y \rightarrow X$

19. A second order phase transition is one in which
(a) the plot of entropy as a function of temperature shows a discontinuity
(b) the plot of specific heat as a function of temperature shows a discontinuity
(c) the plot of volume as a function of pressure shows a discontinuity
(d) the plot of comprehensibility as a function of temperature is continuous

20. Consider the Fermi-Dirac distribution function f(E) at room temperature (300 K) where E refers to energy. If E_F is the Fermi energy, which of the following is true?
(a) f(E) is a step function
(b) $f(E_F)$ has a value of 1/2
(c) States with $E < E_F$ are filled completely
(d) f(E) is large and tends to infinity as E decreases much below E_F

21. If the ionic radii of Mn and S are 0.80 and 0.184 nm respectively, the structure of MnS will be
(a) cubic closed packed
(b) body centered cubic
(c) NaCl type
(d) primitive cubic cell

22. A cubic cell consists of two atoms of masses m_1 and m_2 ($m_1 > m_2$) with m_1 and m_2 atoms situated on alternate planes. Assuming only nearest neighbor interactions, the centre of mass of the two atoms
(a) moves with the atoms in the optical mode and remains fixed in the acoustic mode
(b) remains fixed in the optical mode and moves with the atoms in the acoustic mode
(c) remains fixed in both optical and acoustic modes
(d) moves with the atoms in both optical and acoustic modes

23. In simple metals the phonon contribution to the electrical resistivity at temperature T is
(a) directly proportional to T above Debye temperature and to T^3 below it
(b) inversely proportional to T for all temperatures
(c) independent of T for all temperatures
(d) directly proportional to T above Debye temperature and to T^5 below it

24. The effective mass of an electron in a semiconductor can be
(a) negative near the bottom of the band
(b) a scalar quantity with a small magnitude
(c) zero at the center of the band
(d) negative near the top of the band

25. The dielectric constant of water is 80. However its refractive index is 1.75 invalidating the expression $n = \varepsilon^{1/2}$. This is because
(a) the water molecule has a permanent dipole moment
(b) the boiling point of water is 100°C
(c) the two quantities are measured in different experiments
(d) water is transparent to visible light

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26. The nucleus of the atom $^{9}Be_{4}$ consists of

(a) 13 up quarks and 13 down quarks (b) 13 up quarks and 14 down quarks

(c) 14 up quarks and 13 down quarks (d) 14 up quarks and 14 down quarks

27. Which one of the following nuclear reactions is possible?

(a) $^{14}N_7 \rightarrow {}^{13}C_6 + \beta^+ + \nu_c$ (b) $^{13}N_7 \rightarrow {}^{13}C_6 + \beta^+ + \nu_c$

(c) $^{13}N_7 \rightarrow {}^{13}C_6 + \beta^+$ (d) $^{13}N_7 \rightarrow {}^{13}C_7 + \beta^+ + \nu_c$

28. Suppose that a neutron at rest in free space decays into a proton and an electron. This process would violate

(a) conservation of charge (b) conservation of energy

(c) conservation of linear momentum (d) conservation of angular momentum

29. Which one of the following is TRUE for a semiconductor *pn* junction with no external bias?

(a) The total charge in the junction is not conserved (b) The *p* side of the junction is positively charged

(c) The *p* side of the junction is negative charged (d) No charge develops anywhere in the junction

30. Which one of the set of values given below does NOT satisfy the Boolean relation $R = PQ'$ (where Q' denotes NOT Q)?

(a) $P = 1, Q = 1, R = 0$ (b) $P = 1, Q = 1, R = 1$

(c) $P = 0, Q = 0, R = 0$ (d) $P = 0, Q = 1, R = 1$

Q.31 – Q.90 : Carry ONE mark each.

31. The curl of the vector $A = zi + xj + yk$ is given by

(a) $i + j + k$ (b) $i - j + k$ (c) $i + j - k$ (d) $-i - j - k$

32. Consider the differential equation $d^2x/dt^2 + 2dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by

(a) $1/e$ (b) $2/e$ (c) 1 (d) $3/e$

33. S_{ij} and A_{ij} represent a symmetric and an antisymmetric real-valued tensor respectively in three dimensions. The number of independent components of S_{ij} and A_{ij} are

(a) 3 and 6 respectively (b) 6 and 3 respectively

(c) 6 and 6 respectively (d) 9 and 6 respectively

34. Consider the four statements given below about the function $f(x) = x^4 - x^2$ in the range $-\infty < x < +\infty$. Which one of the following statements is correct?

P the plot of $f(x)$ versus x has two maxima and two minima

Q the plot of $f(x)$ versus x cuts the x axis at four points

R the plot of $f(x)$ versus x has three extrema

S no part of the plot $f(x)$ versus x lies in the fourth quadrant

Pick the right combination of correct choices from those given below

(a) P and R (b) R only (c) R and S (d) P and Q

35. The Fourier transform of the function $f(x)$ is $F(k) = \int e^{ikx} f(x)dx$. The Fourier transform of $df(x)/dx$ is

(a) $dF(k)/dk$ (b) $\int F(k)/dk$ (c) $-ikF(k)$ (d) $ikF(k)$

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36. A particle of mass *m* is moving in a potential of the form $V(x, y, z) = (1/2)m\omega^2 (3x^2 + 3y^2 + 2z^2 + 2xy)$. The oscillation frequencies of the three normal modes of the particle are given by

(a) $\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$ (b) $\sqrt{2}\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$

(c) $\sqrt{2}\omega, \sqrt{2}\omega$ and 2ω (d) $\sqrt{2}\omega, 2\omega$ and 2ω

37. The speed of a particle whose kinetic energy is equal to its rest mass energy is given by (*c* is the speed of light in vacuum)

(a) $c/3$ (b) $\sqrt{2}c/3$ (c) $c/2$ (d) $\frac{\sqrt{3}}{2}c$

38. Electromagnetic waves are propagating along a hollow, metallic waveguide whose cross-section is a square of side W. The minimum frequency of the electromagnetic waves is

(a) c/W (b) $2c/W$ (c) $\pi c/W$ (d) $\sqrt{2}\pi c/W$

39. Consider the given statements about $E(r, t)$ and $B(r,t)$, the electric and magnetic vectors respectively in a region of free space

P Both E and B are conservative vector fields

Q Both E and B are central force fields

R E and B are mutually perpendicular in the region

S Work done by B on a moving charge in the region is zero

Choose the right combination of correct statements from the following

(a) P and R (b) R and S (c) S only (d) P and Q

40. An infinite conducting sheet in the *x*-*y* plane carries a surface current density K along the *y*-axis. The magnetic field B for $z > 0$ is

(a) $B = 0$ (b) $B = \mu_0 K\hat{k}/2$ (c) $B = \mu_0 K\hat{i}/2$ (d) $B = \mu_0 K\hat{j}/(x^2 + z^2)^{0.5}$

41. A parallel beam of infrared radiation of wavelength of 1.01×10^{-6} m is incident normally on a screen with two slits 5×10^{-6} m apart and the resulting interference pattern is observed on a distant screen. What is the largest number of maxima that can be observed on the screen?

(a) 4 (b) 9 (c) 13 (d) infinitely many

42. A parallel beam of electrons of a given momentum pass through a screen S_1 containing a slit and then produces a diffraction pattern on a screen S_2 placed behind it. The width of the central maximum observed on the screen S_2 can be increased by

(a) decreasing the distance between the screens S_1 and S_2

(b) increasing the width of the slit in screen S_1

(c) decreasing the momentum of the electrons

(d) increasing the momentum of the electrons

43. An electron in a time independent potential is in a state which is the superposition of the ground state (E_0 = 11eV) and the first excited state (E_1 = 1eV). The wave function of the electron will repeat itself with a period of

(a) 3.1×10^{-18} s (b) 2.1×10^{-15} s

(c) 1.2×10^{-12} s (d) 1.0×10^{-9} s

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44. A particle has the wave function $\psi(x,t) = A\exp(i\omega t)\cos(kx)$. Which one of the following is correct?
(a) This is an eigenstate of both energy and momentum
(b) This is an eigenstate of momentum and not energy
(c) This is an eigenstate of energy and not momentum
(d) This is not an eigenstate of energy of momentum

45. A free particle with energy E whose wave-function is a plane wave with wavelength λ enters a region of constant potential $V > 0$ where the wavelength of the particle is 2λ. The ratio (V/E) is
(a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5

46. The vibrational spectrum of a molecule exhibits a strong line with P and R branches at a frequency ν_1 and a weaker line at a frequency ν_2. The frequency ν_3 is not shown up. Its vibrational Raman spectrum shows a strongly polarized line at frequency ν_3 and no feature at ν_1 and ν_2.
(a) the molecule could be linear
(b) the molecule lacks a center of inversion
(c) ν_1 arises from a symmetric stretching mode
(d) ν_3 arises from a bending mode

47. Three values of rotational energies of molecules are given below in different units
P $10\ cm^{-1}$
Q 10^{-23} J
R 10^4 MHz
Choose the correct arrangement in the increasing order of energy
(a) P, Q, R (b) R, Q, P (c) R, P, Q (d) Q, R, P

48. The short wavelength cut off of the continuous X-ray spectrum from a nickel target is 0.0825 nm. The voltage required to be applied to an X-ray tube is
(a) 0.15 KV (b) 1.5 KV (c) 15 KV (d) 150 KV

49. The spin-orbit coupling constant for the upper state of sodium atom which emits D lines of wave numbers 16956.2 and 16973.4 cm^{-1} is
(a) $15\ cm^{-1}$ (b) $11.4\ cm^{-1}$ (c) $12.5\ cm^{-1}$ (d) $15.1\ cm^{-1}$

50. Consider the following statements about molecular spectra
P CH_4 does not give pure rotational Raman lines
Q SF_6 could be studied by rotational Raman spectroscopy
R N_2 shows infrared absorption spectrum
S CH_3CH_3 shows vibrational Raman and infrared absorption lines
T H_2O_2 shows pure rotational spectrum
Choose the right combination of correct statements
(a) P and Q (b) P, R and T (c) P, S and T (d) Q and R

51. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?
(a) its energy and the number of photons both increase 8 times
(b) its energy increases 8 times and the number of photons increases 16 times
(c) its energy increases 16 times and the number of photons increases 8 times
(d) its energy and the number of photons both increase 16 times

CAREER ENDEAVOUR

52. A sample of ideal gas with initial pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constant is R. Then the work done by the gas is given by

(a) $(PV\Delta S)/(nR)$ (b) $nR\Delta S$ (c) PV (d) $(P\Delta S)/(nRV)$

53. Hydrogen molecules (mass *m*) are in thermal equilibrium at a temperature T. Assuming classical distribution of velocity, the most probable speed at room temperature is

(a) $(k_BT)/m$ (b) $2k_BT/m$ (c) $(\sqrt{2k_BT/m})$ (d) $m/(\sqrt{2}k_BT)$

54. Consider the energy E in the first Brillouin zone as a function of the magnitude of the wave vector *k* for a crystal of lattice constant *a*. Then

(a) the slope of E versus *k* is proportional to the group velocity

(b) the slope of E versus *k* has its maximum value at $|k| = \pi/a$

(c) the plot of E versus *k* will be parabolic in the interval $(-\pi/a) < k < (\pi/a)$

(d) the slope of E versus *k* is non-zero for all *k* the interval $(-\pi/a) < k < (\pi/a)$

55. An external magnetic field of magnitude H is applied to a Type-I superconductor at a temperature below the transition point. Then which one of the following statements is NOT true for H less than the critical field H_C?

(a) the sample is diamagnetic

(b) it magnetization varies linearly with H

(c) the lines of magnetic induction are pushed out from the sample

(d) the sample exhibits mixed states of magnetization near H_C

56. A ferromagnetic material has a Curie temperature 100K. Then

(a) its susceptibility is doubled when it is cooled from 300K to 200K

(b) all the atomic magnets in it get oriented in the same direction above 100K

(c) the plot of inverse susceptibility versus temperature is linear with a slope T_C

(d) the plot of its susceptibility versus temperature is linear with an intercept T_C

57. The point group symmetrics of the three molecules shown in Figs. 1–3 are respectively

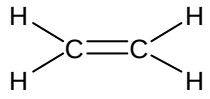

Fig. 1

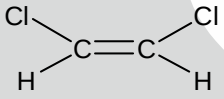

Fig. 2

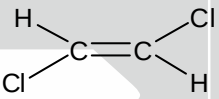

Fig. 3

[notation : C_{2v} = 2mm; C_{2h} = 2/m; D_{2h} = mmm]

(a) C_{2h}, C_{2v}, C_{2h} (b) C_{2v}, C_{2h}, C_{2h}

(c) D_{2h}, C_{2v}, C_{2h} (d) C_{2v}, D_{2h}, C_{2h}

58. The energy density of states of an electron in a one dimensional potential well of infinitely high walls is (the symbols have their usual meaning)

(a) $L\sqrt{m}/[\pi\hbar\sqrt{(2E)}]$ (b) $Lm/(\pi\hbar\sqrt{E})$

(c) $Lm/[\pi\hbar\sqrt{(2E)}]$ (d) $L\sqrt{m}/(2\pi\hbar E)$

59. Which one of the following statements concerning the Compton effect is NOT correct?

(a) The wavelength of the scattered photon is greater than or equal to the wavelength of the incident photon

(b) The electron can acquire a kinetic energy equal to the energy of the incident photon

(c) The energy of the incident photon equals to the kinetic energy of the electron plus the energy of the scattered photon

(d) The kinetic energy acquired by the electron in the largest when the incident and scattered photons move in opposite directions

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60. If the photon were to have a finite mass, then the Coulomb potential between two stationary charges separated by a distance r would

(a) be strictly zero beyond some distance
(b) fall off exponentially for large values of r
(c) fall off as $1/r^3$ for large values of r
(d) fall off as $1/r$ for large values of r

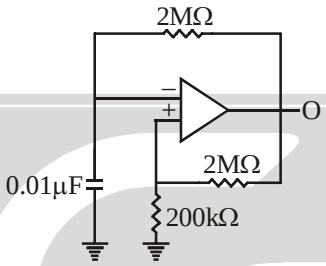
61. A stationary particle in free space is observed to spontaneously decay into two photons. This implies that

(a) the particle carries electric charge
(b) the spin of the particle must be greater than or equal to 2
(c) the particle is a boson
(d) the mass of the particle must be greater than or equal to the mass of the hydrogen atom

62. The masses of a hydrogen atom, neutron and $^{238}U_{92}$ are given by 1.0078, 1.0087 and 238.0508 respectively. The binding energy of $^{238}U_{92}$ is therefore approximately equal to (taking 1 a.m.u. = 931.64 MeV)

(a) 120 MeV (b) 1500 MeV (c) 1600 MeV (d) 1800 MeV

63. A bistable multivibrator with a saturation voltage $\pm 5V$ is shown in the diagram. The positive and negative threshold at the inverting terminal for which the multivibrator will switch to the other state are



(a) $\pm 5/11V$ (b) $\pm 10/11V$ (c) $\pm 5V$ (d) $\pm 11V$

64. An avalanche effect is observed in a diode when

(a) the forward voltage is less than the breakdown voltage
(b) the forward voltage exceeds the breakdown voltage
(c) the reverse voltage exceeds the breakdown voltage
(d) the diode is heavily doped and forward biased

65. Which of the given relations between the Boolean variables P and Q is NOT correct? (In the notation used here, P' denotes NOT P and Q' denotes NOT Q)

(a) $PQ' + PQ = P$
(b) $(PQ)' + P' + Q'$
(c) $PQ' + (P' + Q)'$
(d) $PQ' + Q = P$

Data for Q. No. 66 to 67

Consider the vector $V = r/r^3$

66. The surface integral of this vector over the surface of a cube of size a and centered at the origin

(a) 0 (b) 2π (c) $2\pi a^3$ (d) 4π

67. Which one of the following is NOT correct?

(a) Value of the line integral of this vector around any closed curve is zero
(b) This vector can be written as the gradient of some scalar function
(c) The line integral of this vector from point P to point Q is independent of the path taken
(d) This vector can represent the magnetic field of some current distribution

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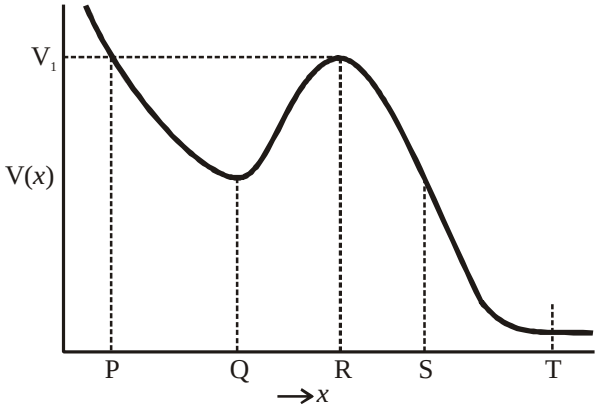
Data for Q. No. 68 to 69

Consider the motion of a particle in the potential $V(x)$ shown in the figure.



68. Suppose the particle has a total energy E = V_1 in the figure. Then the speed of the particle is zero when it is at

(a) point P (b) point Q (c) point S (d) point T

69. Which one of the following statements is NOT correct about the particle?

(a) It experience no force when its position corresponds to the point Q on the curve

(b) It experience no force when its position corresponds to the point R on the curve

(c) Its speed is the largest when it is at S

(d) It will be in a closed orbit between P and R if E < V_1

Data for Q. No. 70 to 71

A particle of mass *m* moving with speed v collides with a stationary particle of equal mass. After the collision, both the particles move. Let θ be the angle between the two velocity vectors

70. If the collision is elastic, then

(a) θ is always less than 90º (b) θ is always equal to 90º

(c) θ is always greater than 90º (d) θ cannot be deduced from the given data

71. If the collision is inelastic, then

(a) θ is always less than 90º

(b) θ is always equal to 90º

(c) θ is always greater than 90º

(d) θ could assume any value in the range 0º to 180º

Data for Q. No. 72 to 73

Consider two conducting plates of infinite extent, one plate at $z = 0$ and the other at $z = L$, both parallel to the *xy* plane. The vector and scalar potential in the region between the plates is given by

$$A(r,t) = A_0\hat{i}\cos(kz+\alpha)\cos(kct);$$

$$\phi(r,t) = 0$$

72. For this to represent a standing wave in the empty region between the plates

(a) $k = \pi / L$ and $\alpha = 0$ (b) $k = 2\pi / L$ and $\alpha = \pi / 2$

(c) $k = \pi / (2L)$ and $\alpha = \pi / 2$ (d) $k = \pi / 2L$ and $\alpha = 0$

CAREER ENDEAVOUR

73. The energy density at $z = 0$ and $t = 0$ is

(a) 0 (b) $\varepsilon_0 c^2 k^2 A_0^2$

(c) $(1/2)\mu_0 A_0^2 k^2$ (d) $(1/2)\mu_0 A_0^2 k^2 + (1/2)\varepsilon_0 c^2 k^2 A_0^2$

Data for Q. No. 74 to 75

A particle is located in a three dimensional cubic well of width L with impenetrable walls.

74. The sum of the energies of the third and the fourth levels is

(a) $10\pi^2\hbar^2 / mL^2$ (b) $10\pi^2\hbar^2 / 3mL^2$

(c) $11\pi^2\hbar^2 / 2mL^2$ (d) $15\pi^2\hbar^2 / 2mL^2$

75. The degeneracy of the fourth level is given by

(a) 1 (b) 2 (c) 3 (d) 4

Data for Q. No. 76 to 77

The normalized wave functions ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$.

76. The expectation value of A for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is

(a) –0.32 (b) 0.0 (c) 0.75 (d) 0.96

77. Which of the following are eigenfunctions of $\hat{A}^2$?

(a) ψ_1 and ψ_2 (b) ψ_2 and not ψ_1

(c) ψ_1 and not ψ_2 (d) neither ψ_1 nor ψ_2

Data for Q. No. 78 to 79

In the presence of an inhomogeneous weak magnetic field, spectral lines due to transitions between two sets of states were observed.

(1) $^5l_5 \rightarrow {}^5H_4$ and (2) $^2D_{5/2} \rightarrow {}^2P_{3/2}$

78. The types of Zeeman effect observed in (1) and (2) respectively are

(a) normal, normal (b) anomalous, anomalous

(c) anomalous, normal (d) normal, anomalous

79. The number of levels into which each of the above four terms split into respectively is

(a) 6, 4, 10, 8 (b) 4, 6, 10, 12 (c) 11, 9, 6, 4 (d) 9, 5, 12, 10

Data for Q. No. 80 to 82

A system consists of three spin-half particles, the *z* components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value +1/2 and –1/2. The total spin of the system is $S_z = S_z(1) + S_z(2) + S_z(3)$.

80. The total number of possible micro-states of this system is

(a) 3 (b) 6 (c) 7 (d) 8

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81. The total number of micro-states with $S_z = 1/2$ is

(a) 3 (b) 5 (c) 6 (d) 7

82. Consider an ensemble of systems where each microstate has equal probability. The ensemble average of S_z is

(a) –1/2 (b) 0 (c) 1/2 (d) 3/2

Data for Q. No. 83 to 84

A gas of N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N \ln\{(V - bN)(k_B T)^{3/2}\}$ where b is a constant with appropriate dimensions

83. If P is the pressure of the gas, the equation of state is given by

(a) $P(V - bN) = Nk_B T$ (b) $P(V - bN) = k_B T$

(c) $P(V - b) = Nk_B T$ (d) $P(V - b) = Nk_B T$

84. The interval energy of the gas is given by

(a) $U = (1/2)k_B T$ (b) $U = Nk_B T$

(c) $U = (3/2)Nk_B T$ (d) $U = 2Nk_B T$

Data for Q. No. 85 to 86

A crystal belongs to a face centered cubic lattice with four atoms in the unit cell. The size of the crystal is 1 cm and its unit cell dimension is 1 nm. f is the scattering factor of the atom.

85. The number of atoms in the crystal is

(a) 2×10^{21} (b) 4×10^{21}

(c) 2×10^{23} (d) 4×10^{24}

86. The structure factors for (0 1 0) and (2 0 0) reflections respectively are

(a) $2f$ and zero (b) zero and $4f$

(c) $2f$ and $2f$ (d) zero and zero

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Data for Q. No. 87 to 88

An atomic bomb consisting of ^{235}U explodes and releases an energy of 10^{14} J. It is known that each ^{235}U which undergoes fission releases 3 neutrons and about 200 MeV of energy. Further, only 20% of the ^{235}U atoms in the bomb undergo fission.

87. The total number of neutrons released is about

(a) 4.7×10^{24} (b) 9.7×10^{24}

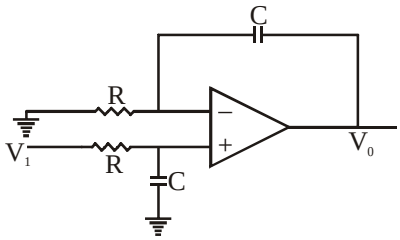
(c) 1.9×10^{25} (d) 3.7×10^{25}

88. The mass of ^{235}U in the bomb is about

(a) 1.5 kg (b) 3.0 kg (c) 6.1 kg (d) 12 kg

Data for Q. No. 89 to 90

The circuit below represents a non-inverting integrator



89. For high frequencies $(\omega \to \infty)$ the input impedance is

(a) 0 (b) R (c) $R/(1+\omega RC)$ (d) ∞

90. For low frequencies $(\omega \to \infty)$ the input impedance is

(a) $V_0 = (1/RC)\int V_1 dt$

(b) The voltages at the inverting and non-inverting terminals of the op-amp are nearly

(c) The voltage at the non-inverting terminal of the op-amp and the current in the resistor attached to it are $\pi/2$ out of phase

(d) The current in the two resistors are in phase

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.30 : Carry ONE mark each.

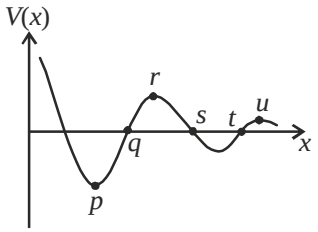
1. For the function $\phi = x^2 y + xy$, the value of $\left|\vec{\nabla}\phi\right|$ at $x = y = 1$ is

(a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

2. The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$

3. Identify the points of unstable equilibrium for the potential shown in the figure.



(a) p and s (b) q and t (c) r and u (d) r and s

4. Which one of the following remains invariant under Lorentz transformations ?

(a) $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} - \frac{1}{c^2}\frac{\partial}{\partial t}$ (b) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

(c) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ (d) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

5. A charge $+q$ is kept at a distance of $2R$ from the centre of a grounded conducting sphere of radius R. The image charge and its distance from the centre are, respectively

(a) $-\frac{q}{2}$ and $\frac{R}{2}$ (b) $-\frac{q}{2}$ and $\frac{R}{4}$ (c) $-q$ and $\frac{R}{2}$ (d) $+\frac{q}{2}$ and $\frac{R}{2}$

6. The state of polarization of light with the electric field vector $\vec{E} = \hat{i}\, E_0 \cos(kz - \omega t) - \hat{j}\, E_0 \cos(kz - \omega t)$ is

(a) linearly polarized along z-direction (b) linearly polarized at –45° to x-axis
(c) circularly polarized (d) elliptically polarized with the major axis along x-axis

7. The resonance widths Γ of ρ, ω and φ particle resonances satisfy the relation $\Gamma_p > \Gamma_\omega > \Gamma_\varphi$. Their life-times τ satisfy the relation

(a) $\tau_p > \tau_\omega > \tau_\varphi$ (b) $\tau_p < \tau_\omega < \tau_\varphi$ (c) $\tau_p < \tau_\omega < \tau_\varphi$ (d) $\tau_p > \tau_\omega < \tau_\varphi$

8. The time-independent Schrödinger equation of a system represents the conservation of the

(a) total binding energy of the system (b) total potential energy of the system
(c) total kinetic energy of the system (d) total energy of the system

9. In a hydrogen atom, the accidental or Coulomb degeneracy for the $n = 4$ state is

(a) 4 (b) 16 (c) 18 (d) 32

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10. The Hamiltonian of a particle is given by $H = \frac{p^2}{2m} + V\left(|\vec{r}|\right) + \phi\left(+|\vec{r}|\right)\vec{L}\cdot\vec{S}$, where $\vec{S}$ is angular momentum. The Hamiltonian does NOT commute with
(a) $\vec{L}+\vec{S}$ (b) $\vec{S}^2$ (c) L_z (d) $\vec{L}^2$

11. The spectral terms for a certain electronic configuration are given by ^3D, ^1D, ^3P, ^1P, ^5S, ^3S. The term with the lowest energy is
(a) ^5S (b) ^3P (c) ^3D (d) ^3S

12. The degeneracy of the spectral term ^3F is
(a) 7 (b) 9 (c) 15 (d) 21

13. The Lande *g* factor for the level $^3D_3$ is
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

14. All vibrations producing a change in the electric dipole moment of a molecule yield
(a) Raman spectra (b) Infrared spectra (c) Ultra-violet spectra (d) X-ray spectra

15. For any process, the second law of thermodynamics requires that the change of entropy of the universe be
(a) positive only (b) positive or zero (c) zero only (d) negative or zero

16. The dimension of phase space of ten rigid diatomic molecules is
(a) 5 (b) 10 (c) 50 (d) 100

17. The specific heat of an ideal Fermi gas in 3-dimension at very low temperatures (T) varies as
(a) T (b) $T^{3/2}$ (c) T^2 (d) T^3

18. Which one of the following is a first order phase transition ?
(a) Vaporization of a liquid at its boiling point (b) Ferromagnetic to paramagnetic
(c) Normal liquid He to superfluid He (d) Superconducting to normal state

19. The *c*/*a* ratio for an ideal hexagonal closed packed structure is
(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{8}$ (c) $\sqrt{5}$ (d) $\sqrt{\frac{8}{3}}$

20. The number of independent elastic constants in an isotropic cubic solid is
(a) 1 (b) 2 (c) 3 (d) 4

21. The effective mass of an electron in a semiconductor
(a) can never be positive (b) can never be negative
(c) can be positive or negative (d) depends on its spin

22. The critical magnetic field for a solid in superconducting state
(a) does not depend upon temperature (b) increases if the temperature increases
(c) increases if the temperature decreases (d) does not depend on the transition temperature

23. The volume of a nucleus in an atom is proportional to the
(a) mass number (b) proton number (c) neutron number (d) electron number

24. As one moves along the of stability from ^{56}Fe to ^{235}U nucleus, the nuclear binding energy per particle decreases from about 8.8 MeV to 7.6 MeV. This trend is mainly due to the
(a) short range nature of the nuclear forces (b) long range nature of the Coulomb forces
(c) tensor nature of the nuclear forces (d) spin dependence of the nuclear forces

CAREER ENDEAVOUR

25. A thermal neutron having speed v impinges on a ^{235}U nucleus. The reaction cross-section is proportional to

(a) v^{-1} (b) v (c) $v^{1/2}$ (d) $v^{-1/2}$

26. Choose the particle with zero Baryon number from the list given below.

(a) pion (b) neutron (c) protron (d) Δ^+

27. A bipolar junction transistor with one junction forward biased and either the collector or emitter open, operates in the

(a) cut-off region (b) saturation region (c) pinch-off region (d) active region

28. A field effect transistor is a

(a) unipolar device
(b) special type of biopolar junction transistor
(c) unijunction device
(d) device with low input impedance

29. The inverting input terminal of an operational amplifier (op-amp) is shorted with the output terminal apart from being grounded. A voltage signal v_i is applied to the non-inverting input terminal of the op-amp. Under this configuration, the op-amp functions as

(a) an open loop inverter
(b) a voltage to current converter
(c) a voltage follower
(d) an oscillator

30. A half-adder is a digital circuit with

(a) three inputs and one output
(b) three inputs and two outputs
(c) two inputs and one output
(d) two inputs and two outputs

Q.31 – Q.90 : Carry TWO marks each.

31. A real traceless 4 × 4 unitary matrix has two eigenvalues –1 and +1. The other eigenvalues are

(a) zero and +2 (b) zero and +1 (c) zero and +2 (d) –1 and +1

32. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are

(a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) –1 and +1

33. The inverse of the complex number $\frac{3+4i}{3-4i}$ is

(a) $\frac{2}{25}+i\frac{24}{25}$ (b) $-\frac{7}{25}+i\frac{24}{25}$ (c) $\frac{7}{25}-i\frac{24}{25}$ (d) $-\frac{7}{25}-i\frac{25}{25}$

34. The value of $\int_C \frac{dz}{(z^2+a^2)}$, where C is a unit circle (anti-clockwise) centered at the origin in the complex z-plane is

(a) π for $a=2$ (b) zero for $a=\frac{1}{2}$ (c) 4π for $a=2$ (d) $\frac{\pi}{2}$ for $a=\frac{1}{2}$

35. The Laplace transform $f(t)=\sin \pi t$ is $F(s)=\frac{\pi}{(s^2+\pi^2)}, s>0$. Therefore, the Laplace transform of $t \sin \pi t$ is

(a) $\frac{\pi}{s^2(s^2+\pi^2)}$ (b) $\frac{2\pi}{s^2(s^2+\pi^2)^2}$ (c) $\frac{2\pi s}{(s^2+\pi^2)^2}$ (d) $\frac{2\pi}{(s^2+\pi^2)^2}$

CAREER ENDEAVOUR

36. A periodic function $f(x) = x$ for $-\pi < x < +\pi$ has the Fourier series representation

$f(x) = \sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin nx$. Using this, one finds the sum $\sum_{n=1}^{\infty} n^{-2}$ to be

(a) $2 \ln 2$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) $\pi \ln 2$

37. The Fourier transform $F(k)$ of a function $f(x)$ is defined as $F(k) = \int_{-\infty}^{\infty} dx\, f(x)\exp(ikx)$. The $F(k)$ for

$f(x) = \exp(-x^2)$ is [Given: $\int_{-\infty}^{\infty} \exp(-x^2)\,dx = \sqrt{x}$]

(a) $\pi \exp(-k)$ (b) $\sqrt{\pi}\exp\left(\frac{-k^2}{4}\right)$ (c) $\frac{\sqrt{\pi}}{2}\exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi}\exp(-k^2)$

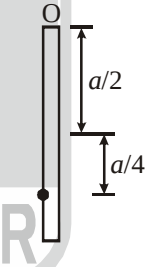
38. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$$

The generalized momenta corresponding to r and θ are given by

(a) $m\dot{r}$ and $mr^2\,\dot{\theta}$ (b) $m\dot{r}$ and $mr\,\dot{\theta}$ (c) $m\dot{r}^2$ and $mr^2\,\dot{\theta}$ (d) $m\dot{r}^2$ and $mr^2\,\dot{\theta}^2$

39. A particle of mass m is attached to a thin uniform rod of length a and mass 4m. The distance of the particle from the center of mass of the rod is $a/4$. The moment of inertia of the combination about an axis passing through O normal to the rod is



(a) $\frac{64}{48}ma^2$ (b) $\frac{91}{48}ma^2$

(c) $\frac{27}{48}ma^2$ (d) $\frac{51}{48}ma^2$

CAREER ENDEAVOUR

40. A rigid frictionless rod rotates anticlockwise in a vertical plane with angular velocity $\vec{\omega}$. A bead of mass m moves outward along the rod with constant velocity $\vec{u}_0$. The bead will experience a coriolis force

(a) $2mu_0\omega\hat{\theta}$ (b) $-2mu_0\omega\hat{\theta}$ (c) $4mu_0\omega\hat{\theta}$ (d) $-mu_0\omega\hat{\theta}$

41. The Hamiltonian corresponding to the Langrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is

(a) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + kxy$ (b) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$ (c) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$ (d) $\frac{p_x^2 + p_y^2}{4ab} + kxy$

42. The value of the Poisson bracket $\left[\vec{a}\cdot\vec{r},\, \vec{b}\cdot\vec{p}\right]$, where $\vec{a}$ and $\vec{b}$ are constant vectors, is

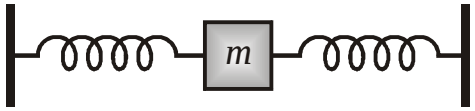
(a) $\vec{a}\,\vec{b}$ (b) $\vec{a} - \vec{b}$ (c) $\vec{a} + \vec{b}$ (d) $\vec{a}\cdot\vec{b}$

43. A mass *m* is connected on either side with a spring each of spring constant k_1 and k_2. The free ends of springs are tied to rigid supports. The displacement of the mass is *x* from equilibrium position. Which one of the following is TRUE ?

(a) The force acting on the mass is $-(k_1 k_2)^{1/2} x$

(b) The angular momentum of the mass is zero about the equilibrium point and its Lagrangian is $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}(k_1 + k_2)x^2$

(c) The total energy of the system is $\frac{1}{2}m\dot{x}^2$



(d) The angular momentum of the mass is $mx\dot{x}$ and the Lagrangian of the system is $\frac{m}{2}\dot{x}^2 + \frac{1}{2}(k_1 + k_2)x^2$

44. An electron gains energy so that its mass becomes $2m_0$. Its speed is

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{3}{4}c$ (c) $\frac{3}{2}c$ (d) $\sqrt{\frac{3}{2}}c$

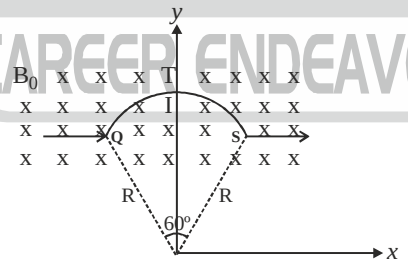
45. A conducting sphere of radius *R* has charge $+Q$ on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will

(a) increase four times (b) increase eight times (c) remain the same (d) decrease four times

46. A conducting sphere of radius *R* is placed in a uniform electric field $\vec{E}_0$ directed along $+z$ axis. The electric potential for outside points is given as $V_{out} = -E_0\left(1 - \frac{R^3}{r^3}\right) r\cos\theta$, where *r* is the distance from the centre and θ is the polar angle. The charge density on the surface of the sphere is

(a) $3\varepsilon_0 E_0 \cos\theta$ (b) $\varepsilon_0 E_0 \cos\theta$ (c) $3\varepsilon_0 E_0 \cos\theta$ (d) $\frac{\varepsilon_0}{3} E_0 \cos\theta$

47. A circular arc QTS is kept in an external magnetic field $\vec{B}_0$ as shown in figure. The arc carries a current *I*. The magnetic field is directed normal and into the page. The force acting on the arc is



(a) $2I B_0 R\hat{j}$ (b) $I B_0 R\hat{j}$ (c) $-2I B_0 R\hat{j}$ (d) $-I B_0 R\hat{j}$

48. A plane electromagnetic wave of frequency ω is incident on an air-dielectric interface. The dielectric is linear, isotropic, non-magnetic and its refractive index is *n*. The reflectance (*R*) and transmittance (*T*) from the interface are

(a) $R = \left(\frac{n-1}{n+1}\right)^2,\ T = \frac{4n}{(n+1)^2}$ (b) $R = -\left(\frac{n-1}{n+1}\right),\ T = \frac{2}{(n+1)^2}$

(c) $R = -\left(\frac{n-1}{n+1}\right)^3,\ T = \frac{4n^3}{(n+1)^3}$ (d) $R = -\left(\frac{n-1^2}{n+1}\right),\ T = \frac{4n^2}{(n+1)^2}$

49. The electric field of a plane e.m. wave is $\vec{E} = \vec{E}_0 \exp[i(xk\cos\alpha + yk\sin\alpha - \omega t)]$. If $\hat{x}$, $\hat{y}$ and $\hat{z}$ are cartesian unit vectors, the wave vector $\hat{k}$ of the e.m. wave is

(a) $\hat{z}k$ (b) $\hat{x}k\sin\alpha + \hat{y}k\cos\alpha$

(c) $\hat{x}k\cos\alpha + \hat{y}k\sin\alpha$ (d) $-\hat{z}k$

50. The dispersion relation for a low density plasma is $\omega^2 = \omega_0^2 + c^2k^2$, where ω_0 is the plasma frequency and c is the speed of light in free space. The relationship between the group velocity (v_g) and phase velocity (v_p) is

(a) $v_p = v_g$ (b) $v_p = v_g^{1/2}$ (c) $v_p\, v_g = c^2$ (d) $v_g = v_p^{1/2}$

51. A Michelson interferometer is illuminated with monochromatic light. When one of the mirrors is moved through a distance of 25.3 μm, 92 fringes pass through the cross-wire. The wavelength of the monochromatic light is

(a) 500 nm (b) 550 nm (c) 600 nm (d) 650 nm

52. A beam of mono-energetic particles having speed v is described by the wave function $\psi(x) = u(x)\exp(ikz)$, where $u(x)$ is a real function. This corresponds to a current density

(a) $u^2(x)v$ (b) v (c) zero (d) $u^2(x)$

53. The wave function of a spin-less particle of mass m in a one-dimensional potential $V(x)$ is $\psi(x) = A\exp(-\alpha^2x^2)$ corresponding to an eigenvalue $E_0 = \hbar^2\,\alpha^2/m$. The potential $V(x)$ is

(a) $2E_0(1-\alpha^2x^2)$ (b) $2E_0(1+\alpha^2x^2)$ (c) $2E_0\alpha^2x^2$ (d) $2E_0(1+2\alpha^2x^2$

54. Two spin $\vec{S}_1$ and $\vec{S}_2$ interact via a potential $V(r) = \vec{S}_1\cdot\vec{S}_2\,V_0(r)$. The contribution of this potential in the singlet and triplet states, respectively, are

(a) $-\frac{3}{2}V_0(r)$ and $\frac{1}{2}V_0(r)$ (b) $\frac{1}{2}V_0(r)$ and $-\frac{3}{2}V_0(r)$

(c) $\frac{1}{4}V_0(r)$ and $-\frac{3}{4}V_0(r)$ (d) $-\frac{3}{4}V_0(r)$ and $\frac{1}{4}V_0(r)$

CAREER ENDEAVOUR

55. The wave function of a one-dimensonal harmonic oscillator is $\psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$ for the ground state $E_0 = \frac{\hbar\omega}{2}$, where $\alpha^2 = \frac{mw}{\hbar}$. In the presence of a perturbing potential of $E_0\left(\frac{\alpha x}{10}\right)^4$, the first order change in the ground state energy is

$$\left[\text{Given}: \Gamma(x+1) = \int_0^\infty t^x\exp(-t)\,dt\right]$$

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)10^{-4}$ (c) $\left(\frac{3}{4}E_0\right)10^{-4}$ (d) $(E_0)10^{-4}$

56. The L, S and J quantum numbers corresponding to the ground state electronic configuration of Boron ($Z = 5$) are

(a) $L = 1,\ S = \frac{1}{2},\ J = \frac{3}{2}$ (b) $L = 1,\ S = \frac{1}{2},\ J = \frac{1}{2}$

(c) $L = 1,\ S = \frac{3}{2},\ J = \frac{1}{2}$ (d) $L = 1,\ S = \frac{3}{2},\ J = \frac{3}{2}$

57. The degeneracies of the J-states arising from the ^3P term with spin-orbit interaction are
(a) 1, 3, 5 (b) 1, 2, 3 (c) 3, 5, 7 (d) 2, 6, 10

58. Assuming that the L-S coupling scheme is valid, the number of permitted transitions from $^2P_{3/2}$ to $^2S_{1/2}$ due to a weak magnetic field is
(a) 2 (b) 4 (c) 6 (d) 10

59. Consider the pure rotational spectrum of a diatomic rigid rotor. The separation between two consecutive lines $(\Delta\bar{\nu})$ in the spectrum
(a) is directly proportional to the moment of inertial of the rotor
(b) in inversely proportional to the moment of inertia of the rotor
(c) depends on the angular momentum
(d) is directly proportional to the square of the interatomic separation

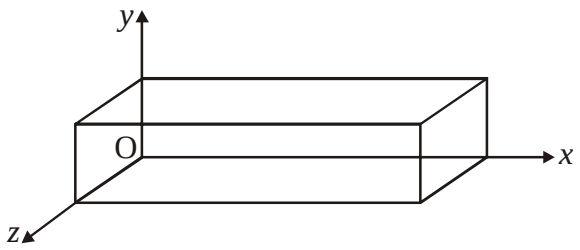
60. Light of wavelength 1.5 μm incident on a material with a characteristic Raman frequency of 20×10^{12} Hz results in a Stokes-shifted line of wavelength
[Given : $c = 3 \times 10^8$ m.s^{-1}]
(a) 1.47 μm (b) 1.57 μm (c) 1.67 μm (d) 1.77 μm

61. Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 cm^3 to 640 cm^3, the temperature of the cavity changes to
(a) 800 K (b) 700 K (c) 600 K (d) 500 K

62. The equation of state of a dilute gas at very high temperature is described by $\frac{pv}{k_B T} \approx 1 + \frac{B(T)}{v}$, where v is the volume per particle and $B(T)$ is a negative quantity. One can conclude that this is a property of
(a) a van der Waals gas (b) an ideal Fermi gas
(c) an ideal Bose gas (d) an ideal inert gas

63. In the region of co-existence of a liquid and vapor phases of a material
(a) C_p and C_v are both infinite (b) C_v and $\beta \left[= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \right]$ are both finite
(c) C_v and $K \left[= -\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_T \right]$ are both finite (d) C_p, β and K are all infinite

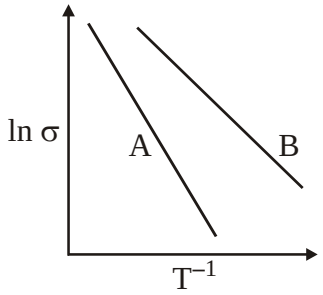
64. A doped Germanium crystal of length 2 cm, breadth 1 cm and width 1 cm, carries a current of 1 mA along its length parallel to +x-axis. A magnetic field of 0.5 T is applied along +z-axis. Hall voltage of 6 mV is measured with negative polarity at $y = 0$ plane. The sign and concentration of the majority charge carrier are, respectively. [Given : $e = 1.6 \times 10^{-19}$ C]



(a) positive and 5.2×10^{19} m^{-3} (b) negative and 5.2×10^{19} m^{-3}

(c) positive and 10.4×10^{19} m^{-3} (d) negative and 10.4×10^{19} m^{-3}

65. The temperature dependence of the electrical conductivity σ of two intrinsic semiconductors *A* and *B* is shown in the figure. If E_A and E_B are the band gaps of *A* and *B* respectively, which one of the following is TRUE ?



(a) $E_A > E_B$ (b) $E_A < E_B$

(c) $E_A = E_B$ (d) E_A and E_B both depend on temperature

66. If the static dielectric constant of NaCl crystal is 5.6 and its optical refractive index is 1.5, the ratio of its electric polarizability to its total polarizability is

(a) 0.5 (b) 0.7 (c) 0.8 (d) 0.9

67. Which one of the following statements is TRUE ?

(a) Magnetic tapes are made of Iron

(b) Permanent magnets are made from ferrites

(c) Ultrasonic transducers are made from quartz crystals

(d) Optoelectronic devices are made from soft ferrites

68. Which one of the following statements is NOT TRUE ?

(a) Entropy decreases markedly on cooling a superconductor below the critical temperature, T_c

(b) The electronic contribution to the heat capacity in the superconducting state has an exponential form with an argument proportional to T^{-1}, suggestive of an energy gap

(c) A type I superconductor is a perfect diamagnet

(d) Critical temperature of superconductors does not vary with the isotopic mass

CAREER ENDEAVOUR

69. The form factor $F(\vec{q}) = \int \exp\left(i\vec{q}\cdot\frac{\vec{r}}{\hbar}\right)\rho(\vec{r})\,d^3r$ of Rutherford scattering is obtained by choosing a delta function for the charge density $\rho(\vec{r})$. The value of the form factor is

(a) unity (b) infinity (c) zero (d) undefined

70. Deuteron in its ground state has a total angular momentum $J=1$ and a positive parity. The corresponding orbital angular momentum *L* and spin *S* combinations are

(a) $L=0,\ S=1$ and $L=2,\ S=0$ (b) $L=0,\ S=1$ and $L=1,\ S=1$

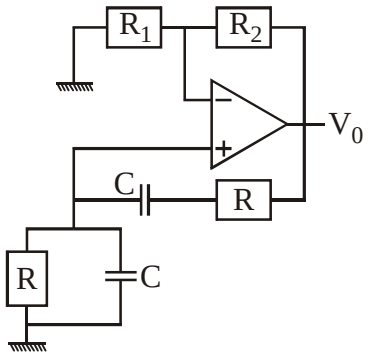
(c) $L=0,\ S=1$ and $L=2,\ S=1$ (d) $L=1,\ S=1$ and $L=2,\ S=1$

71. Which one of the following reaction is allowed ?

(a) $p\to n+e^+$ (b) $p\to e^+ + \nu_e$ (c) $p\to \pi^+ + \gamma$ (d) $\bar{p}+n\to \pi^- + \pi^0$

72. What should be the values of the components R and R_2 such that the frequency of the Wien Bride oscillator is 300 Hz ?

[Given : $C = 0.01\,\mu F$ and $R_1 = 12\,k\Omega$]

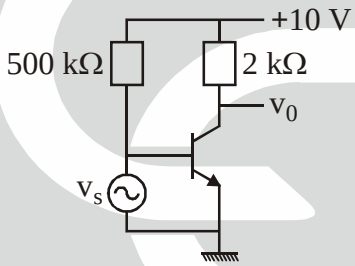


(a) $R = 48\,k\Omega$ and $R_2 = 12\,k\Omega$ (b) $R = 26\,k\Omega$ and $R_2 = 24\,k\Omega$

(c) $R = 530\,\Omega$ and $R_2 = 1\,M\Omega$ (d) $R = 53\,k\Omega$ and $R_2 = 24\,k\Omega$

73. Figure shows a common emitter amplifier with $\beta = 100$. What is the maximum peak to peak input signal (v_s) for which is distortion-free output may be obtained ?

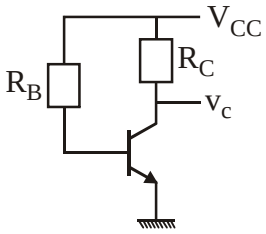
[Assume $V_{BE} = 0$ and $r_e = 20\,\Omega$]



(a) 40 mV (b) 60 mV (c) 80 mV (d) 100 mV

74. Calculate the collector voltage (v_c) of the transistor circuit is shown in the figure.

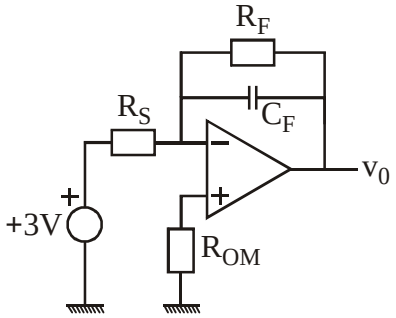
[Given : $\alpha = 0.96$, $I_{CB0} = 20\,\mu A$, $V_{BE} = 0.3V$, $R_B = 100\,k\Omega$, $V_{CC} = +10V$ and $R_C = 2.2\,k\Omega$]



(a) 3.78 V (b) 3.82 V (c) 4.72 V (d) 9.7 V

CAREER ENDEAVOUR

75. Figure shows a practical integrator with $R_S = 30\ \text{M}\Omega$, $R_F = 20\ \text{M}\Omega$ and $C_F = 0.1\ \mu\text{F}$. If a step (dc) voltage of +3 V is applied as input for $0 \le t \le 4$ (t is in seconds), the output voltage is



(a) a ramp function of –6 V
(b) a step function of –12 V
(c) a ramp function of –15 V
(d) a ramp function of –4 V

76. The Boolean expression $Y = \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BCD$ reduces to

(a) $\overline{A}B$ (b) D (c) $\overline{A}$ (d) $\overline{A}D$

Common Data for Q. 77 and Q. 78

Consider the differential equation $y'' + p(x)\,y' + q(x)\,y(x) = 0$.

77. If $xp(x)$ and $x^2q(x)$ have the Taylor series expansions

$xp(x) = 4 + x + x^2 + ...$

$x^2q(x) = 2 + 3x + 5x^2 + ...$

then the roots of the incidicial equation are

(a) –1, 0 (b) –1, –2 (c) –1, 1 (d) –1, 2

78. If $p(x) = 0$ with the Wronskian at $x = 0$ as $W(x = 0) = 1$ and one of the solutions is x, then the other linearly independent solution which vanishes at $x = 1/2$ is

(a) 1 (b) $1 - 4x^2$ (c) x (d) $-1 + 2x$

Common Data for Q. 79 and Q. 80

Consider a comet of mass m moving in a parabolic orbit around the Sun. The closest distance between the comet and the Sun is b, the mass of the Sun is M and the universal gravitation constant is G.

79. The angular momentum of the comet is

(a) $M\sqrt{Gmb}$ (b) $b\sqrt{GmM}$ (c) $G\sqrt{mMb}$ (d) $m\sqrt{2GMb}$

80. Which one of the following is TRUE for the above system ?

(a) The acceleration of the comet is maximum when it is closest to the Sun
(b) The linear momentum of the comet is a constant
(c) The comet will return to the solar system after a specified period
(d) The kinetic energy of the comet is a constant

CAREER ENDEAVOUR

Common Data for Q. 81 and Q. 82

Let $\tilde{E} = \hat{x}E_0 \exp\left[i\vec{k}\cdot\vec{r} - \omega t\right]$, where $\vec{k} = \hat{z}(k\cos\phi + ik\sin\phi)$, $k = 1\left|\vec{k}\right|$ and $\hat{x}$, $\hat{y}$ and $\hat{z}$ are cartesian unit vectors, represent an electric field of plane electro magnetic wave of frequency ω.

81. Which one of the following statements is TRUE ?
(a) The magnitude of the electric field is attenuated as the wave propagates
(b) The energy of the e.m. wave flows along the *x*-direction
(c) The magnitude of the electric field of the wave is a constant
(d) The speed of the wave is the same as *c* (speed of light in free space)

82. The magnetic field $\tilde{B}$ of the wave is

(a) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp[i(zk\cos\phi - \omega t)]$

(b) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\sin\phi)\exp[i(zk\cos\phi - \omega t + \phi)]$

(c) $\hat{y}\frac{k}{\omega}E_0 \exp[i(zk\cos\phi - \omega t + \phi)]$

(d) $\hat{y}\frac{k}{\omega}E_0 \exp(-zk\cos\phi)\exp[i(zk\sin\phi - \omega t)]$

Common Data for Q. 83 and Q. 84

A particle is confined to the region $0 < x < L$ in one dimension

83. If the particle is in the first excited state, then the probability of finding the particle is maximum at

(a) $x = \frac{L}{6}$ (b) $x = \frac{L}{2}$ (c) $x = \frac{L}{3}$ (d) $x = \frac{L}{4}$ and $\frac{3L}{4}$

84. If the particle is in the lowest energy state, then the probability of finding the particle in the region $0 < x < \frac{L}{4}$ is

(a) $\frac{1}{4} - \frac{1}{(2\pi)}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4} + \frac{1}{(2\pi)}$ (d) $\frac{1}{2}$

Common Data for Q. 85 and Q. 86

The one-electron states for non-interacting electrons confined in a cubic box of side *a* are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc.

85. The energy of the lowest state is

(a) zero (b) $\frac{\hbar^2\pi^2}{2ma^2}$ (c) $\frac{\hbar^2\pi^2}{ma^2}$ (d) $\frac{3\hbar^2\pi^2}{2ma^2}$

86. The degeneracy (including spin) of the level ε_3 is

(a) 2 (b) 4 (c) 6 (d) 8

CAREER ENDEAVOUR

Common Data for Q. 87 and Q. 88

An ensemble of N three level systems with energies $\varepsilon = -\varepsilon_0,\ 0,\ +\varepsilon_0$ is in thermal equilibrium at temperature T. Let $\beta = (k_B T)^{-1}$.

87. If $\beta\varepsilon_0 = 2$, the probability of finding the system in the level $\varepsilon = 0$ is

(a) $\dfrac{(\cosh 2)}{2}$ (b) $(\cosh 2)^{-1}$ (c) $(2\cosh 2)^{-1}$ (d) $(1+2\cosh 2)^{-1}$

88. The free energy of the system at high temperature (i.e., $x \equiv \beta\varepsilon_0 \ll 1$) is approximately

(a) $-Nk_B T x^2$ (b) $-Nk_B T\,[\ln 2 + x^2/2]$

(c) $-Nk_B T\,[\ln 3 + x^2/3]$ (d) $-Nk_B T \ln 3$

Common Data for Q. 89 and Q. 90

The nucleus ^{41}Ca can be described by the single particle shell model.

89. The single particle states occupied by the last proton and the last neutron, respectively, are given by

(a) $d_{5/2}$ and $f_{7/2}$ (b) $d_{3/2}$ and $f_{5/2}$ (c) $d_{5/2}$ and $f_{5/2}$ (d) $d_{3/2}$ and $f_{7/2}$

90. The ground state angular momentum and parity of ^{41}Ca are

(a) $\dfrac{7}{2^-}$ (b) $\dfrac{3}{2^+}$ (c) $\dfrac{5}{2^+}$ (d) $\dfrac{5}{2^-}$

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.30 : Carry ONE mark each.

1. The average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is
(a) 15 (b) 20 (c) 40 (d) 80

2. The unit normal to the curve $x^3 y^2 + xy = 17$ at the point (2, 0) is
(a) $\frac{(\hat{i} + \hat{j})}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

3. The value of the integral $\int_C \frac{dz}{z+3}$, where C is a circle (anticlockwise) with $|z| = 4$, is
(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$

4. The determinant of a 3 × 3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvalue is
(a) 4 (b) 6 (c) 8 (d) 9

5. For a particle moving in a central field
(a) the kinetic energy is a constant of motion (b) the potential energy is velocity dependent
(c) the motion is confined in a plane (d) the total energy is not conserved

6. A bead of mass m slides along a straight frictionless rigid wire rotating in a horizontal plane with a constant angular speed ω. The axis of rotation is perpendicular to the wire and passes through one end of the wire. If r is the distance of the mass from the axis of rotation and v is its speed then the magnitude of the Coriolis force is
(a) $\frac{mv^2}{r}$ (b) $\frac{2mv^2}{r}$ (c) $mv\omega$ (d) $2mv\omega$

7. If for a system of N particles of different masses $m_1, m_2, ..., m_N$ with position vectors $\vec{r}_1, \vec{r}_2, ..., \vec{r}_N$ and corresponding velocities $\vec{v}_1, \vec{v}_2, ..., \vec{v}_N$, respectively, such that $\sum_i \vec{v}_i = 0$, then
(a) the total momentum MUST be zero
(b) the total angular momentum MUST be independent of the choice of the origin
(c) the total force on the system MUST be zero
(d) the total torque on the system MUST be zero

8. Although mass-energy equivalence of special relativity allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because
(a) the mass is not conserved (b) the energy is not conserved
(c) the momentum is not conserved (d) the charge is not conserved

9. Three infinitely long wires are placed equally apart on the circumference of a circle of radius a, perpendicular to its plane. Two of the wires carry current I each, in the same direction, while the third carries current $2I$ along the direction opposite to the other two. The magnitude of the magnetic induction $\vec{B}$ at a distance r from the centre of the circle, for $r > a$, is
(a) 0 (b) $\frac{2\mu_0}{\pi}\frac{I}{r}$ (c) $-\frac{2\mu_0}{\pi}\frac{I}{r}$ (d) $\frac{2\mu_0}{\pi}\frac{Ia}{r^2}$

CAREER ENDEAVOUR

10. A solid sphere of radius R carries a uniform volume charge density ρ. The magnitude of electric field inside the sphere at a distance r from the centre is

(a) $\frac{r\rho}{3\varepsilon_0}$ (b) $\frac{R\rho}{2\varepsilon_0}$ (c) $\frac{R^2\rho}{r\varepsilon_0}$ (d) $\frac{R^3\rho}{r^2\varepsilon_0}$

11. The electric field $\vec{E}(\vec{r}, t)$ for a circularly polarized electromagnetic wave propagating along the position z-direction is

(a) $E_0(\hat{x}+\hat{y})\exp[i(kz-\omega t)]$ (b) $E_0(\hat{x}+i\hat{y})\exp[i(kz-\omega t)]$

(c) $E_0(\hat{x}+i\hat{y})\exp[i(kz+\omega t)]$ (d) $E_0(\hat{x}+\hat{y})\exp[i(kz+\omega t)]$

12. The electric (E) and magnetic (B) field amplitudes associated with an electromagnetic radiation from a point source behave at a distance r from the source as

(a) E = constant, B = constant (b) $E \propto \frac{1}{r}$, $B \propto \frac{1}{r}$

(c) $E \propto \frac{1}{r^2}$, $B \propto \frac{1}{r^2}$ (d) $E \propto \frac{1}{r^3}$, $B \propto \frac{1}{r^3}$

13. The parities of the wave functions

(i) $\cos(kx)$, and

(ii) and $\tan h(kx)$ are

(a) (i) odd, (ii) odd (b) (i) even, (ii) even (c) (i) odd, (ii) even (d) (i) even, (ii) odd

14. The commutator, $[L_z, Y_{lm}(\theta, \phi)]$, where L_z is the z-component of the orbital angular momentum and $Y_{lm}(\theta, \phi)$ is a spherical harmonic, is

(a) $l(l+1)\hbar Y_{lm}(\theta, \phi)$ (b) $-m\hbar Y_{lm}(\theta, \phi)$ (c) $m\hbar Y_{lm}(\theta, \phi)$ (d) $+l\hbar Y_{lm}(\theta, \phi)$

15. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$, with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigenstates of the system, requires that the constants c_1 and c_2 must satisfy the condition

(a) $|c_1|\cdot|c_2| = 1$ (b) $|c_1| + |c_2| = 1$ (c) $(|c_1| + |c_2|)^2 = 1$ (d) $|c_1|^2 + |c_2|^2 = 1$

16. A one dimensional harmonic oscillator carrying a charge $-q$ is placed in a uniform electric field $\vec{E}$ along the positive x-axis. The corresponding Hamiltonian operator is

(a) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$ (b) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

(c) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$ (d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

17. The L_β line of X-rays emitted from an atom with principal quantum numbers $n = 1, 2, 3, \ldots$, arises from the transition

(a) $n = 4 \to n = 2$ (b) $n = 3 \to n = 2$ (c) $n = 5 \to n = 2$ (d) $n = 3 \to n = 1$

18. For an electron in hydrogen atom, the states are characterized by the usual quantum numbers n, l, m_l. The electric dipole transition between any two states requires that

(a) $\Delta l = 0,\ \Delta m_l = 0, \pm 1$ (b) $\Delta l = \pm 1,\ \Delta m_l = \pm 1, \pm 2$

(c) $\Delta l = \pm 1,\ \Delta m_l = 0, \pm 1$ (d) $\Delta l = \pm 1,\ \Delta m_l = 0, \pm 2$

CAREER ENDEAVOUR

19. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is

(a) $pV^{1/3} = \text{constant}$ (b) $pV^{2/3} = \text{constant}$ (c) $pV^{4/3} = \text{constant}$ (d) $pV^{3/5} = \text{constant}$

20. The total number of accessible states of N non-interacting particles of spin-$\frac{1}{2}$ is

(a) 2^N (b) N^2 (c) $2^{N/2}$ (d) N

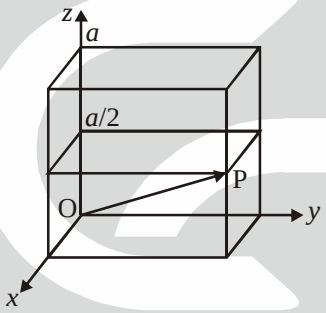
21. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is

(a) $p = \frac{3}{2}\frac{U}{V}$ (b) $p = \frac{2}{3}\frac{U}{V}$ (c) $p = \frac{3}{5}\frac{U}{V}$ (d) $p = \frac{1}{2}\frac{U}{V}$

22. A system of non-interacting Fermi particles with Fermi energy E_F has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature $T = 0$ is

(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$ (c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$

23. In crystallographic notations the vector $\overline{OP}$ in the cubic cell shown in the figure is



(a) [221] (b) [122] (c) [121] (d) [112]

24. Match the following and choose the correct combination

Group-I	Group-II
P. Atomic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$	1. Na
Q. Strongly electropositive	2. Si
R. Strongly electronegative	3. Ar
S. Covalent bonding	4. Cl

(a) P-1, Q-2, R-3, S-4 (b) P-3, Q-2, R-4, S-1
(c) P-3, Q-1, R-4, S-2 (d) P-3, Q-4, R-1, S-2

25. The evidence for the non-conservation of parity in β- decay has been obtained from the observation that the β intensity

(a) antiparallel to the nuclear spin directions is same as that along the nuclear spin direction
(b) antiparallel to the nuclear spin direction is not the same as that along the nuclear spin direction
(c) shows a continuous distribution as a function of momentum
(d) is independent of the nuclear spin direction

CAREER ENDEAVOUR

26. Which of the following expressions for total binding energy B of a nucleus is correct $(a_1, a_2, a_3, a_4 > 0)$?

(b) $B = a_1 A - a_2 A^{2/3} - a_3 \dfrac{Z(Z-1)}{A^{1/3}} - a_4 \dfrac{(A-2Z)^2}{A} + \delta$

(b) $B = a_1 A + a_2 A^{2/3} - a_3 \dfrac{Z(Z-1)}{A^{1/3}} - a_4 \dfrac{(A-2Z)^2}{A} + \delta$

(c) $B = a_1 A + a_2 A^{1/3} - a_3 \dfrac{Z(Z-1)}{A^{1/3}} - a_4 \dfrac{(A-2Z)^2}{A} + \delta$

(d) $B = a_1 A - a_2 A^{1/3} - a_3 \dfrac{Z(Z-1)}{A^{1/3}} - a_4 \dfrac{(A-2Z)^2}{A} + \delta$

27. Which of the following decay is forbidden ?

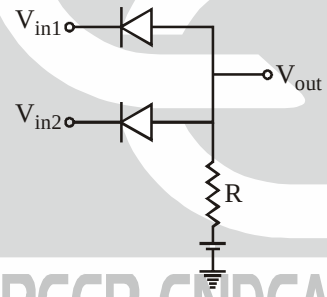
(a) $\mu^- \to e^- + \nu_\mu + \overline{\nu}_c$ (b) $\pi^+ \to \mu^+ + \nu_\mu$

(c) $\pi^+ \to e^+ + \nu_e$ (d) $\mu^- \to e^+ + e^- + e^-$

28. With reference to nuclear forces which of the following statements is NOT true ?
The nuclear forces are
(a) short range (b) charge independent (c) velocity dependent (d) spin independent

29. A junction field effect transistor behaves as a
(a) voltage controlled current source (b) voltage controlled voltage source
(c) current controlled voltage source (d) current controlled current source

30. The circuit shown can be used as



(a) NOR gate (b) OR gate (c) NAND gate (d) AND gate

CAREER ENDEAVOUR

Q.31 – Q.80 : Carry TWO marks each.

31. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{F}\right)$ is

(a) 0 (b) $\hat{i}$ (c) $2\hat{j}$ (d) $3\hat{k}$

32. All solutions of the equation $e^z = -3$ are

(a) $z = i n\pi \ln 3,\ n = \pm 1, \pm 2, ...$ (b) $z = \ln 3 + i(2n+1)\pi,\ n = 0, \pm 1, \pm 2, ...$

(c) $z = \ln 3 + i\, 2n\pi,\ n = 0, \pm 1 \pm 2, ...$ (d) $z = i\, 3n\pi,\ n = \pm 1 \pm 2, ...$

33. If $\overline{f}(s)$ is the Laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constant, is

(a) $\dfrac{1}{a}\overline{f}(s)$ (b) $\dfrac{1}{a}\overline{f}(s/a)$ (c) $\overline{f}(s)$ (d) $\overline{f}(s/a)$

34. Given the four vectors

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \end{pmatrix}, \quad u_4 = \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix}$$

the linearly dependent pair is

(a) $u_1\ u_2$ (b) $u_1\ u_3$ (c) $u_1\ u_4$ (d) $u_3\ u_4$

35. Consider the following function: $f(z) = \dfrac{\sin z}{z}$. Which of the following statemens is are TRUE?

(a) z = 0 is pole of order 1
(b) z = 0 is a removable singular point
(c) z = 0 is a pole order 3
(d) z = 0 is an essential singular point

36. Eigen values of the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix} \text{ are}$$

(a) –2, –1, 1, 2 (b) –1, 1, 0, 2 (c) 1, 0, 2, 3 (d) –1, 1, 0, 3

37. If a particle moves outward in a plane along a curved trajectory described by $r = a\theta$, $\theta = \omega t$, where a and ω are constants, then its

(a) kinetic energy is conserved
(b) angular momentum is conserved
(c) total momentum is conserved
(d) radial momentum is conserved

38. A circular hoop of mass M and radius a rolls without slipping with constant angular speed ω along the horizontal x-axis in the xy-plane. When the centre of the hoop is at a distance $d = \sqrt{2}a$ from the origin, the magnitude of the total angular momentum of the hoop about the origin is

(a) $Ma^2\omega$ (b) $\sqrt{2}Ma^2\omega$ (c) $2Ma^2\omega$ (d) $3Ma^2\omega$

39. Two solid spheres of radius R and mass M each are connected by a thin rigid rod of negligible mass. The distance between the centres is $4R$. The moment of inertia about an axis passing through the centre of symmetry and perpendicular to the line joining the spheres is

(a) $\dfrac{11}{5}MR^2$ (b) $\dfrac{22}{5}MR^2$ (c) $\dfrac{44}{5}MR^2$ (d) $\dfrac{88}{5}MR^2$

40. A car is moving with constant linear acceleration a along horizontal x-axis. A solid sphere of mass M and radius R is found rolling without slipping on the horizontal floor of the car in the same direction as seen from an inertial frame outside the car. The acceleration of the sphere in the inertial frame is

(a) $\dfrac{a}{7}$ (b) $\dfrac{2a}{7}$ (c) $\dfrac{3a}{7}$ (d) $\dfrac{5a}{7}$

41. A rod of length l_0 makes an angle θ_0 with the y-axis in its rest frame, while the rest frame moves to the right along the x-axis with relativistic speed v with respect to the lab frame. If $\gamma = \left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$, the angle θ in the lab frame is

(a) $\theta = \tan^{-1}(\gamma \tan\theta_0)$
(b) $\theta = \tan^{-1}(\gamma \cot\theta_0)$
(c) $\theta = \tan^{-1}\left(\dfrac{1}{\gamma}\tan\theta_0\right)$
(d) $\theta = \tan^{-1}\left(\dfrac{1}{\gamma}\cot\theta_0\right)$

CAREER ENDEAVOUR

42. A particle of mass m moves in a potential $V(x) = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\mu v^2$, where x is the position coordinate, v is the speed, and ω and μ are constants. The canonical (conjugate) momentum of the particle is

(a) $p = m(1+\mu)v$ (b) $p = mv$ (c) $p = m\mu v$ (d) $p = m(1-\mu)v$

43. Consider the following three independent cases :

(i) Particle A of charge $+q$ moves in free space with a constant velocity $\vec{v}$ ($v \ll$ speed of light)

(ii) Particle B of charge $+q$ moves in free space in a circle of radius R with same speed v as in case (i)

(iii) Particle C having charge $-q$ moves as in case (ii)

If the power radiated by A, B and C are P_A, P_B and P_C, respectively, then

(a) $P_A = 0,\ P_B > P_C$ (b) $P_A = 0,\ P_B = P_C$ (c) $P_A > P_B > P_C$ (d) $P_A = P_B = P_C$

44. If the electrostatic potential were given by $\phi = \phi_0(x^2 + y^2 + z^2)$, where ϕ_0 is constant, then the charge density giving rise to the above potential would be

(a) 0 (b) $-6\phi_0\,\varepsilon_0$ (c) $-2\phi_0\,\varepsilon_0$ (d) $-\frac{6\phi_0}{\varepsilon_0}$

45. The work done in bringing a charge $+q$ from infinity in free space, to a position at a distance d in front of a semi-infinite grounded metal surface is

(a) $-\frac{q^2}{4\pi\varepsilon_0(d)}$ (b) $-\frac{q^2}{4\pi\varepsilon_0(2d)}$ (c) $-\frac{q^2}{4\pi\varepsilon_0(4d)}$ (d) $-\frac{q^2}{4\pi\varepsilon_0(6d)}$

46. A plane electromagnetic wave travelling in vaccum is incident normally on a non-magnetic, non-absorbing medium of refractive index n. The incident (E_i), reflected (E_r) and transmitted (E_t) electric fields are given as, $E_i = E\exp[i(kz-\omega t)]$, $E_r = E_{0r}\exp[i(k_r z-\omega t)]$, $E_t = E_{0t}\exp[i(k_t z-\omega t)]$. If $E = 2$ V/m and $n = 1.5$, then the application of appropriate boundary conditions leads to

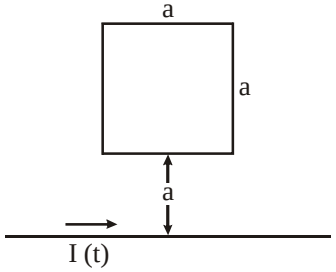
(a) $E_{0r} = -\frac{3}{5}$ V/m, $E_{0t} = \frac{7}{5}$ V/m (b) $E_{0r} = -\frac{1}{5}$ V/m, $E_{0t} = \frac{8}{5}$ V/m

(c) $E_{0r} = -\frac{2}{5}$ V/m, $E_{0t} = \frac{8}{5}$ V/m (d) $E_{0r} = \frac{4}{5}$ V/m, $E_{0t} = \frac{6}{5}$ V/m

47. For a vector potential $\vec{A}$, the divergence of $\vec{A}$ is $\vec{\nabla}\cdot\vec{A} = -\frac{\mu_0}{4\pi}\frac{Q}{r^2}$, where Q is a constant of appropriate dimension. The corresponding scalar potential $\varphi(\vec{r}, t)$ that makes $\vec{A}$ and φ Lorentz gauge invariant is

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r^2}$

48. An infinitely long wire carrying a current $I(t) = I_0\cos(\omega t)$ is placed at a distance a from a square loop of side a as shown in the figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is



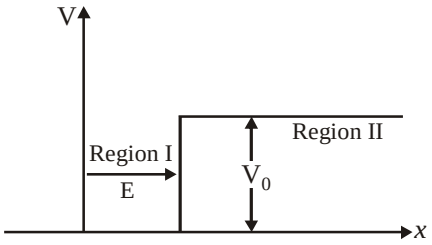
(a) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}\ln 2$ (b) $\frac{\mu_0}{\pi}\frac{aI_0\omega}{R}\ln 2$ (c) $\frac{2\mu_0}{\pi}\frac{aI_0\omega}{R}\ln 2$ (d) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}$

CAREER ENDEAVOUR

49. The de Broglie wavelength λ for an electron of energy 150 eV is
(a) 10^{-8} m (b) 10^{-10} m (c) 10^{-12} m (d) 10^{-14} m

50. A particle is incident with a constant energy E on a one-dimensional potential barrier as shown in the figure. The wavefunctions in regions I and II are respectively
(a) decaying, oscillatory (b) oscillatory, oscillatory
(c) oscillatory, decaying (d) decaying, decaying

51. The expectation value of the z-coordinate, (z) in the ground state of the hydrogen atom (wavefunction : $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is



(a) a_0 (b) $\frac{a_0}{2}$ (c) $\frac{a_0}{4}$ (d) 0

52. The degeneracy of the $n = 2$ level for a three dimensional isotropic oscillator is
(a) 4 (b) 6 (c) 8 (d) 10

53. For a spin-$\frac{1}{2}$ particle, the expectation value of $s_x s_y s_z$, where s_x, s_y and s_z are spin operators, is

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$ (c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

54. An atom emits a photon of wavelength $\lambda = 600$ nm by transition from an excited state of lifetime 8×10^{-9} s. If $\Delta\nu$ represents the minimum uncertainty in the frequency of the photon, the fractional width $\Delta\nu/\nu$ of the spectral line is of the order of
(a) 10^{-4} (b) 10^{-6} (c) 10^{-8} (d) 10^{-10}

55. The sodium doublet lines are due to transitions from $^2P_{3/2}$ and $^2P_{1/2}$ levels to $^2S_{1/2}$ level. On application of a weak magnetic field, the total number of allowed transitions becomes
(a) 4 (b) 6 (c) 8 (d) 10

56. A three level system of atoms has N_1 atoms in level E_1, N_2 in level E_2, and N_3 in level $E_3 (N_2 > N_1 > N_3$ and $E_1 < E_2 < E_3)$. Laser emission is possible between the levels
(a) $E_3 \to E_1$ (b) $E_2 \to E_1$ (c) $E_3 \to E_2$ (d) $E_2 \to E_3$

57. In an Raman scattering experiment, light of frequency ν from a laser is scattered by diatomic molecules having the moment of intertia I. The typical Raman shifted frequency depends on
(a) ν and I (b) only ν (c) only I (d) neither v nor I

58. For a diatomic molecule with the vibrational quantum number n and rotational quantum number J, the vibrational level spacing $\Delta E_n = E_n - E_{n-1}$ and the rotational level spacing $\Delta E_J = E_J - E_{J-1}$ are approximately
(a) ΔE_n = constant, ΔE_J = constant (b) ΔE_n = constant, $\Delta E_J \propto J$
(c) $\Delta E_n \propto n$, $\Delta E_J \propto J$ (d) $\Delta E_n \propto n$, $\Delta E_J \propto J^2$

CAREER ENDEAVOUR

59. The typical wavelengths emitted by diatomic molecules in purely vibrational and purely rotational transitions are respectively in the region of

(a) infrared and visible (b) visible and infrared

(c) infrared and microwave (d) microwave and infrared

60. In a two electron atomic system having orbital and spin angular momenta l_1, l_2 and s_1, s_2 respectively, the coupling strengths are defined as $\Gamma_{l_1 l_2}$, $\Gamma_{s_1 s_2}$, $\Gamma_{l_1 s_1}$, $\Gamma_{l_2 s_2}$, $\Gamma_{l_1 s_2}$ and $\Gamma_{l_2 s_1}$. For the *jj* coupling scheme to be applicable, the coupling strengths MUST satisfy the condition

(a) $\Gamma_{l_1 s_2}, \Gamma_{s_1 s_2} > \Gamma_{l_1 s_1}, \Gamma_{l_2 s_2}$ (b) $\Gamma_{l_1 s_1}, \Gamma_{l_2 s_2} > \Gamma_{l_1 l_2}, \Gamma_{s_1 s_2}$

(c) $\Gamma_{l_1 s_2}, \Gamma_{l_2 s_1} > \Gamma_{l_1 l_2}, \Gamma_{s_1 s_2}$ (d) $\Gamma_{l_1 s_2}, \Gamma_{l_2 s_1} > \Gamma_{l_1 s_1}, \Gamma_{l_2 s_2}$

61. If the probability that x lies between x and $x + dx$ is $p(x)\,dx = ae^{-ax}dx$, where $0 < x < \infty$, $a > 0$, then probability that x lies between x_1 and x_2 $(x_2 > x_1)$ is

(a) $\left(e^{-ax_1} - e^{-ax_2}\right)$ (b) $a\left(e^{-ax_1} - e^{-ax_2}\right)$

(c) $e^{-ax_2}\left(e^{-ax_1} - e^{-ax_2}\right)$ (d) $e^{-ax_1}\left(e^{-ax_1} - e^{-ax_2}\right)$

62. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{kT}{\hbar\omega}$, then the free energy of N such independent oscillator is

(a) $\frac{3}{2}NkT$ (b) $kT \ln\frac{\hbar\omega}{kT}$ (c) $NkT \ln\frac{\hbar\omega}{kT}$ (d) $NkT \ln\frac{\hbar\omega}{2kT}$

63. The partition function of two Bose particles each of which can occupy any of the two energy levels 0 and ε is

(a) $1 + e^{-2\varepsilon/kT} + 2e^{-\varepsilon/kT}$ (b) $1 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

(c) $2 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$ (d) $e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

64. A one dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origin is back to origin after N even number of steps is

(a) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$ (c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

65. The number of states for a system of N identical free particles in a three dimensional space having total energy between E and $E + \delta E$ $(\delta E \ll E)$, is proportional to

(a) $\left(E^{\frac{3N}{2}-1}\right)\delta E$ (b) $E^{N/2}\,\delta E$ (c) $NE^{1/2}\,\delta E$ (d) $E^N\,\delta E$

66. The energy of a ferromagnet as a function of magnetization M is given by

$F(M) = F_0 + 2(T - T_c)\,M^2 + M^4, \quad F_0 > 0.$

The number of minima in the function $F(M)$ for $T > T_c$ is

(a) 0 (b) 1 (c) 3 (d) 4

CAREER ENDEAVOUR

67. For a closed packed BCC structure of hard spheres, the lattice constant a is related to the sphere radius R as

(a) $a = \frac{4R}{\sqrt{3}}$ (b) $a = 4R\sqrt{3}$ (c) $a = 4R\sqrt{2}$ (d) $a = 2R\sqrt{2}$

68. An n-type semiconductor has an electron concentration of 3×10^{20} m^{-3}. If the electron drift velocity is 100 ms^{-1} in an electric field of 200 Vm^{-1}, the conductivity (in Ω^{-1} m^{-1}) of this material is

(a) 24 (b) 36 (c) 48 (d) 96

69. Density of states of free electrons in a solid moving with an energy 0.1 eV is given by 2.15×10^{21} eV^{-1} cm^{-3}. The density of states (in eV^{-1} cm^{-3}) for electrons moving with an energy of 0.4 eV will be

(a) 1.07×10^{21} (b) 1.52×10^{21} (c) 3.04×10^{21} (d) 4.30×10^{21}

70. The effective density of states at the conduction band edge of Ge is 1.04×10^{19} cm^{-3} at room temperature (300 K). Ge has an optical bandgap of 0.66 eV. The intrinsic carrier concentration (in cm^{-3}) in Ge at room temperature (300 K) is approximately

(a) 3×10^{10} (b) 3×10^{13} (c) 3×10^{16} (d) 3×10^{16}

71. For a conventional superconductor, which of the following statements is NOT true ?

(a) Specific heat is discontinuous at transition temperature T_c

(b) The resistivity falls sharply at T_c

(c) It is diamagnetic below T_c

(d) It is paramagnetic below T_c

72. A nucleus having mass number 240 decays by α emission to the ground state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy (in MeV) of the α particle is

(a) 5.26 (b) 5.17 (c) 5.13 (d) 5.09

73. The threshold temperature above which the thermonuclear reaction

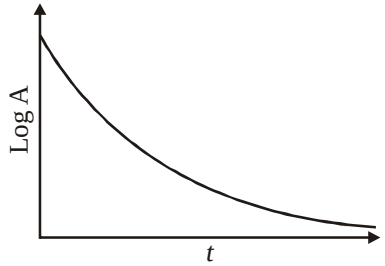
$^3_2\text{He} + ^3_2\text{He} \rightarrow ^4_2\text{He} + 2^1_1\text{H} + 12.86 \text{ MeV}$ can occur is (use $e^{2/4\pi\varepsilon_0} = 1.44 \times 10^{-15}$ MeVm)

(a) 1.28×10^{10} K (b) 1.28×10^{9} K (c) 1.28×10^{8} K (d) 1.28×10^{7} K

74. According to the shell model, the ground state of $^{15}_8\text{O}$ nucleus is

(a) $\frac{3^+}{2}$ (b) $\frac{1^+}{2}$ (c) $\frac{3^-}{2}$ (d) $\frac{1^-}{2}$

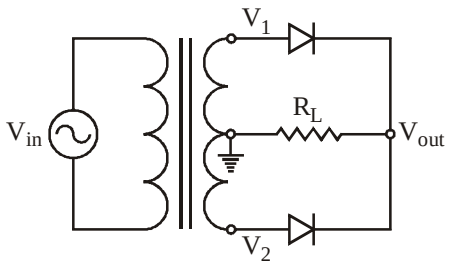
75. The plot of log A vs. time t, where A is activity, as shown in the figure, corresponds to decay



(a) from only one kind of radioactive nuclei having same half life
(b) from only neutron activated nuclei
(c) from a mixture of radioactive nuclei having different half lives
(d) which is unphysical

CAREER ENDEAVOUR

76. For the rectifier circuit shown in the figure, the sinusoidal voltage (V_1 or V_2) at the output of the transformer has a maximum value of 10 V. The load resistance R_L is $k\Omega$. If I_{ave} is the average current through the resistor R_L the circuit corresponds



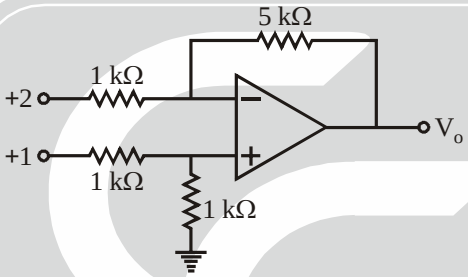
(a) full wave rectifier with $I_{av} = 20/\pi$ mA (b) half wave rectifier with $I_{av} = 20/\pi$ mA

(c) half wave rectifier with $I_{av} = 10/\pi$ mA (d) full wave rectifier with $I_{av} = 10/\pi$ mA

77. The Boolean expression : $B(A + B) + A.\ (\bar{B} + A)$ can be realized using minimum number of

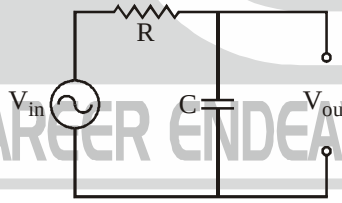
(a) 1 AND gate (b) 2 AND gates (c) 1 OR gate (d) 2 OR gates

78. The output V_0 of the ideal opamp circuit shown in the figure is



(a) –7 V (b) –5 V (c) 5 V (d) 7 V

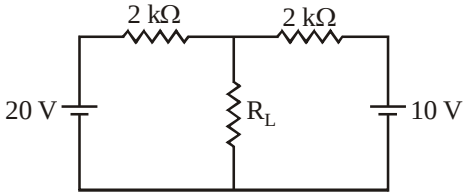
79. The circuit shown in the figure can be used as a



(a) high pass filter or a differentiator (b) high pass filter or an integrator

(c) low pass filter or a differentiator (d) low pass filter or an integrator

80. In the circuit shown in the figure the Thevenin voltage V_{Th} and Thevenin resistance R_{Th} as seen by the load resistance R_L $(= 1\,k\Omega)$ are respectively



(a) 15 V, 1 kΩ (b) 30 V, 4 kΩ (c) 20 V, 2 kΩ (d) 10 V, 5 kΩ

Linked Answer Questions : Q. 81a to Q. 85b carry two marks each.

Statement for Linked Answer Q.81a and Q.81b :

For the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

81a. One of the solutions is

(a) e^x (b) $\ln x$ (c) e^{-x^2} (d) e^{x^2}

81b. The second linearly independent solution is

(a) e^{-x} (b) xe^x (c) x^2e^x (d) x^2e^{-x}

Statement for Linked Answer Q.82a and Q.82b :

The Langrangian of two coupled oscillators of mass m each is

$$L = \frac{1}{2}m\left(x_1^2 + x_2^2\right) - \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right) + m\omega_0^2\,\mu\,x_1\,x_2$$

82a. The equation of motion are

(a) $\ddot{x}_1 + \omega_0^2\,\mu\,x_1,\ \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\,\mu\,x_2$ (b) $\ddot{x}_1 + \omega_0^2\,x_2 - \omega_0^2\mu x_1 = 0,\ \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\,\mu\,x_1$

(c) $\ddot{x}_1 + \omega_0^2\,\mu\,x_1,\ \ddot{x}_2 + \omega_0^2 x_2 = -\omega_0^2\,\mu\,x_2$ (d) $\ddot{x}_1 + \omega_0^2\,\mu\,x_1,\ \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\,\mu\,x_1$

82b. The normal modes of the system are

(a) $\omega_0\sqrt{\mu^2 - 1},\ \omega_0\sqrt{\mu^2 + 1}$ (b) $\omega_0\sqrt{1-\mu^2},\ \omega_0\sqrt{1+\mu^2}$

(c) $\omega_0\sqrt{\mu - 1},\ \omega_0\sqrt{\mu + 1}$ (d) $\omega_0\sqrt{1-\mu},\ \omega_0\sqrt{1+\mu}$

Statement for Linked Answer Q.83a and Q.83b :

An infinitely long hollow cylinder of radius R carrying a surface density σ is rotated about its cylindrical axis with a constant angular speed ω

83a. The magnitude of the surface current is

(a) $\sigma R^2\,\omega$ (b) $2\sigma R\,\omega$ (c) $\pi\sigma R\,\omega$ (d) $2\pi\sigma R\,\omega$

83b. The magnitude of vector potential inside the cylindrical at a distance are from its axis is

(a) $2\mu_0\sigma R\omega r$ (b) $\mu_0\sigma R\omega r$ (c) $\frac{1}{2}\mu_0\sigma R\omega r$ (d) $\frac{1}{4}\mu_0\sigma R\omega r$

Statement for Linked Answer Q.84a and Q.84b :

A particle is scattered by a spherically symmetric potential. In the centre of mass (CM) frame the wavefunction of the incoming particle is $\psi = Ae^{ikz}$, where k is the wavevector and A is a constant.

84a. If $f(\theta)$ is an angular function then in the asymtotic region the scattered wavefunction has the form

(a) $\frac{A\,f(\theta)e^{ikr}}{r}$ (b) $\frac{A\,f(\theta)e^{-ikr}}{r}$ (c) $\frac{A\,f(\theta)e^{ikr}}{r^2}$ (d) $\frac{A\,f(\theta)e^{-ikr}}{r^2}$

84b. The differential scattering cross section $\sigma(\theta)$ in CM frame is

(a) $\sigma(\theta) = |A|^2\,\frac{|f(\theta)|^2}{r^2}$ (b) $\sigma(\theta) = |A|^2\,|f(\theta)|^2$

(c) $\sigma(\theta) = |f(\theta)|^2$ (d) $\sigma(\theta) = |A|\,|f(\theta)|$

CAREER ENDEAVOUR

Statement for Linked Answer Q.85(a) and Q.85(b) :

Lead has atomic weight of 207.2 amu and density of 11.35 gm cm^{-3}

85.a (A) Number of atoms per cm^3 for lead is

(a) 1.1×10^{25} (b) 3.3×10^{22} (c) 1.1×10^{22} (d) 3.3×10^{25}

85.b (B) If the energy of vacancy formation in lead is 0.55 eV/atom, the number of vacancies/cm^3 at 500 K is

(a) 3.2×10^{16} (b) 3.2×10^{19} (c) 9.5×10^{19} (d) 9.5×10^{16}

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.20 : Carry ONE mark each.

1. The trace of a 3 × 3 matrix is 2. Two of its eigenvalues are 1 and 2. The third eigenvalue is
(a) –1 (b) 0 (c) 1 (d) 2

2. The value of $\oint_c \vec{A}\cdot d\vec{l}$ along a square loop of side L in a uniform field $\vec{A}$ is
(a) 0 (b) 2LA (c) 4LA (d) L^2A

3. A particle of charge q, mass m and linear momentum $\vec{p}$ enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential ϕ. The Hamiltonian of the particle is

(a) $\frac{p^2}{2m} + q\phi + \frac{A^2}{2m}$ (b) $\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi$

(c) $\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + \vec{p}\cdot\vec{A}$ (d) $\frac{p^2}{2m}q\phi - \vec{p}\cdot\vec{A}$

4. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is
(a) circular (b) elliptical (c) parabolic (d) hyperbolic

5. In an electromagnetic field, which one of the following remains invariant under Lorentz transformation ?
(a) $\vec{E}\times\vec{B}$ (b) $E^2 - c^2B^2$ (c) B^2 (d) E^2

6. A sphere of radius R has uniform volume charge density. The electric potential at a point $r(r < R)$ is
(a) due to the charge inside a sphere of radius r only
(b) due to the entire charge of the sphere
(c) due to charge in the spherical shell of inner and outer radii r and R, only
(d) independent of r

7. A free particle is moving in +x-direction with a linear momentum p. The wavefunction of the particle normalised in a length L is

(a) $\frac{1}{\sqrt{L}}\sin\frac{p}{\hbar}x$ (b) $\frac{1}{\sqrt{L}}\cos\frac{p}{\hbar}x$ (c) $\frac{1}{\sqrt{L}}e^{-i\frac{p}{\hbar}x}$ (d) $\frac{1}{\sqrt{L}}e^{i\frac{p}{\hbar}x}$

8. Which one of the following relations is true for Pauli matrices σ_x, σ_y and σ_z ?
(a) $\sigma_x\sigma_y = \sigma_y\sigma_x$ (b) $\sigma_x\sigma_y = \sigma_z$ (c) $\sigma_x\sigma_y = i\sigma_z$ (d) $\sigma_x\sigma_y = -\sigma_y\sigma_x$

9. The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3}a\mathrm{VT}^{-4}$, where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is
(a) 0 (b) $\frac{4}{3}a\mathrm{VT}^3$ (c) $\frac{1}{3}a\mathrm{T}^{-4}$ (d) $a\mathrm{VT}^{-4}$

10. The wavefunctions of two identical particles in state n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two particle system is expressed as

(a) $\phi_n(r_1) + \phi_s(r_2)$ (b) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)\,\phi_s(r_2) + \phi_n(r_2)\,\phi_s(r_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)\,\phi_s(r_2) - \phi_n(r_2)\,\phi_s(r_1)\right]$ (d) $\phi_n(r_1)\,\phi_s(r_2)$

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11. The target of an X-ray tube is subjected to an excitation voltage V. The wavelength of the emitted X-rays is proportional to

(a) $\frac{1}{\sqrt{V}}$ (b) $\sqrt{V}$ (c) $\frac{1}{V}$ (d) V

12. The principal series of spectral lines of lithium is obtained by transitions between

(a) nS and 2P, $n > 2$ (b) nD and 2P, $n > 2$ (c) nP and 2S, $n > 2$ (d) nF and 3D, $n > 3$

13. Which one of the following is NOT a correct statement about semiconductors ?

(a) The electrons and holes have different mobilities in a semiconductor
(b) In an n-type semiconductor, the Fermi level lies closer to the conduction band edge
(c) Silicon is a direct band gap semiconductor
(d) Silicon has diamond structure

14. Which one of the following axes of rotational symmetry is NOT permissible in single crystals ?

(a) two-fold axis (b) three-fold axis (c) four-fold axis (d) five-fold axis

15. Weak nuclear forces act on

(a) both hadrons and leptons (b) hadrons only
(c) all particles (d) all charged particle

16. Which one of the following disintegration series of the heavy elements will give ^{209}Bi as a stable nucleus ?

(a) Thorium series (b) Neptunium series (c) Uranium series (d) Actinium series

17. The order of magnitude of the binding energy per nucleon in a nucleus is

(a) 10^{-5} MeV (b) 10^{-3} MeV (c) 0.1 MeV (d) 10 MeV

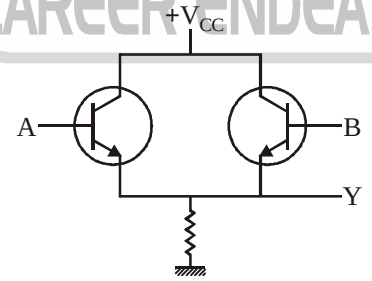
18. The interaction potential between two quarks, separated by a distance r inside a nucleon, can be described by (a, b and β are positive constants)

(a) $ae^{-\beta r}$ (b) $\frac{a}{r} + br$ (c) $-\frac{a}{r} + br$ (d) $\frac{a}{r}$

19. The high input impedance of field effect transistor (FET) amplifier is due to

(a) the pinch-off voltage (b) its very low gate current
(c) the source and drain being far apart (d) the geometry of the FET

20. The circuit shown in the figure function as



(a) an OR gate (b) an AND gate (c) a NOR gate (d) a NAND gate

CAREER ENDEAVOUR

Q.21 – Q.85 : Carry TWO marks each.

21. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\vec{x}$ for a 3-dimensional real space to a 2-dimensional real space. The transformation matrix T is

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

22. The value of $\oint_S \frac{\vec{r} \cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and *S* is a closed surface enclosing the origin, is

(a) 0 (b) π (c) 4π (d) 8π

23. The value of $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where *C* is a circle defined by $|z| = 3$, is

(a) $\frac{8\pi i}{3} e^{-2}$ (b) $\frac{8\pi i}{3} e^{-1}$ (c) $\frac{8\pi i}{3} e$ (d) $\frac{8\pi i}{3} e^{2}$

24. The *k*th Fourier component of $f(x) = \delta(x)$ is

(a) 1 (b) 0 (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

25. An atom with net magnetic moment $\vec{\mu}$ and net angular momentum $\vec{L}\left(\vec{\mu} = \gamma\vec{L}\right)$ is kept in a uniform magnetic induction $\vec{B} = B_0\hat{k}$. The magnetic moment $\vec{\mu}(= \mu_x)$ is

(a) $\frac{d^2\mu_x}{dt^2} + \gamma B_0 \mu_x = 0$ (b) $\frac{d^2\mu_x}{dt^2} + \gamma B_0 \frac{d\mu_x}{dt} + \mu_x = 0$

(c) $\frac{d^2\mu_x}{dt^2} + \gamma^2 B_0^2 \mu_x = 0$ (d) $\frac{d^2\mu_x}{dt^2} + 2\gamma B_0 \mu_x = 0$

CAREER ENDEAVOUR

26. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a positive constant. In a stationary state, the expectation value of the kinetic energy $\langle T \rangle$ of the particle is

(a) $\langle T \rangle = \langle V \rangle$ (b) $\langle T \rangle = 2\langle V \rangle$ (c) $\langle T \rangle = 3\langle V \rangle$ (d) $\langle T \rangle = 4\langle V \rangle$

27. A particle of mass 2 kg is moving such that at time *t*, its position, in metre, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at $t = 2$ s about the origin, in kg m^2 s^{-1}, is

(a) $-40\hat{k}$ (b) $-80\hat{k}$ (c) $80\hat{k}$ (d) $40\hat{k}$

28. A system of four particles is in *xy*-plane. Of these, two particles each of mass *m* are located at (1, 1) and (–1, –1). The remaining two particles each of mass 2*m* are located at (–1, 1) and (1, –1). The *xy*-component of the moment of inertia tensor of this system of particles is

(a) $10m$ (b) $-10m$ (c) $2m$ (d) $-2m$

29. For the given transformations (i) $Q = p, P = -q$ *and* (ii) $Q = p, P = q$, where p and q are canonically conjugate variables, which one of the following statement is true ?

(a) Both (i) and (ii) are canonical
(b) Only (i) is canonical
(c) Only (ii) is canonical
(d) Neither (i) nor (ii) is canonical

30. The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where m_0 is its rest mass. The linear momentum of the particle is

(a) $2m_0c$ (b) $\frac{2m_0}{\sqrt{3}}$ (c) m_0c (d) $\frac{m_0c}{\sqrt{3}}$

31. Three point charges q, q and $-2q$ are located at $(0, -a, a)$, $(0, a, a)$ and $(0, 0, -a)$, respectively. The net dipole moment of this charge distribution is

(a) $4qa\,\hat{k}$ (b) $2qa\,\hat{k}$ (c) $-4qa\,\hat{i}$ (d) $-2qa\,\hat{j}$

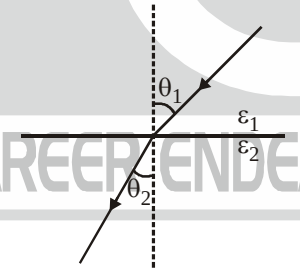
32. A long cylindrical kept along z-axis carries a current density $\hat{j} = J_0 r\hat{k}$, where J_0 is a constant and r is the radial distance form the axis of the cylinder. The magnetic induction $\hat{B}$ inside the conductor at a distance d from the axis of the cylinder is

(a) $\mu_0 J_0 \hat{\phi}$ (b) $-\frac{\mu_0 J_0 d}{2}\hat{\phi}$ (c) $\frac{\mu_0 J_0 d^2}{2}\hat{\phi}$ (d) $-\frac{\mu_0 J_0 d^3}{4}\hat{\phi}$

33. The vector potential in a region is given as $\vec{A}(x, y, z) = -y\,\hat{i} + 2x\,\hat{j}$. The associated magnetic induction is $\vec{B}$ is

(a) $\hat{i} + \hat{k}$ (b) $3\hat{k}$ (c) $-\hat{i} + 2\hat{j}$ (d) $-\hat{i} + \hat{j} + \hat{k}$

34. At the interface between two linear dielectrics (with dielectric constants ε_1 and ε_2), the electric field lines bend, as shown in the figure. Assume that there are no free charges at the interface. The ratio $\varepsilon_1/\varepsilon_2$ is



(a) $\frac{\tan\theta_1}{\tan\theta_2}$ (b) $\frac{\cos\theta_1}{\cos\theta_2}$ (c) $\frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{\cot\theta_1}{\cot\theta_2}$

35. Which one of the following sets of Maxwell's equations for time-independent charge density ρ and current density $\hat{J}$ is correct ?

(a) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$, $\vec{\nabla}\cdot\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(b) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$, $\vec{\nabla}\cdot\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\hat{J}$

(c) $\vec{\nabla}\cdot\vec{E} = 0$, $\vec{\nabla}\cdot\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\hat{J}$

(d) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$, $\vec{\nabla}\cdot\vec{B} = \mu_0\hat{J}$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

36. A classical charged particle moving with frequency ω in a circular orbit of radius a, centred at the origin in the xy-plane, electromagnetic radiation. At points $(b, 0, 0)$ and $(0, 0, b)$, where $b \gg a$, the electromagnetic waves are
(a) circularly polarized and elliptically polarized, respectively
(b) plane polarized and elliptically polarized, respectively
(c) plane polarized and circularly polarized, respectively
(d) circularly polarized and plane polarized, respectively

37. A particle of mass m is represented by the wavefunction $\psi(x) = Ae^{ikx}$, where k is the wavevector and A is a constant. The magnitude of the probability current density of the particle is

(a) $|A|^2 \frac{\hbar k}{m}$ (b) $|A|^2 \frac{\hbar k}{2m}$ (c) $|A|^2 \frac{(\hbar k)^2}{m}$ (d) $|A|^2 \frac{(\hbar k)^2}{2m}$

38. A one-dimensional harmonic oscillator is in the state $\psi(x) = \frac{1}{\sqrt{14}}\left[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)\right]$, where $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is

(a) 0 (b) $\frac{3}{\sqrt{14}}$ (c) $\frac{9}{14}$ (d) 1

39. The wavefunction of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t=0) = \frac{1}{\sqrt{5}}\left[2\psi_0(x) - \psi_1(x)\right],$$

where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wavefunction of the particle at a time t is

(a) $\frac{1}{\sqrt{5}} e^{\frac{-i(E_0E_1)t}{2\hbar}} \left[2\psi_0(x) - \psi_1(x)\right]$ (b) $\frac{1}{\sqrt{5}} e^{\frac{-iE_0t}{\hbar}} \left[2\psi_0(x) - \psi_1(x)\right]$

(c) $\frac{1}{\sqrt{5}} e^{\frac{-iE_1t}{\hbar}} \left[2\psi_0(x) - \psi_1(x)\right]$ (d) $\frac{1}{\sqrt{5}}\left[2\psi_0(x) e^{\frac{-E_0t}{\hbar}} - \psi_1(x) e^{\frac{-E_1t}{\hbar}}\right]$

40. The commutator $[L_x, y]$, where L_x is the x-component of the angular momentum operator and y is the y-component of the position operator, is equal to
(a) 0 (b) $i\hbar x$ (c) $i\hbar y$ (d) $i\hbar z$

41. In hydro genic states, the probability of finding an electron at $r = 0$ is
(a) zero in state $\phi_{1s}(r)$ (b) non-zero in state $\phi_{1s}(r)$
(c) zero in state $\phi_{2s}(r)$ (d) zero in state $\phi_{2p}(r)$

42. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperatures of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contact. At equilibrium, the change in entropy is

(a) $\frac{3}{2} N k_B \ln\left[\frac{T_1^2 + T_2^2}{4T_1T_2}\right]$ (b) $N k_B \ln\left[\frac{T_2}{T_1}\right]$

(c) $\frac{3}{2} N k_B \ln\left[\frac{(T_1 + T_2)^2}{4T_1T_2}\right]$ (d) $2N k_B$

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43. The internal energy of n moles of a gas is given $E = \frac{3}{2} n\mathrm{RT} - \frac{a}{V}$, where V is the volume of the gas at temperature T and a is a positive constant. One mole of the gas in state $(\mathrm{T}_1, \mathrm{V}_1)$ is allowed to expand adiabatically into vacuum to a final state $(\mathrm{T}_2, \mathrm{V}_2)$. The temperature T_2 is

(a) $\mathrm{T}_1 + \mathrm{R}a\left(\frac{1}{\mathrm{V}_2} + \frac{1}{\mathrm{V}_1}\right)$ (b) $\mathrm{T}_1 - \frac{2}{3}\mathrm{R}a\left(\frac{1}{\mathrm{V}_2} - \frac{1}{\mathrm{V}_1}\right)$

(c) $\mathrm{T}_1 + \frac{2}{3}\mathrm{R}a\left(\frac{1}{\mathrm{V}_2} - \frac{1}{\mathrm{V}_1}\right)$ (d) $\mathrm{T}_1 - \frac{1}{3}\mathrm{R}a\left(\frac{1}{\mathrm{V}_2} - \frac{1}{\mathrm{V}_1}\right)$

44. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is

(a) $\frac{1}{2}k_B T$ (b) $k_B T$ (c) $\frac{3}{2}k_B T$ (d) $3k_B T$

45. A monatomic crystalline solid comprises of N atms, out of which n atoms are in interstitial positions. If the available interstitial sites are N', then number of possible microstates is

(a) $\frac{(\mathrm{N}'+n)!}{n!\mathrm{N}!}$ (b) $\frac{\mathrm{N}!}{n!(\mathrm{N}'+n)!}\frac{\mathrm{N}!}{n!(\mathrm{N}'+n)!}$

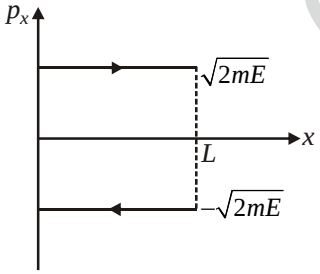
(c) $\frac{\mathrm{N}!}{n!(\mathrm{N}'-n)!}$ (d) $\frac{\mathrm{N}!}{n!(\mathrm{N}'-n)!}\frac{\mathrm{N}!}{n!(\mathrm{N}'-n)!}$

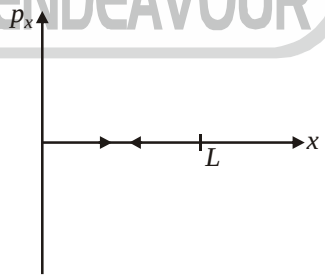
46. A system of N localized, non-interacting spin-$\frac{1}{2}$ ions of magnetic moment μ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the system is

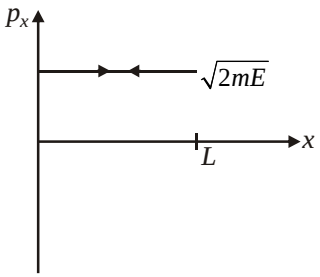
(a) $N k_B T \ln\left(\cosh\frac{\mu H}{k_B T}\right)$ (b) $-N k_B T \ln\left(2\cosh\frac{\mu H}{k_B T}\right)$

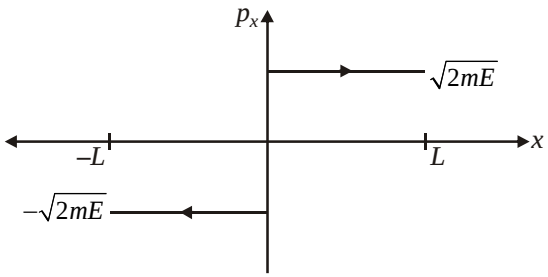
(c) $N k_B T \ln\left(2\cosh\frac{\mu H}{k_B T}\right)$ (d) $-N k_B T \ln\left(2\sinh\frac{\mu H}{k_B T}\right)$

47. The phase diagram of a free particle of mass m kinetic energy E, moving in one-dimensional box with perfectly elastic walls at $x = 0$ and $x = L$, is given by

(a)


(b)


(c)


(d)


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48. In the microwave spectrum of identical rigid diatomic molecules, the separation between the spectral lines is recorded to be 0.7143 cm^{-1}. The moment of inertia of the molecule, in kg m^2, is

(a) 2.3×10^{-36} (b) 2.3×10^{-40} (c) 7.8×10^{-42} (d) 7.8×10^{-46}

49. Which one of the following electronic transitions in Neon is NOT responsible for LASER action in a helium-neon laser ?

(a) $6s \rightarrow 5p$ (b) $5s \rightarrow 4p$ (c) $5s \rightarrow 3p$ (d) $4s \rightarrow 3p$

50. In the linear Stark effect, the application of an electric field

(a) completely lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into four levels

(b) partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into three levels

(c) partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into two levels

(d) does not affect the $n = 2$ levels

51. In hyperfine interaction, there is coupling between the electron angular momentum $\vec{J}$ and nuclear angular momentum $\vec{I}$, forming resultant angular momentum $\vec{F}$. The selection rules for the corresponding quantum number F in hyperfine transitions are

(a) $\Delta F = \pm 2$ only (b) $\Delta F = \pm 1$ only (c) $\Delta F = 0, \pm 1$ (d) $\Delta F = \pm 1, \pm 2$

52. A vibrational electronic spectrum of homonuclear binary molecules, involving electronic ground state ε'' and excited ε', exhibits a continuum at $\bar{\nu}$ cm^{-1}. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by E_{ex} cm^{-1}, then dissociation energy of the molecule in the ground state is

(a) $\frac{(\bar{\nu} + E_{ex})}{2}$ (b) $\frac{(\bar{\nu} - E_{ex})}{2}$ (c) $(\bar{\nu} - E_{ex})$ (d) $\sqrt{(\bar{\nu}^2 - E_{ex}^2)}$

53. The NMR spectrum of ethanol (CH_3 CH_2 OH) comprises of three bunches of spectral lines. The number of spectral lines in the bunch corresponding to CH_2 group is

(a) 1 (b) 2 (c) 3 (d) 4

54. The energy $E(\vec{k})$ of electrons of wavevector $\vec{k}$ in a solid is given by $E(\vec{k}) = Ak^2 + Bk^4$, where A and B are constants. The effective mass of the electron at $|\vec{k}| = k_0$ is

(a) Ak_0^2 (b) $\frac{\hbar^2}{2A}$ (c) $\frac{\hbar^2}{2A + 12Bk_0^2}$ (d) $\frac{\hbar^2}{Bk_0^2}$

55. Which one of the following statements is NOT correct about the Brillouin zones (BZ) of a square lattices with constant a ?

(a) The first BZ is a square of side $2\pi/a$ in $k_x - k_y$ plane

(b) The areas of the first BZ and third BZ are the same

(c) The k-points are equidistant in k_x as well as in k_y directions

(d) The area of the second BZ is twice that of the first BZ

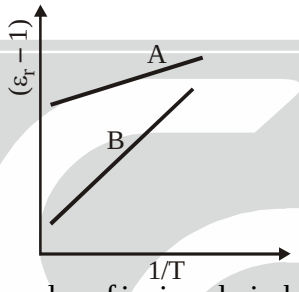
CAREER ENDEAVOUR

56. In a crystal of N primitive cells, each cell contains two monovalent atoms. The highest occupied energy band of the crystal is
(a) one-fourth filled (b) one-third filled (c) half filled (d) completely filled

57. If the number density of a free electron gas changes from 10^{28} to 10^{26} electrons/m^3, the value of plasma frequency (in Hz) changes from 5.7×10^{15} to
(a) 5.7×10^{13} (b) 5.7×10^{14} (c) 5.7×10^{16} (d) 5.7×10^{17}

58. Which one of the following statements about superconductors is NOT true ?
(a) A type I superconductor is completely diamagnetic
(b) A type II superconductor exhibits Meissner effect upto the second critical magnetic field (H_{c_2})
(c) A type II superconductor exhibits zero resistance upto the second critical magnetic field
(d) Both type I and type II superconductors exhibits sharp fall in resistance at the superconducting transition temperature

59. Two dielectric materials A and B exhibit both ionic and orientational polarizabilities. The variation of their susceptibilities $\chi(=\varepsilon_r, -1)$ with temperature T is shown in the figure, where ε_r is the relative dielectric constant. It can be inferred from the figure that



(a) A is more polar and it has a smaller value of ionic polarizability than that of B
(b) A is more polar and it has a higher value of ionic polarizability than that of B
(c) B is more polar and it has a higher value of ionic polarizability than that of A
(d) B is more polar and it has a smaller value of ionic polarizability than that of A

60. The experimentally measured spin *g* factors of proton and a neutron indicate that
(a) Both proton and neutron are elementary point particles
(b) Both proton and neutron are not elementary point particles
(c) While proton is an elementary point particle, neutron is not
(d) While neutron is an elementary point particle, proton is not

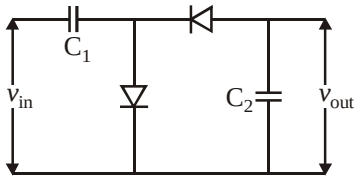
61. By capturing an electron, $^{54}_{25}Mn_{29}$ transform into $^{54}_{25}Cr_{30}$ releasing
(a) a neutrino (b) an antineutrino (c) an α-particle (d) a positron

62. Which one of the following nuclear processes is forbidden ?
(a) $\bar{\nu} + p \rightarrow n + e^+$ (b) $\pi^- \rightarrow e^- + \nu_e + \pi^0$
(c) $\pi^- + p^- \rightarrow n + K^+ + K^-$ (d) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

63. To explain the observed magnetic moment of deuteron (0.8574 μ_N), its ground state wavefunction is taken to be an admixture of S and D states. The expectation values of the *z*-component of the magnetic moment in pure S and pure D states are 0.8797 μ_N and 0.3101 μ_N respectively. The contribution of the D state to the mixed ground state is approximately
(a) 40% (b) 4% (c) 0.4% (d) 0.04%

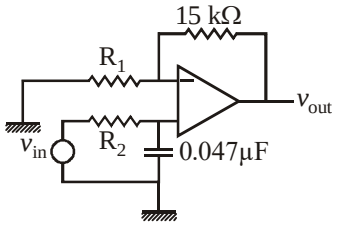
CAREER ENDEAVOUR

64. A sinusoidal input voltage v_{in} of frequency ω is fed to the circuit shown in the figure, where $C_1 \gg C_2$. If v_m is the peak value of the input voltage, then output voltage (v_{out}) is



(a) $2v_m$ (b) $2v_0 \sin \omega t$ (c) $\sqrt{2}v_m$ (d) $\frac{v_m}{2}\sin \omega t$

65. The low-pass active filter shown in the figure has a cut-off frequency of 2 kHz and a pass band gain of 1.5. The values of the resistors are

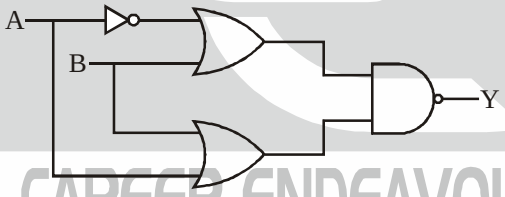


(a) $R_1 = 10\ k\Omega;\ R_2 = 1.3\ \Omega$ (b) $R_1 = 30\ k\Omega;\ R_2 = 1.3\ \Omega$

(c) $R_1 = 10\ k\Omega;\ R_2 = 1.7\ \Omega$ (d) $R_1 = 30\ k\Omega;\ R_2 = 1.7\ \Omega$

66. In order to obtain a solution of the differential equation $\frac{d^2v}{dt^2} - 2\frac{dv}{dt} + v_1 = 0$, involving voltages $v(t)$ and v_1, an operational amplifier (Op-Amp) circuit would require at least
(a) two Op-Amp integrators and one Op-Amp adder
(b) two Op-Amp differentiators and one Op-Amp adder
(c) one Op-Amp integrator and one Op-Amp adder
(d) one Op-Amp integrator, one Op-Amp differentiator and one Op-Amp adder

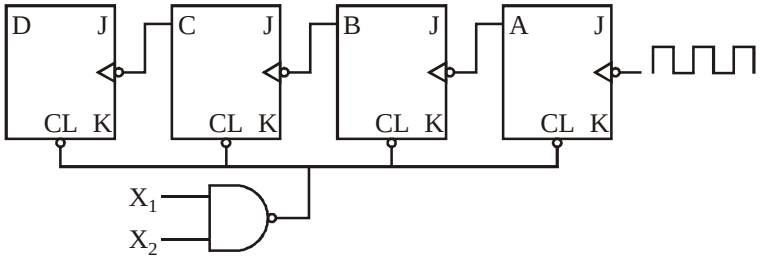
67. In the given digital logic circuit, A and B form the input. The output Y is



(a) $Y = \bar{A}$ (b) $Y = A\bar{B}$ (c) $Y = A \oplus B$ (d) $Y = \bar{B}$

68. The largest analog output voltage from a 6-bit digital to analog converter (DAC) which produces 1.0 V output for a digital input of 010100, is
(a) 1.6 V (b) 2.9 V (c) 3.15 V (d) 5.0 V

69. A ripple counter designed with JK flip-flops provided with CLEAR (CL) input is shown in the figure. In order that this circuit functions as a MOD-12 counter, the NAND gate input (X_1 and X_2) should be



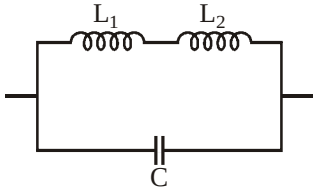
(a) A and C (b) A and D (c) B and D (d) C and D

CAREER ENDEAVOUR

70. The tank circuit of a Hartley oscillator is shown in the figure. If M is the mutual inductance between the inductors, the oscillation frequency is

(a) $\frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}}$ (b) $\frac{1}{2\pi\sqrt{(L_1+L_2-2M)C}}$

(c) $\frac{1}{2\pi\sqrt{(L_1+L_2+M)C}}$ (d) $\frac{1}{2\pi\sqrt{(L_1+L_2-M)C}}$



Common Data for Q.71, Q.72 and Q.73 :

An unperturbed two-level system has energy eigenvalues E_1 and E_2, and eigen functions $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. When perturbed, its Hamiltonian is represented by $\begin{pmatrix}E_1 & A\\ A^* & E_2\end{pmatrix}$

71. The first-order correct to E_1 is

(a) 4A (b) 2A (c) A (d) 0

72. The second-order correction to E_1 is

(a) 0 (b) A (c) $\frac{A^2}{E_2-E_1}$ (d) $\frac{A^2}{E_1-E_2}$

73. The first-order correction to the eigenfunction $\begin{pmatrix}1\\0\end{pmatrix}$ is

(a) $\begin{pmatrix}0\\A^*/(E_1-E_2)\end{pmatrix}$ (b) $\begin{pmatrix}0\\1\end{pmatrix}$ (c) $\begin{pmatrix}A^*/(E_1-E_2)\\0\end{pmatrix}$ (d) $\begin{pmatrix}1\\1\end{pmatrix}$

Common Data for Q.74 and Q.75 :

One of the eigenvalues of the matrix $\begin{pmatrix}2 & 3 & 0\\ 3 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}$ is 5.

CAREER ENDEAVOUR

74. The other two eigenvalues are

(a) 0 and 0 (b) 1 and 1 (c) 1 and –1 (d) –1 and –1

75. The normalized eigenvector corresponding to the eigenvalue 5 is

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$

Linked Answer Questions : Q. 76 to Q. 85 carry two marks each.

Statement for Linked Answer Q.76 and Q.77 :

The powder diffraction pattern of a body centred cubic crystal is recorded by using Cu K_α X-rays of wavelength 1.54 Å.

76. If the (002) planes diffract at 60°, then lattice parameter is

(a) 2.67 Å (b) 3.08 Å (c) 3.56 Å (d) 5.34 Å

77. Assuming the atomic mass of the constituent atoms to be 50.94 amu, then density of the crystal in units of kg m^{-3} is

(a) 3.75×10^3 (b) 4.45×10^3 (c) 5.79×10^3 (d) 8.89×10^3

Statement for Linked Answer Q.78 and Q.79 :

A particle of mass *m* is constrained to move in a vertical plane along a trajectory given by $x = \mathrm{A}\cos\theta$, $y = \mathrm{A}\sin\theta$, where A is a constant.

78. The Lagrangian of the particle is

(a) $\frac{1}{2}m\mathrm{A}^2\dot{\theta}^2 - mg\,\mathrm{A}\cos\theta$ (b) $\frac{1}{2}m\mathrm{A}^2\dot{\theta}^2 - mg\,\mathrm{A}\sin\theta$

(c) $\frac{1}{2}m\mathrm{A}^2\dot{\theta}^2$ (d) $\frac{1}{2}m\mathrm{A}^2\dot{\theta}^2 + mg\,\mathrm{A}\cos\theta$

79. The equation of motion of the particle is

(a) $\ddot{\theta} - \frac{g}{\mathrm{A}}\cos\theta = 0$ (b) $\ddot{\theta} + \frac{g}{\mathrm{A}}\sin\theta = 0$ (c) $\ddot{\theta} = 0$ (d) $\ddot{\theta} - \frac{g}{\mathrm{A}}\sin\theta = 0$

Statement for Linked Answer Q.80 and Q. 81 :

A dielectric sphere of radius R carries polarization $\vec{\mathrm{P}} = kr^2\,\hat{r}$, where *r* is the distance from the centre and *k* is a constant. In the spherical polar coordinate system, $\hat{r}$, $\hat{\theta}$ and $\hat{\varphi}$ are the unit vectors.

80. The bound volume charge density inside the sphere at a distance *r* from the centre is

(a) $-4k\mathrm{R}$ (b) $-4kr$ (c) $-4kr^2$ (d) $-4kr^3$

81. The electric field inside the sphere at a distance *d* from the centre is

(a) $\frac{-kd^2}{\varepsilon_0}\hat{r}$ (b) $\frac{-k\mathrm{R}^2}{\varepsilon_0}\hat{r}$ (c) $\frac{-kd^2}{\varepsilon_0}\hat{\theta}$ (d) $\frac{-k\mathrm{R}^2}{\varepsilon_0}\hat{\theta}$

Statement for Linked Answer Q. 82 and Q. 83 :

Consider Fermi theory of β -decay.

82. The number of final states states of electrons corresponding to momenta between p and $p + dp$ is

(a) independent of p (b) proportional to pdp

(c) proportional to p^2dp (d) proportional to p^3dp

83. The number of emitted electrons with momentum *p* and energy E, in the allowed approximation, is proportional to (E_0 is the total energy given up by the nucleus).

(a) $(\mathrm{E}_0 - \mathrm{E})$ (b) $p(\mathrm{E}_0 - \mathrm{E})$ (c) $p^2(\mathrm{E}_0 - \mathrm{E})^2$ (d) $p(\mathrm{E}_0 - \mathrm{E})^2$

Statement for Linked Answer Question 84 and 85 :

Consider a radiation cavity of volume V at temperature T.

84. The density of states at energy E of the quantized radiation (photons) is

(a) $\frac{8\pi\mathrm{V}}{h^3c^3}\mathrm{E}^2$ (b) $\frac{8\pi\mathrm{V}}{h^3c^3}\mathrm{E}^{3/2}$ (c) $\frac{8\pi\mathrm{V}}{h^3c^3}\mathrm{E}$ (d) $\frac{8\pi\mathrm{V}}{h^3c^3}\mathrm{E}^{1/2}$

85. The average number of photons in equilibrium inside the cavity is proportional to

(a) T (b) T^2 (c) T^3 (d) T^4

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.20 : Carry ONE mark each.

1. The eigenvalues of a matrix are i, $-2i$ and $3i$. The matrix is
   (a) unitary (b) anti-unitary
   (c) Hermitian (d) anti-Hermitian

2. A space station moving in a circular orbit around the Earth goes into a new bound orbit by firing its engine radially outwards. This orbit is
   (a) a larger circle (b) a smaller circle
   (c) an ellipse (d) a parabola

3. A power amplifier gives 150W output for an input of 1.5W. The gain, in dB, is
   (a) 10 (b) 20 (c) 54 (d) 100

4. Four point charges are placed in a plane at the following positions: $+Q$ at (1, 0), $-Q$ at (–1, 0), $+Q$ at (0, 1) and $-Q$ at (0, –1). At large distances the electrostatic potential due to this charge distribution will be dominated by the
   (a) monopole moment (b) dipole moment
   (c) quadrupole moment (d) octopole moment

5. A charged capacitor (C) is connected in series with an inductor (L). When the displacement current reduces to zero, the energy of the LC circuit is
   (a) stored entirely in its magnetic field
   (b) stored entirely in its electric field
   (c) distributed equally among its electric and magnetic fields
   (d) radiated out of the circuit

6. Match the following

P. Frank-Hertz experiment	1. electronic excitation of molecules
Q. Hartree-Fock method	2. wave function of atoms
R. Stern-Gerlach experiment	3. spin angular momentum of atoms
S. Franck-Condon principle	4. energy levels in atoms

(a)	(b)	(c)	(d)
P-4	P-1	P-3	P-4
Q-2	Q-4	Q-2	Q-1
R-3	R-3	R-4	R-3
S-1	S-2	S-1	S-2

7. The wavefunction of a particle, moving in a one-dimensional time-independent potential $V(x)$, is given by $\psi(x) = e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form.
   (a) $V(x) \propto x$ (b) $V(x) \propto x^2$ (c) $V(x) = 0$ (d) $V(x) \propto e^{-ax}$

8. The D_1 and D_2 lines of Na ($3^2P_{1/2} \rightarrow 3^2S_{1/2}$, $3^2P_{3/2} \rightarrow 3^2S_{1/2}$) will split on the application of a weak magnetic field into
   (a) 4 and 6 lines respectively (b) 3 lines each
   (c) 6 and 4 lines respectively (d) 6 lines each

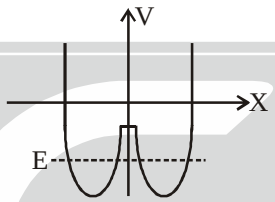
CAREER ENDEAVOUR

9. In a He-Ne laser, the laser transition takes place in
   (a) He only (b) Ne only
   (c) Ne first, then in He (d) He first, then in Ne

10. The partition function of a single gas molecule is Z_α. The partition function of N such non-interacting gas molecules is then given by
   (a) $\frac{(Z_\alpha)^N}{N!}$ (b) $(Z_\alpha)^N$ (c) $N(Z_\alpha)$ (d) $\frac{(Z_\alpha)^N}{N}$

11. A solid superconductor is placed in an external magnetic field and then cooled below its critical temperature. The superconductor
   (a) retains its magnetic flux because the surface current supports it
   (b) expels out its magnetic flux because it behaves like a paramagnetic materials
   (c) expels out its magnetic flux because it behaves like an anti-ferromagnetic material
   (d) expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field

12. A particle with energy E is in a time-independent double well potential as shown in the figure.



   Which of the following statements about the particle is NOT correct?
   (a) The particle will always be in a bound state
   (b) The probability of finding the particle in one well will be time-dependent
   (c) The particle will be confined to any one of the wells
   (d) The particle can tunnel from one well to the other, and back

13. It is necessary to apply quantum statistics to a system of particles if
   (a) there is substantial overlap between the wavefunctions of the particles
   (b) the mean free path of the particles is comparable to the inter-particle separation
   (c) the particles have identical mass and charge
   (d) the particles are interacting

14. When liquid oxygen is poured down close to a strong bar magnet, the oxygen stream is
   (a) repelled towards the lower field because it is diamagnetic
   (b) attracted towards the higher field because it is diamagnetic
   (c) repelled towards the lower field because it is paramagnetic
   (d) attracted towards the higher field because it is paramagnetic

15. Fission fragments are generally radioactive as
   (a) they have excess of neutrons
   (b) they have excess of protons
   (c) they are products of radioactive nuclides
   (d) their total kinetic energy is of the order of 200 MeV

CAREER ENDEAVOUR

16. In a typical *npn* transistor the doping concentrations in emitter, base and collector regions are C_E, C_B and C_C respectively. These satisfy the relation

(a) $C_E > C_C > C_B$ (b) $C_E > C_B > C_C$

(c) $C_C > C_B > C_E$ (d) $C_E = C_C > C_B$

17. The allowed states for He ($2p^2$) configuration are

(a) $^1S_0$, $^3S_1$, $^1P_1$, $^3P_{0,1,2}$, $^1D_2$ and $^3D_{1,2,3}$ (b) $^1S_0$, $^3P_{0,1,2}$ and $^1D_2$

(c) $^1P_1$ and $^3P_{0,1,2}$ (d) $^1S_0$ and $^1P_1$

18. The energy levels of a particle of mass *m* in a potential of the form

$$V(x) = \infty, \; x \le 0$$

$$= \frac{1}{2} m\omega^2 x^2, \; x > 0$$

are given, in terms of quantum number $n = 0,1,2,3....., $ by

(a) $\left(n + \frac{1}{2}\right)\hbar\omega$ (b) $\left(2n + \frac{1}{2}\right)\hbar\omega$

(c) $\left(2n + \frac{3}{2}\right)\hbar\omega$ (d) $\left(n + \frac{3}{2}\right)\hbar\omega$

19. The electromagnetic field due to a point charge must be described by Lienard Weichert potentials when

(a) the point charge is highly accelerated

(b) the electric and magnetic fields are not perpendicular

(c) the point charge is moving with velocity close to that of light

(d) the calculation is done for the radiation zone, i.e. far away from the charge

20. The strangeness quantum number is conserved in

(a) strong, weak and electromagnetic interactions

(b) weak and electromagnetic interactions only

(c) strong and weak interactions only

(d) strong and electromagnetic interactions only

CAREER ENDEAVOUR

Q.21 – Q.75 : Carry TWO marks each.

21. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

22. A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side *L*, with one vertex at the origin and sides along the positive X, Y, and Z axes, is

(a) $2L^3$ (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

23. If $\vec{r} = x\hat{i} + y\hat{j}$, then

(a) $\vec{\nabla}\cdot\vec{r} = 0$ and $\vec{\nabla}\,|\vec{r}\,| = \vec{r}$

(b) $\vec{\nabla}\cdot\vec{r} = 2$ and $\vec{\nabla}\,|\vec{r}\,| = \hat{r}$

(c) $\vec{\nabla}\cdot\vec{r} = 2$ and $\vec{\nabla}\,|\vec{r}\,| = \dfrac{\hat{r}}{r}$

(d) $\vec{\nabla}\cdot\vec{r} = 3$ and $\vec{\nabla}\,|\vec{r}\,| = \dfrac{\hat{r}}{r}$

24. Consider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is

(a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$

(b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$

(d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

25. The contour integral $\oint \dfrac{dz}{z^4 + a^4}$ is to be evaluated on a circle of radius $2a$ centered at the origin. It will have contributions only from the points

(a) $\dfrac{1+i}{\sqrt{2}}a$ and $-\dfrac{1+i}{\sqrt{2}}a$

(b) ia and $-ia$

(c) ia, $-ia$, $\dfrac{1-i}{\sqrt{2}}a$ and $-\dfrac{1-i}{\sqrt{2}}a$

(d) $\dfrac{1+i}{\sqrt{2}}a$, $-\dfrac{1+i}{\sqrt{2}}a$, $\dfrac{1-i}{\sqrt{2}}a$ and $-\dfrac{1-i}{\sqrt{2}}a$

26. Inverse Laplace transform of $\dfrac{s+1}{s^2-4}$ is

(a) $\cos 2x + \dfrac{1}{2}\sin 2x$

(b) $\cos x + \dfrac{1}{2}\sin x$

(c) $\cosh x + \dfrac{1}{2}\sinh x$

(d) $\cosh 2x + \dfrac{1}{2}\sinh 2x$

27. The points, where the series solution of the Legendre differential equation $(1-x^2)\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + \dfrac{3}{2}\left(\dfrac{3}{2}+1\right)y = 0$ will diverge, are located at

(a) 0 and 1 (b) 0 and –1 (c) –1 and 1 (d) $\dfrac{3}{2}$ and $\dfrac{5}{2}$

28. Solution of the differential equation $x\dfrac{dy}{dx} + y = x^4$, with the boundary condition that $y = 1$, at $x = 1$, is

(a) $y = 5x^4 - 4$

(b) $y = \dfrac{x^4}{5} + \dfrac{4x}{5}$

(c) $y = \dfrac{4x^4}{5} + \dfrac{1}{5x}$

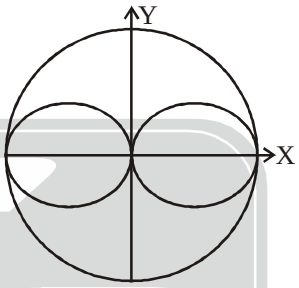
(d) $y = \dfrac{x^4}{5} + \dfrac{4}{5x}$

CAREER ENDEAVOUR

29. Match the following

P. rest mass		1. time like vector	
Q. charge		2. Lorentz invariant	
R. four-momentum		3. tensor of rank 2	
S. electromagnetic field		4. conserved and Lorentz invariant	
(a)	(b)	(c)	(d)
P-2	P-4	P-2	P-4
Q-4	Q-2	Q-4	Q-2
R-3	R-1	R-1	R-3
S-1	S-3	S-3	S-1

30. The moment of inertia of a uniform sphere of radius r about an axis passing through its centre is given by $\frac{2}{5}\left(\frac{4\pi}{3}r^5\rho\right)$. A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radius $R/2$ hollowed out of it, as shown in the figure. The moment of inertia of the resulting body about the Y axis is



(a) $\frac{\pi\rho R^5}{4}$ (b) $\frac{5\pi\rho R^5}{12}$

(c) $\frac{7\pi\rho R^5}{12}$ (d) $\frac{3\pi\rho R^5}{4}$

31. The Lagrangian of a particle of mass m is

$$L=\frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right]-\frac{V}{2}(x^2+y^2)+W\sin\omega t,$$ where V, W and ω are constants.

The conserved quantities are

(a) energy and z-component of linear momentum only

(b) energy and z-component of angular momentum only

(c) z-components of both linear and angular momenta only

(d) energy and z-components of both linear and angular momenta

32. Three particles of mass m each situated at $x_1(t), x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length l. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is

(a) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-l)^2+\frac{k}{2}(x_3-x_2-l)^2$

(b) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_3-l)^2+\frac{k}{2}(x_3-x_2-l)^2$

(c) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2+l)^2-\frac{k}{2}(x_3-x_2+l)^2$

(d) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-l)^2-\frac{k}{2}(x_3-x_2-l)^2$

CAREER ENDEAVOUR

33. The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq,$ where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is

(a) $\frac{m}{2}\left(\frac{dq}{dt} + q\right)^2$ (b) $\frac{m}{2}\left(\frac{dq}{dt} - q\right)^2$

(c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 + q\frac{dq}{dt} - q^2\right]$ (d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 - q\frac{dq}{dt} + q^2\right]$

34. A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permittivity μ_r. The magnitude of magnetic induction $\vec{B}$ inside the toroid, at a radial distance r from the axis, is given by

(a) $\mu_r\mu_0 NIr$ (b) $\frac{\mu_r\mu_0 NI}{r}$

(c) $\frac{\mu_r\mu_0 NI}{2\pi r}$ (d) $2\pi\mu_r\mu_0 NIr$

35. An electromagnetic wave with $\vec{E}(z,t) = E_0\cos(\omega t - kz)\hat{i}$ is travelling in free space and crosses a disc of radius 2m placed perpendicular to the z-axis. If $E_0 = 60Vm^{-1},$ the average power, in Watt, crossing the disc along the z-direction is

(a) 30 (b) 60 (c) 120 (d) 270

36. Can the following scalar and vector potentials describe an electromagnetic field?

$\phi(\vec{x},t) = 3xyz - 4t$

$\vec{A}(\vec{x},t) = (2x - \omega t)\hat{i} + (y - 2z)\hat{j} + (z - 2xe^{iax})\hat{k}$

where ω is a constant

(a) Yes, in the Coulomb gauge (b) Yes, in the Lorentz gauge

(c) Yes, provided $\omega = 0$ (d) No

37. For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2x^2,$ the first excited energy eigenstate is $\psi(x) = xe^{-ax^2}$. The value of a is

(a) $m\omega/4\hbar$ (b) $m\omega/3\hbar$

(c) $m\omega/2\hbar$ (d) $2m\omega/3\hbar$

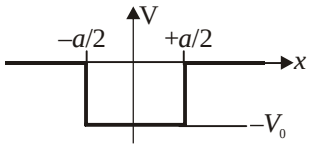
38. If $[x, p] = i\hbar,$ the value of $[x^3, p]$ is

(a) $2i\hbar x^2$ (b) $-2i\hbar x^2$

(c) $3i\hbar x^2$ (d) $-3i\hbar x^2$

CAREER ENDEAVOUR

39. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies



(a) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$ (d) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

40. An atomic state of hydrogen is represented by the following wavefunction:

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where a_0 is a constant. The quantum numbers of the state are

(a) $l=0,\ m=0,\ n=1$ (b) $l=1,\ m=1,\ n=2$

(c) $l=1,\ m=0,\ n=2$ (d) $l=2,\ m=0,\ n=3$

41. Three operators X, Y and Z satisfy the commutation relations

$[X,Y]=i\hbar Z$, $[Y,Z]=i\hbar X$ and $[Z,X]=i\hbar Y$.

The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is

(a) $\{0, \pm1, \pm2, \pm3, \ldots\}$ (b) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\right\}$

(c) $\left\{0, \pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm2, \pm\frac{5}{2}, \ldots\right\}$ (d) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

42. A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22ºC by supplying 450 kJ s^{-1}. If the outside temperature is 0ºC, the heat taken, in kJ s^{-1}, from the outside air is approximately

(a) 487 (b) 470 (c) 467 (d) 417

43. The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by $\ln p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in Kelvin) of the material is

(a) 185 (b) 190 (c) 195 (d) 200

44. The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^4$, where a is a constant. The entropy S and the pressure P of the photon gas are

(a) $S = \frac{4}{3}aVT^3, P = \frac{a}{3}T^4$ (b) $S = \frac{1}{3}aVT^4, P = \frac{4a}{3}T^3$

(c) $S = \frac{4}{3}aVT^4, P = \frac{a}{3}T^3$ (d) $S = \frac{1}{3}aVT^3, P = \frac{4a}{3}T^4$

CAREER ENDEAVOUR

45. A system has energy levels E_0, $2E_0$, $3E_0$,..., where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is

(a) 2 (b) 3 (c) 4 (d) 5

46. In accordance with the selection rules for electric dipole transitions, the 4^3P_1 state of helium can decay by photon emission to the states

(a) 2^1S_0, 2^1P_1 and 3^1D_2 (b) 3^1P_1, 3^1D_2 and 3^1S_0

(c) 3^3P_2, 3^3D_3 and 3^3P_0 (d) 2^3S_1, 3^3D_2 and 3^3D_1

47. If an atom is in the $^3D_3$ state, the angle between its orbital and spin angular momentum vectors ($\vec{L}$ and $\vec{S}$) is

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{2}{\sqrt{3}}$ (c) $\cos^{-1}\frac{1}{2}$ (d) $\cos^{-1}\frac{\sqrt{3}}{2}$

48. The hyperfine structure of Na($3^2P_{3/2}$) with nuclear spin $I = 3/2$ has

(a) 1 state (b) 2 states (c) 3 states (d) 4 states

49. The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_j = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.

In a system of such diatomic molecules of reduced mass μ; some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is

(a) 0.475 B (b) 0.50 B (c) 0.95 B (d) 1.0 B

50. The number of fundamental vibrational modes of CO_2 molecule is

(a) four : 2 are Raman active and 2 are infrared active

(b) four : 1 is Raman active and 3 are infrared active

(c) three : 1 is Raman active and 2 are infrared active

(d) three : 2 are Raman active and 1 is infrared active

51. A piece of paraffin is placed in a uniform magnetic field H_0. The sample contains hydrogen nuclei of mass m_p, which interact only with external magnetic field. An additional oscillating magnetic field is applied to observe resonance absorption. If g_i is the g-factor of the hydrogen nucleus, the frequency, at which resonance absorption takes place, is given by

(a) $\frac{3g_i eH_0}{2\pi m_p}$ (b) $\frac{3g_i eH_0}{4\pi m_p}$ (c) $\frac{g_i eH_0}{2\pi m_p}$ (d) $\frac{g_i eH_0}{4\pi m_p}$

52. The solid phase of an element follows van der Waals bonding with inter-atomic potential $V(r) = -\frac{P}{r^6} + \frac{Q}{r^{12}}$, where P and Q are constants. The bond length can be expressed as

(a) $\left(\frac{2Q}{P}\right)^{-6}$ (b) $\left(\frac{Q}{P}\right)^{-6}$ (c) $\left(\frac{P}{2Q}\right)^{-6}$ (d) $\left(\frac{P}{Q}\right)^{-6}$

CAREER ENDEAVOUR

53. Consider the atomic packing factor (APF) of the following crystal structures:

P. Simple Cubic

Q. Body Centred Cubic

R. Face Centred Cubic

S. Diamond

T. Hexagonal Close Packed

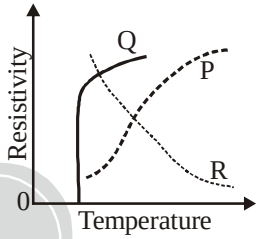
Which two of the above structures have equal APF?

(a) P and Q (b) S and T (c) R and S (d) R and T

54. In a powder diffraction pattern recorded from a face-centred cubic sample using x-rays, the first peak appears at 30°. The second peak will appear at

(a) 32.8° (b) 33.7° (c) 34.8° (d) 35.3°

55. Variation of electrical resistivity ρ with temperature T of three solids is sketched (on different scales) in the figure, as curves P, Q and R.



Which one of the following statements describes the variations most appropriately?

(a) P is for a superconductor, and R for a semiconductor

(b) Q is for a superconductor, and P for a conductor

(c) Q is for a superconductor, and R for a conductor

(d) R is for a superconductor, and P for a conductor

56. An extrinsic semiconductor sample of cross-section A and length L is doped in such a way that the doping concentration varies as $N_D(x) = N_0 \exp\left(-\frac{x}{L}\right)$, where N_0 is a constant. Assume that the mobility μ of the majority carriers remains constant. The resistance R of the sample is given by

(a) $R = \frac{L}{A\mu e N_0}[\exp(1.0) - 1]$ (b) $R = \frac{L}{\mu e N_0}[\exp(1.0) - 1]$

(c) $R = \frac{L}{A\mu e N_0}[\exp(-1.0) - 1]$ (d) $R = \frac{L}{A\mu e N_0}$

CAREER ENDEAVOUR

57. A ferromagnetic mixture of iron and copper having 75% atoms of Fe exhibits a saturation magnetization of 1.3×10^6 Am^{-1}. Assume that the total number of atoms per unit volume is 8×10^{28} m^{-3}. The magnetic moment of an iron atom, in terms of the Bohr Magneton, is

(a) 1.7 (b) 2.3 (c) 2.9 (d) 3.8

58. Half life of a radio-isotope is 4×10^8 years. If there are 10^3 radioactive nuclei in a sample today, the number of such nuclei in the sample 4×10^9 years ago were

(a) 128×10^3 (b) 256×10^3 (c) 512×10^3 (d) 1024×10^3

59. In the deuterium + tritium ($d + t$) fusion more energy is released as compared to deuterium + deuterium ($d + d$) fusion because

(a) tritium is radioactive

(b) more nucleons participate in fusion

(c) the Coulomb barrier is lower for the $d + t$ system than $d + d$ system

(d) the reaction product ^{4}He is more tightly bound

60. According to the shell model the ground state spin of the ^{17}O nucleus is

(a) $\frac{3^+}{2}$ (b) $\frac{5^+}{2}$ (c) $\frac{3^-}{2}$ (d) $\frac{5^-}{2}$

61. A relativistic particle travels a length of 3×10^{-3} m in air before decaying. The decay process of the particle is dominated by

(a) strong interactions (b) electromagnetic interactions
(c) weak interactions (d) gravitational interactions

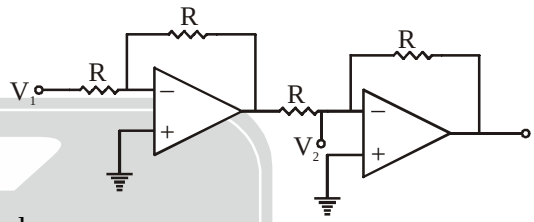
62. The strange baryon Σ^+ has the quark structure

(a) *uds* (b) *uud* (c) *uus* (d) $u\bar{s}$

63. A neutron scatters elastically from a heavy nucleus. The initial and final states of the neutron have the

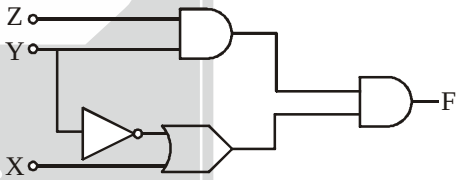
(a) same energy (b) same energy and linear momentum
(c) same energy and angular momentum (d) same linear and angular momenta

64. The circuit shown is based on ideal operational amplifiers. It acts as a



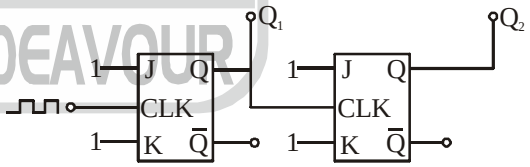
(a) subtractor (b) buffer amplifier
(c) adder (d) divider

65. Identify the function F generated by the logic network shown



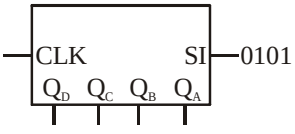
(a) $F = (X + Y)Z$ (b) $F = Z + Y + \bar{Y}X$
(c) $F = ZY(Y + X)$ (d) $F = XYZ$

66. In the circuit shown, the ports Q_1 and Q_2 are in the state $Q_1 = 1$, $Q_2 = 0$. The circuit is now subjected to two complete clock pulses. The state of these ports now becomes



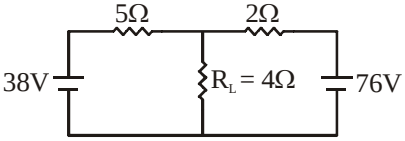
(a) $Q_2 = 1, Q_1 = 0$ (b) $Q_2 = 0, Q_1 = 1$
(c) $Q_2 = 1, Q_1 = 1$ (d) $Q_2 = 0, Q_1 = 0$

67. The registers Q_D, Q_C, Q_B and Q_A shown in the figure are initially in the state 1010 respectively. An input sequence SI = 0101 is applied. After two clock pulses, the state of the shift registers (in the same sequence Q_D Q_C Q_B Q_A) is



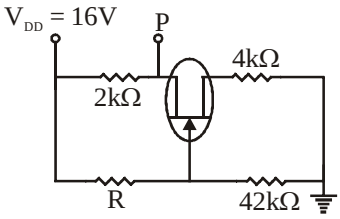
(a) 1001 (b) 0100
(c) 0110 (d) 1010

68. For the circuit shown, the potential difference (in Volts) across R_L is



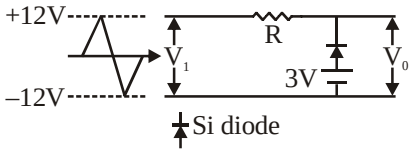
(a) 48 (b) 52
(c) 56 (d) 65

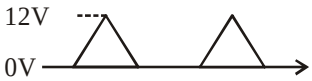
69. In the circuit shown, the voltage at test point P is 12V and the voltage between gate and source is –2V. The value of R (in $k\Omega$) is

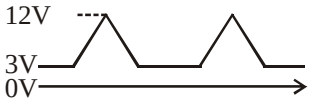


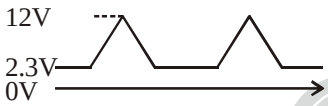
(a) 42 (b) 48
(c) 56 (d) 70

70. When an input voltage V_i, of the form shown, is applied to the circuit given below, the output voltage V_0 is of the form



(a) 

(b) 

(c) 

(d)

Common Data Questions

Common Data for Questions 71, 72, 73:

A particle of mass m is confined in the ground state of a one-dimensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$, where ψ_0 is a constant

71. The normalization factor ψ_0 of this wavefunction is

(a) $\sqrt{\frac{2}{L}}$ (b) $\sqrt{\frac{1}{4L}}$ (c) $\sqrt{\frac{1}{2L}}$ (d) $\sqrt{\frac{1}{L}}$

72. The energy eigenvalue corresponding to this state is

(a) $\frac{\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{4mL^2}$ (c) $\frac{\hbar^2\pi^2}{16mL^2}$ (d) $\frac{\hbar^2\pi^2}{32mL^2}$

73. The expectation value of p^2 (p is the momentum operator) in this state is

(a) 0 (b) $\frac{\hbar^2\pi^2}{32L^2}$ (c) $\frac{\hbar^2\pi^2}{16L^2}$ (d) $\frac{\hbar^2\pi^2}{8L^2}$

Common Data for Questions 74, 75:

The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index μ, are

$$E_{\parallel}^{reflected} = \frac{\cos r - \mu \cos i}{\cos r + \mu \cos i} E_{\parallel}^{incident} \quad \text{and} \quad E_{\perp}^{reflected} = \frac{\mu \cos r - \cos i}{\mu \cos r + \cos i} E_{\perp}^{incident}$$

The subscripts $\parallel$ and $\perp$ refer to polarization, parallel and normal to the plane of incidence respectively. Here, i and r are the angles of incidence and refraction respectively

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74. The coordination for the reflected ray to be completely polarized is

(a) $\mu \cos i = \cos r$ (b) $\cos i = \mu \cos r$

(c) $\mu \cos i = -\cos r$ (d) $\cos i = -\mu \cos r$

75. For normal incidence at an air-glass interface with $\mu = 1.5$ the fraction of energy reflected is given by

(a) 0.40 (b) 0.20 (c) 0.16 (d) 0.04

Linked Answer Questions : Q. 76 to Q.85 carry two marks each.

Statement for Linked Answer Questions 76 & 77:

In the laboratory frame, a particle P of rest mass m_0 is moving in the positive *x* direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q, moving in the negative *x* direction with a speed of $\frac{2c}{5}$.

76. The speed of the particle P in the rest frame of the particle Q is

(a) $\frac{7c}{95}$ (b) $\frac{13c}{85}$ (c) $\frac{3c}{5}$ (d) $\frac{63c}{95}$

77. The energy of the particle P in the rest frame of the particle Q is

(a) $\frac{1}{2}m_0c^2$ (b) $\frac{5}{4}m_0c^2$ (c) $\frac{19}{13}m_0c^2$ (d) $\frac{11}{9}m_0c^2$

Statement for Linked Answer Questions 78 & 79

The atomic density of a solid is 5.85×10^{28} m^{-3}. Its electrical resistivity is 1.6×10^{-8} Ω m. Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

78. The drift mobility (in m^2 V^{-1} s^{-1}) of the conduction electrons is

(a) 6.67×10^{-3} (b) 6.67×10^{-6} (c) 7.63×10^{-3} (d) 7.63×10^{-6}

79. The relaxation time (mean free time), in seconds, of the conduction electrons is

(a) 3.98×10^{-15} (b) 3.79×10^{-14} (c) 2.84×10^{-12} (d) 2.64×10^{-11}

Statement for Linked Answer Questions 80 & 81:

A sphere of radius *R* carries a polarization $\vec{P} = k\vec{r}$, where *k* is a constant and $\vec{r}$ is measured from the centre of the sphere.

80. The bound surface and volume charge densities are given, respectively, by

(a) $-k|\vec{r}|$ and $3k$ (b) $k|\vec{r}|$ and $-3k$

(c) $k|\vec{r}|$ and $-4\pi kR$ (d) $-k|\vec{r}|$ and $4\pi kR$

81. The electric field $\vec{E}$ at a point $\vec{r}$ outside the sphere is given by

(a) $\vec{E} = 0$ (b) $\vec{E} = \frac{kR(R^2 - r^2)}{\varepsilon_0 r^3}\hat{r}$

(c) $\vec{E} = \frac{kR(R^2 - r^2)}{\varepsilon_0 r^5}\hat{r}$ (d) $\vec{E} = \frac{3k(r - R)}{4\pi\varepsilon_0 r^4}\hat{r}$

CAREER ENDEAVOUR

Statement for Linked Answer Questions 82 & 83:

An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = 1/k_B\beta$

82. The partition function of a single oscillator with energy levels $(n+\frac{1}{2})\hbar\omega$ is given by

(a) $Z = \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$ (b) $Z = \frac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$

(c) $Z = \frac{1}{1-e^{-\beta\hbar\omega}}$ (d) $Z = \frac{1}{1+e^{-\beta\hbar\omega}}$

83. The average number of energy quanta of the oscillations is given by

(a) $< n >= \frac{1}{e^{\beta\hbar\omega}-1}$ (b) $< n >= \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$

(c) $< n >= \frac{1}{e^{\beta\hbar\omega}+1}$ (d) $< n >= \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

Statement for Linked Answer Questions 84 & 85:

A 16 μA beam of alpha particles, having cross-sectional area 10^{-4} m^2, is incident on a rhodium target of thickness 1 μm. This produces neutrons through the reaction

$\alpha + {}^{100}Rh \rightarrow {}^{101}Pd + 3n$

84. The number of alpha particles hitting the target per second is

(a) 0.5×10^{14} (b) 1.0×10^{14}

(c) 2.0×10^{20} (d) 4.0×10^{20}

85. The neutrons are observed at the rate of 1.806×10^8 s^{-1}. If the density of rhodium is approximated as 10^4 kg m^{-3} the cross-section for the reaction (in barns) is

(a) 0.1 (b) 0.2 (c) 0.4 (d) 0.8

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.20 : Carry ONE mark each.

1. For arbitrary matrices E, F, G and H, if $EF - FE = 0$ then $Trace(EFGH)$ is equal to

(a) $Trace(HGFE)$ (b) $Trace(E).Trace(F).Trace(G).Trace(H)$

(c) $Trace(GFEH)$ (d) $Trace(EGHF)$

2. An unitary matrix $\begin{pmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{pmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is

(a) $\begin{pmatrix} ae^{i\alpha} & -ce^{j\beta} \\ b & d \end{pmatrix}$ (b) $\begin{pmatrix} ae^{i\alpha} & ce^{j\beta} \\ b & d \end{pmatrix}$ (c) $\begin{pmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{pmatrix}$ (d) $\begin{pmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{pmatrix}$

3. The curl of a vector field $\vec{F}$ is $2\hat{x}$. Identify the appropriate vector field $\vec{F}$ from the choices given below

(a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$

(c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2\hat{x} + 5y\hat{z}$

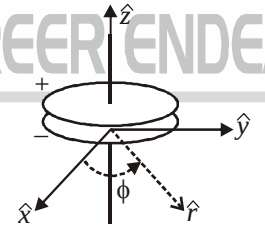
4. A rigid body is rotating about its centre of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is $\vec{L}$, the rate of change of its kinetic energy is

(a) $\frac{1}{2}\vec{\tau}.\vec{\omega}$ (b) $\frac{1}{2}\vec{L}.\vec{\omega}$ (c) $\frac{1}{2}(\vec{\tau}.\vec{\omega} + \vec{L}.\vec{\alpha})$ (d) $\frac{1}{2}\vec{L}.\vec{\alpha}$

5. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ. The number of generalized coordinates required to describe the motion of this system is

(a) 1 (b) 2 (c) 4 (d) 6

6. A parallel plate capacitor is being discharged. What is the direction of the energy flow in terms of the Poynting vector in the space between the plates?



(a) Along the wire in the positive z-axis (b) Radially inward ($-\hat{r}$)

(c) Radially outward ($\hat{r}$) (d) Circumferential (ϕ)

7. Unpolarized light falls from air to a planar air-glass interface (refractive index of glass is 1.5) and the reflected light is observed to be plane polarized. The polarization vector and the angle of incidence θ_i are

(a) perpendicular to the plane of incidence and $\theta_i = 42^o$

(b) parallel to the plane of incidence and $\theta_i = 56^o$

(c) perpendicular to the plane of incidence and $\theta_i = 56^o$

(d) parallel to the plane of incidence and $\theta_i = 42^o$

CAREER ENDEAVOUR

8. A finite wave train, of an unspecified nature, propagates along the positive *x*-axis with a constant speed v and without any change of shape. The differential equation among the four listed below.

(a) $\left(\frac{\partial^2}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t)=0$ (b) $\left(\nabla^2-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t)=0$

(c) $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}-i\hbar\frac{\partial^2}{\partial t}\right)\psi(x,t)=0$ (d) $\left(\nabla^2+a\frac{\partial}{\partial t}\right)\psi(\vec{r},t)=0$

9. Let $|\psi_0\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below:

(a) $[L_x,L_y]|\psi_0\rangle=0$ (b) $J^2|\psi_0\rangle=0$

(c) $\vec{L}.\vec{S}|\psi_0\rangle\neq 0$ (d) $[S_x,S_y]|\psi_0\rangle=0$

10. Thermodynamic variables of a system can be volume *V*, pressure *P*, temperature *T*, number of particles *N*, internal energy *E* and chemical potential μ, etc. For a system to be specified by Microcanonical (MC), Canonical (CE) and Grand Canonical (GC) ensembles, the parameters required for the respective ensembles are:

(a) $MC:(N,V,T);CE:(E,V,N);GC:(V,T,\mu)$ (b) $MC:(E,V,N);CE:(N,V,T);GC:(V,T,\mu)$

(c) $MC:(V,T,\mu);CE:(N,V,T);GC:(E,V,N)$ (d) $MC:(E,V,N);CE:(V,T,\mu);GC:(N,V,T)$

11. The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in the region. The conclusion one can get from the above is

(a) the entropy of solid is zero in this temperature region

(b) the entropy increases when the system goes from liquid to solid phase in this temperature region

(c) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature

(d) the change in entropy is zero in the liquid-to-solid transition region

12. The radial wave function of the electrons in the state of $n=1$ and $l=0$ in a hydrogen atom is $R_{10}=\frac{2}{a_0^{3/2}}\exp\left(-\frac{r}{a_0}\right)$, a_0 is the Bohr radius. The most probable value of *r* for an electron is

(a) a_0 (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

13. The last two terms of the electronic configuration of manganese (Mn) atom is $3d^5 4s^2$. The term factor of Mn^{4+} ion is

(a) $^4D_{1/2}$ (b) $^4F_{3/2}$ (c) $^3F_{9/2}$ (d) $^3D_{7/2}$

14. The coherence length of laser light is

(a) directly proportional to the length of the active lasing medium

(b) directly proportional to the width of the spectral line

(c) inversely proportional to the width of the spectral line

(d) inversely proportional to the length of the active lasing medium

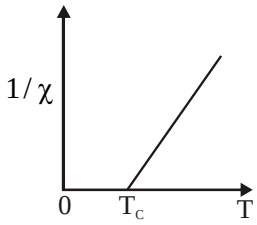
CAREER ENDEAVOUR

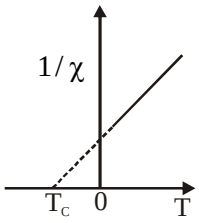
15. Metallic monovalent sodium crystallizes in body centered cubic structure. If the length of the unit cell is 4×10^{-8} cm, the concentration of conduction electrons in metallic sodium is

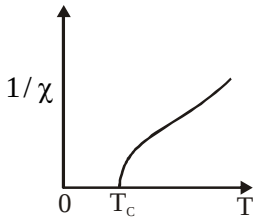
(a) 6.022×10^{23} cm^{-3} (b) 3.125×10^{22} cm^{-3}

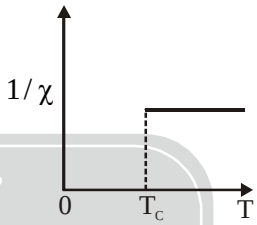
(c) 2.562×10^{21} cm^{-3} (d) 1.250×10^{20} cm^{-3}

16. The plot of inverse magnetic susceptibility $1/\chi$ versus temperature T of an antiferromagnetic sample corresponds to

(a) 

(b) 

(c) 

(d) 

17. According to the quark model, the K^+ meson is composed of the following quarks:

(a) $u\ u\ d$ (b) $u\,\vec{c}$ (c) $u\,\vec{s}$ (d) $s\,\vec{u}$

18. An O^{16} nucleus is spherical and has a charge radius R and a volume $V \equiv \frac{4}{3}\pi R^3$. According to the empirical observations of the charge radii, the volume of the $_{54}Xe^{128}$ nucleus, assumed to be spherical, is

(a) 8V (b) 2V (c) 6.75V (d) 1.89V

19. A common emitter transistor amplifier circuit is operated under a fixed bias. In this circuit, the operating point.

(a) remains fixed with an increase in temperature

(b) moves towards cut-off region with an increase in temperature

(c) moves towards the saturation region with a decrease in temperature

(d) moves towards the saturation region with an increase in temperature

20. Under normal operating conditions, the gate terminal of an *n*-channel junction field effect transistor (JFET) and an *n*-channel metal oxide semiconductor field effect transistor (MOSFET) are

(a) both biased with positive potentials

(b) both biased with negative potentials

(c) biased with positive and negative potentials, respectively

(d) biased with negative and positive potentials, respectively

CAREER ENDEAVOUR

Q.21 – Q.75 : Carry TWO marks each.

21. The eigenvalues of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ are

(a) $\frac{1}{2}(\sqrt{3} \pm i)$ when $\theta = 45^o$

(b) $\frac{1}{2}(\sqrt{3} \pm i)$ when $\theta = 30^o$

(c) ± 1 since the matrix is unitary

(d) $\frac{1}{\sqrt{2}}(1 \pm i)$ when $\theta = 30^o$

22. If the Fourier transform $F[\delta(x-a)] = \exp(-i2\pi va)$, then $F^{-1}(\cos 2\pi av)$ will correspond to

(a) $\delta(x-a) - \delta(x+a)$

(b) a constant

(c) $\frac{1}{2}[\delta(x-a) + i\delta(x+a)]$

(d) $\frac{1}{2}[\delta(x-a) + \delta(x+a)]$

23. If $I \equiv \oint_C dz Ln(z)$, where C is the unit circle taken anticlockwise and $Ln(z)$ is the principal branch of the Logarithm function, which one of the following is correct?

(a) $I = 0$ by residue theorem

(b) I is not defined since $Ln(z)$ has a branch cut

(c) $I \neq 0$

(d) $\oint_C dz Ln(z^2) = 2I$

24. The value of $\int_{-i}^{i} \pi(z+1)dz$ is

(a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) $(-1+2i)\pi$

25. Consider the Bessel equation $(\nu = 0), \frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} + y = 0$. Which one of the following statements is correct?

(a) Equation has regular singular points at $z = 0$ and $z = \infty$

(b) Equation has 2 linearly independent solutions that are entire

(c) Equation has an entire solution and a second linearly independent solution singular at $z = 0$

(d) Limit $z \to \infty$, taken along x-axis, exists for both the linearly independent solutions

26. Under a certain rotation of coordinate axes, a rank-1 tensor $v_a (a = 1, 2, 3)$ transforms according to the orthogonal transformation defined by the relations $v_1' = \frac{1}{\sqrt{2}}(v_1 + v_2)$; $v_2' = \frac{1}{\sqrt{2}}(-v_1 + v_2)$; $v_3' = v_3$. Under the same rotation a rank-2 tensor $T_{a,b}$ would transform such that

(a) $T_{1,1}' = T_{1,1}T_{1,2}$

(b) $T_{1,1}' = T_{1,1}$

(c) $T_{1,1}' = T_{1,1} + 2T_{2,2} - T_{2,3}$

(d) $T_{1,1}' = \frac{1}{2}(T_{1,1} + T_{2,2} + T_{1,2} + T_{2,1})$

CAREER ENDEAVOUR

27. The Lagrangian of a system is given by $L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$. It describes the motion of

(a) a harmonic oscillator
(b) a damped harmonic oscillator
(c) an anharmonic oscillator
(d) a system with unbounded motion

28. The moment of inertia tensor of a rigid body is given by $I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$. The magnitude of the moment of inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is

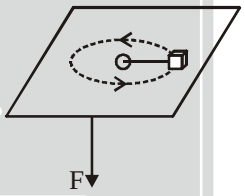
(a) 6 (b) 5 (c) 2 (d) 8/3

29. A hoop of radius R is pivoted at a point on the circumference. The period of small oscillations in the plane of the hoop is

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{4g}}$ (c) $2\pi\sqrt{\frac{R}{g}}$ (d) $2\pi\sqrt{\frac{9R}{7g}}$

30. A mass *m* is constrained to move on a horizontal frictionless surface. It is set in circular motion with radius r_0 and angular speed ω_0 by an applied force $\vec{F}$ communicated through an inextensible thread that passes through a hole on the surface as shown in the figure. This force is then suddenly doubled. The magnitude of the radial velocity of the mass

(a) increases till the mass falls into the hole
(b) decreases till the mass falls into the hole
(c) remains constant
(d) becomes zero at a radius r_1 where $0 < r_1 < r_0$



31. For a simple harmonic oscillator the Lagrangian is given by $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$. If $A(p,q) = \frac{p+iq}{\sqrt{2}}$ and $H(p,q)$ is the Hamiltonian of the system, the Poisson bracket $\{A(p,q), H(p,q)\}$ is given by

(a) $iA(p,q)$ (b) $A^*(p,q)$ (c) $-iA^*(p,q)$ (d) $-iA(p,q)$

32. A plane electromagnetic wave is given by $E_0(\hat{x} + e^{i\delta}\hat{y})\exp\{i(kz - \omega t)\}$. At a given location, the number of times $\vec{E}$ vanishes in one second is

(a) An integer near $\frac{\omega}{\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi, n$ is integer

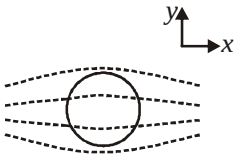
(b) An integer near $\frac{\omega}{\pi}$ and is independent of δ

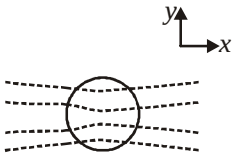
(c) An integer near $\frac{\omega}{2\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi, n$ is integer

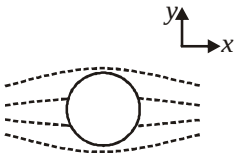
(d) An integer near $\frac{\omega}{2\pi}$ and is independent of δ

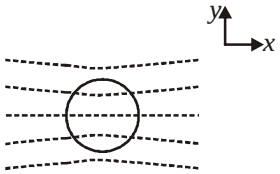
CAREER ENDEAVOUR

33. A dielectric sphere is placed in a uniform electric field directed along the positive *y*-axis. Which one of the following represents the correct equipotential surfaces?

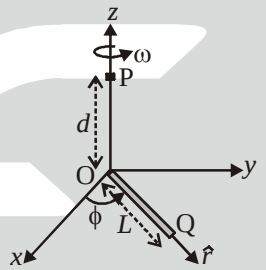
(a) 

(b) 

(c) 

(d) 

34. A rod of length L with uniform charge density λ per unit length is in the *xy*-plane and rotating about *z*-axis passing through one of its edge with an angular velocity $\vec{\omega}$ as shown in the figure below. $(\hat{r}, \hat{\phi}, \hat{z})$ refer to the unit vectors at *Q*, $\vec{A}$ is the vector potential at a distance *d* from the origin O along z-axis for $d >> L$ and $\vec{J}$ is the current density due to the motion of the road. Which one of the following statements is correct?



(a) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \frac{1}{d}$

(b) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d^2}$

(c) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \frac{1}{d^2}$

(d) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d}$

35. A circular disc of radius *a* on the *xy* plane has a surface charge density $\sigma = \frac{\sigma_0 r \cos\theta}{a}$. The electric dipole moment of this charge distribution is

(a) $\frac{\sigma_0 \pi a^4}{4}\hat{x}$ (b) $\frac{\sigma_0 \pi a^3}{4}\hat{x}$ (c) $-\frac{\sigma_0 \pi a^3}{4}\hat{x}$ (d) $-\frac{\sigma_0 \pi a^4}{4}\hat{x}$

36. At time $t = 0$, a charge distribution $\rho(\vec{r}, 0)$ exists within an ideal homogeneous conductor of permittivity ε and conductivity σ. At a later time $\rho(\vec{r}, t)$ is given by

(a) $\rho(\vec{r}, t) = \rho(\vec{r}, 0)\exp\left(-\frac{\sigma t}{\varepsilon}\right)$ (b) $\rho(\vec{r}, t) = \frac{\rho(\vec{r}, 0)}{1 + (\sigma t / \varepsilon)^2}$

(c) $\rho(\vec{r}, t) = \rho(\vec{r}, 0)\exp\left[-\left(\frac{\sigma t}{\varepsilon}\right)^2\right]$ (d) $\rho(\vec{r}, t) = \rho(\vec{r}, 0)\frac{\varepsilon}{\sigma t}\sin\left(\frac{\sigma t}{\varepsilon}\right)$

CAREER ENDEAVOUR

37. A nonrelativistic charged particle moves along the positive *x*-axis with a constant positive acceleration $a\hat{x}$. The particle is at the origin at $t = 0$. Radiation is observed at $t = 0$ at a distant point $(0, d, 0)$ on the *y*-axis. Which one of the following statements is correct?

(a) The radiation is unpolarized

(b) The radiation is plane polarized with polarization parallel to the *x*-axis

(c) The radiation is plane polarized with polarization parallel to the *xy* plane along a line inclined to the *x*-axis

(d) The radiation is elliptically polarized

38. For a physical system, two observer O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below:

(a) Every eigenstate of O_1 must necessarily be an eigenstate of O_2

(b) Every non-degenerate eigenstate of O_1 must necessarily be an eigenstate of O_2

(c) When an observation of O_1 is carried out on an arbitrary state $|\psi\rangle$ of the physical system, a subsequent observation of O_2 leads to an unambiguous result

(d) Observation of O_1 and O_2, carried out on an arbitrary state $|\psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observations are made.

39. An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x = x_0$. [The SHO has energy levels E_n (n = 0, 1, 2...) and associated normalized wave-functions ψ_0]. Subsequently, an exact measurement of energy E is made. Using the general notation $\Pr(E = E')$ denoting the probability that a result E' is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

(a) $\Pr(E = E_0) = 0$

(b) $\Pr(E = E_n) = 1$ for some value of n

(c) $\Pr(E = E_n) \propto \psi_0(x)$

(d) $\Pr(E > E'') > 0$ for any E''

40. Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let I denote the quantum number associated with the total angular momentum and let $\langle \mu_j \rangle$ denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B is Bohr magneton)?

(a) $I = 0, \langle \mu_j \rangle = 0$

(b) $I = \frac{1}{2}, \langle \mu_j \rangle = 1\mu_B$

(c) $I = 1, \langle \mu_j \rangle \approx 1\mu_B$

(d) $I = 0, \langle \mu_j \rangle = 2\mu_B$

41. A particle is placed in a one dimensional box of size L along the *x*-axis $(0 < x < L)$. Which of the following is true?

(a) In the ground state, the probability of finding the particle in the interval $(L/4, 3L/4)$ is half.

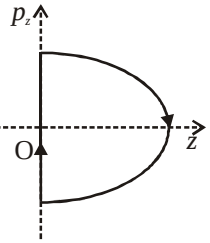
(b) In the first excited state, the probability of finding the particle in the interval $(L/4, 3L/4)$ is half. This also holds for states with $n = 4, 6, 8, ...$

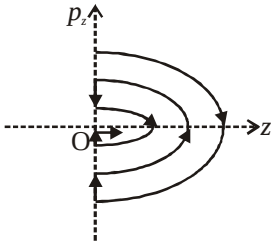
(c) For an arbitrary state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half

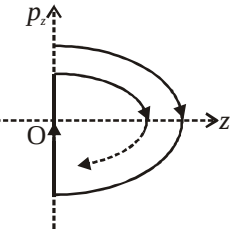
(d) In the ground state, the particle has a definite momentum.

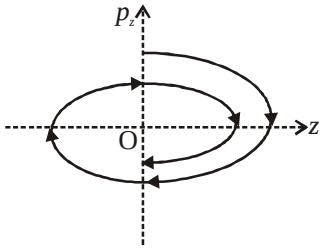
CAREER ENDEAVOUR

42. An inelastic ball of mass *m* has been thrown vertically upwards from the ground at $z = 0$. The initial kinetic energy of the ball is E. The phase trajectory of the ball after successive bouncing on the ground is

(a) 

(b) 

(c) 

(d) 

43. A system containing *N* non-interacting localized particles of spin $\frac{1}{2}$ and magnetic moment μ each is kept in constant external magnetic field B and in thermal equilibrium at temperature *T*. The magnetization of the system is,

(a) $N\mu \coth\left(\frac{\mu B}{k_B T}\right)$

(b) $N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$

(c) $N\mu \sinh\left(\frac{\mu B}{k_B T}\right)$

(d) $N\mu \cosh\left(\frac{\mu B}{k_B T}\right)$

44. Two identical particles have to be distributed among three energy levels. Let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given energy state. The subscripts B, F and C correspond to whether the particles are bosons, fermions and classical particles, respectively. Then, $r_B : r_F : r_C$ is equal to

(a) $\frac{1}{2} : 0 : 1$ (b) $1 : \frac{1}{2} : 1$ (c) $1 : \frac{1}{2} : \frac{1}{2}$ (d) $1 : 0 : \frac{1}{2}$

45. A photon gas is at thermal equilibrium at temperature *T*. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is

(a) $\exp\left(\frac{\hbar\omega}{k_B T}\right) + 1$

(b) $\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1$

(c) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right) + 1\right)^{-1}$

(d) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1\right)^{-1}$

CAREER ENDEAVOUR

46. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium.

(a) the total entropy of the final system is the same as the sum of the entropy of the individual system always

(b) the entropy of mixing is $2Nk_B \ln 2$

(c) the entropy of the final system is less than that of sum of the initial entropies of the two gases

(d) the entropy of mixing is non-zero when the atoms A and B are of the same type.

47. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0,$ and ε and 2ε having degeneracies $g(E) = 1, 2$ and 4 respectively. The mean energy of the system is

(a) $\varepsilon\left(\dfrac{4\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$

(b) $\varepsilon\left(\dfrac{2\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$

(c) $\varepsilon\left(\dfrac{2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)^2$

(d) $\varepsilon\left(\dfrac{\exp\left(-\varepsilon/k_BT\right)+2\exp\left(-2\varepsilon/k_BT\right)}{1+\exp\left(-\varepsilon/k_BT\right)+\exp\left(-2\varepsilon/k_BT\right)}\right)$

48. Three consecutive absorption lines at 64.275 cm^{-1}, 77.130 cm^{-1} and 89.985 cm^{-1} have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia I_A and I_B are (I_A is with respect to the bond axis passing through the centre of mass and I_B is with respect to an axis passing through the centre of mass and perpendicular to bond axis).

(a) both equal to $\dfrac{\hbar^2}{12.855hc}$ gm cm^2

(b) zero and $\dfrac{\hbar^2}{12.855hc}$ gm cm^2

(c) both equal to $\dfrac{\hbar^2}{6.427hc}$ gm cm^2

(d) zero and $\dfrac{\hbar^2}{6.427hc}$ gm cm^2

49. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency ν_c. The frequency of two consecutive Stokes lines are

(a) $\nu_c - 10B, \nu_c - 14B$

(b) $\nu_c - 2B, \nu_c - 4B$

(c) $\nu_c + 10B, \nu_c + 14B$

(d) $\nu_c + 2B, \nu_c + 4B$

50. Which one of the following statement is **INCORRECT** in vibrational spectroscopy with anharmonicity?

(a) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, ...$

(b) Anharmonicity leads to multiple absorption lines

(c) The intensities of hot band lines are stronger than the fundamental absorption

(d) The frequencies of hot band lines are smaller than the fundamental absorption

51. The molecular spectra of two linear molecules O-C-O and O-C-S are recorded in the microwave region. Which one of the following statement is correct?

(a) Both the molecules would show absorption lines

(b) Both the molecules would not show absorption lines

(c) O-C-O would show absorption lines, but not O-C-S

(d) O-C-S would show absorption lines, but not O-C-O

CAREER ENDEAVOUR

52. When the refractive index μ of the active medium changes by $\Delta\mu$ in a laser resonator of length L, the change in the spectral spacing between the longitudinal modes of the laser is (c is the speed of light in free space)

(a) $\dfrac{c}{2(\mu+\Delta\mu)L}$ (b) $\dfrac{c}{2\Delta\mu L}$ (c) $\dfrac{c}{2L}\left[\dfrac{\Delta\mu}{\mu(\mu+\Delta\mu)}\right]$ (d) zero

53. The primitive translation vectors of the body centered cubic lattice are $\vec{a}=\dfrac{a}{2}(\hat{x}+\hat{y}-\hat{z})$, $\vec{b}=\dfrac{a}{2}(-\hat{x}+\hat{y}+\hat{z})$ and $\vec{c}=\dfrac{a}{2}(\hat{x}-\hat{y}+\hat{z})$. The primitive translation vectors $\vec{A},\vec{B}$ and $\vec{C}$ of the reciprocal lattice are

(a) $\vec{A}=\dfrac{2\pi}{a}(\hat{x}-\hat{y});\ \vec{B}=\dfrac{2\pi}{a}(\hat{y}+\hat{z});\vec{C}=\dfrac{2\pi}{a}(\hat{x}+\hat{z})$

(b) $\vec{A}=\dfrac{2\pi}{a}(\hat{x}+\hat{y});\ \vec{B}=\dfrac{2\pi}{a}(\hat{y}-\hat{z});\vec{C}=\dfrac{2\pi}{a}(\hat{x}+\hat{z})$

(c) $\vec{A}=\dfrac{2\pi}{a}(\hat{x}+\hat{y});\ \vec{B}=\dfrac{2\pi}{a}(\hat{y}+\hat{z});\vec{C}=\dfrac{2\pi}{a}(\hat{x}-\hat{z})$

(d) $\vec{A}=\dfrac{2\pi}{a}(\hat{x}+\hat{y});\ \vec{B}=\dfrac{2\pi}{a}(\hat{y}+\hat{z});\vec{C}=\dfrac{2\pi}{a}(\hat{x}+\hat{z})$

54. The structure factor of a single cell of identical atoms of form factor f is given by $S_{hkl}=f\sum_{f}\exp(-i2\pi(x_jh+y_jk+z_jl))$ where (x_j,y_j,z_j) is the coordinate of an atom, and hkl are the Miller indices. Which one of the following statement is correct for the diffraction peaks of body centered cubic (BCC) and face centered cubic (FCC) lattices?

(a) BCC : (200); (110); (222)
FCC : (111); (311); (400)

(b) BCC : (210); (110); (222)
FCC : (111); (311); (400)

(c) BCC : (200); (110); (222)
FCC : (111); (211); (400)

(d) BCC : (200); (210); (222)
FCC : (111); (211); (400)

55. The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model, Einstein model and Debye model. At low temperature $\hbar\omega >> k_BT$, which one of the following statements is true (a and A are constants)

(a) Dulong Petit : $C\propto\exp(-a/T)$; Einstein : C = constant; Debye: $C\propto\left(\dfrac{T}{A}\right)^3$

(b) Dulong Petit : C = constant; Einstein : $C\propto\left(\dfrac{T}{A}\right)^3$; Debye: $C\propto\exp(-a/T)$

(c) Dulong Petit : C = constant; Einstein : $C\propto\dfrac{e^{-a/T}}{T^2}$; Debye: $C\propto\left(\dfrac{T}{A}\right)^3$

(d) Dulong Petit : $C\propto\left(\dfrac{T}{A}\right)^3$; Einstein : $C\propto\dfrac{e^{-a/T}}{T^2}$; Debye: C = constant

CAREER ENDEAVOUR

56. A linear diatomic lattice of lattice constant a with masses M and $m(M > m)$ are coupled by a force constant C. The dispersion relation is given by

$$\omega_2^2 = C\left(\frac{M+m}{Mm}\right) \pm \left[C^2\left(\frac{M+m}{Mm}\right)^2 - \frac{4C^2}{Mm}\sin^2\frac{ka}{2}\right]^{1/2}$$

Which one of the following statements is **INCORRECT**?

(a) The atoms vibrating in transverse mode correspond to the optical branch

(b) The maximum frequency of the acoustic branch depends on the mass of the lighter atom m

(c) The dispersion of frequency in the optical branch is smaller than that in the acoustic branch

(d) No normal modes exist in the acoustic branch for any frequency greater than the maximum frequency at $k = \pi / a$

57. The kinetic energy of a free electron at a corner of the first Brillouin zone of a two dimensional square lattice is larger than that of an electron at the mid-point of a side of the zone by a factor b. The value of b is

(a) $b = \sqrt{2}$ (b) $b = 2$ (c) $b = 4$ (d) $b = 8$

58. An intrinsic semiconductor with mass of a hole m_h and mass of an electron m_e is at a finite temperature T. If the top of the valence band energy is E_v and the bottom of the conduction band energy is E_c, the Fermi energy of the semiconductor is

(a) $E_F = \left(\frac{E_v + E_c}{2}\right) - \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$ (b) $E_F = \left(\frac{k_B T}{2}\right) + \frac{3}{4}(E_v + E_c)\ln\left(\frac{m_h}{m_e}\right)$

(c) $E_F = \left(\frac{E_v + E_c}{2}\right) + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$ (d) $E_F = \left(\frac{k_B T}{2}\right) - \frac{3}{4}(E_v + E_c)\ln\left(\frac{m_h}{m_e}\right)$

59. Choose the correct statement from the following

(a) The reaction $K^+K^- \to p\bar{p}$ can proceed irrespective of the kinetic energies of K^+ and K^-.

(b) The reaction $K^+K^- \to p\bar{p}$ is forbidden by the baryon number conservation

(c) The reaction $K^+K^- \to 2\gamma$ is forbidden by strangeness conservation

(d) The decay $K^0 \to \pi^+\pi^-$ proceeds via weak interactions

60. The following gives a list of pairs containing (*i*) a nucleus (*ii*) one of its properties. Find the pair which is INAPPROPRIATE.

(a) (*i*) $_{10}Ne^{20}$ nucleus; (*ii*) stable nucleus

(b) (*i*) A spheroidal nucleus; (*ii*) an electric quadrupole moment

(c) (*i*) $_8O^{16}$ nucleus; (*ii*) nuclear spin J = 1/2

(d) (*i*) U^{238} nucleus; (*ii*) Binding energy = 1785 MeV (approximately)

61. The four possible configurations of neutrons in the ground state of $_4Be^9$ nucleus, according to the shell model, and the associated nuclear spin are listed below. Choose the correct one:

(a) $(1s_{1/2})^2(1p_{3/2})^3; J = 3/2$ (b) $(1s_{1/2})^2(1p_{1/2})^2(1p_{3/2})^1; J = 3/2$

(c) $(1s_{1/2})^1(1p_{3/2})^4; J = 1/2$ (d) $(1s_{1/2})^2(1p_{3/2})^2(1p_{1/2})^1; J = 1/2$

CAREER ENDEAVOUR

62. The mass difference between the pair of mirror nuclei ${}_6C^{11}$ and ${}_5B^{11}$ is given to be $\Delta MeV/c^2$. According to the semi-empirical mass formula, the mass difference between the pair of mirror nuclei ${}_9F^{17}$ and ${}_8O^{17}$ will approximately be (rest mass of proton m_p = 938.27 MeV/c^2 and rest mass of neutrons m_n = 939.57 MeV/c^2)

(a) $1.39\,\Delta MeV/c^2$ (b) $(1.39\Delta + 0.5)MeV/c^2$

(c) $0.86\Delta MeV/c^2$ (d) $(1.6\Delta + 0.78)MeV/c^2$

63. A heavy nucleus is found to contain more neutrons than protons. This fact is related to which one of the following statements

(a) The nuclear force between neutrons is stronger than that between protons

(b) The nuclear force between protons is of a shorter range than those between neutrons, so that a smaller number of protons are held together by the nuclear force

(c) Protons are unstable, so their number in a nucleus diminishes

(d) It costs more energy to add a proton to a (heavy) nucleus than a neutron because of the Coulomb repulsion between protons

64. A neutral pi meson (π^0) has a rest-mass of approximately 140 MeV/c^2 and a lifetime of τ sec. A π^0 produced in the laboratory is found to decay after 1.25τ sec into two photons. Which of the following sets represents a possible set of energies of the two photons as seen in the laboratory?

(a) 70 MeV and 70 MeV (b) 35 MeV and 100 MeV

(c) 75 MeV and 100 MeV (d) 25 MeV and 150 MeV

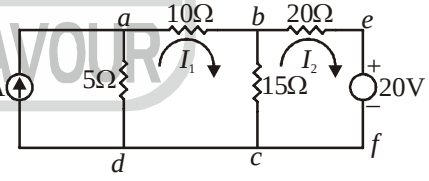
65. An a.c. voltage of 220 V_{rms} is applied to the primary of a 10:1 step-down transformer. The secondary of the transformer is centre tapped and connected to a full wave rectifier with a load resistance. The d.c. voltage appearing across the load is

(a) $\frac{22}{\pi}$ (b) $\frac{31}{\pi}$ (c) $\frac{62}{\pi}$ (d) $\frac{44}{\pi}$

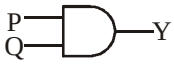
66. Let I_1 and I_2 represent mesh currents in the loop *abcda* and *befcb* respectively. The correct expression describing Kirchoff's voltage loop law in one of the following loops is

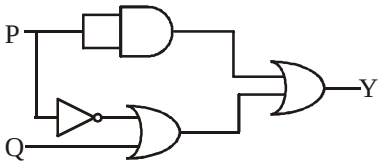
(a) $30I_1 - 15I_2 = 10$ (b) $-15I_1 + 20I_2 = -20$

(c) $30I_1 - 15I_2 = -10$ (d) $-15I_1 + 20I_2 = 20$

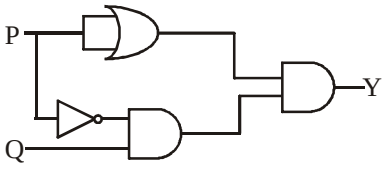


67. The simplest logic gate circuit corresponding to the Boolean expression, $Y = P + \bar{P}Q$ is

(a) 

(b) 

(c)

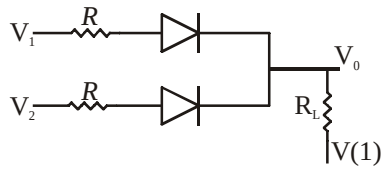
(d) 

68. An analog voltage V is converted into 2-bit binary number. The minimum number of comparators required and their reference voltage are

(a) $3, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}\right)$ (b) $3, \left(\frac{V}{3}, \frac{2V}{3}, V\right)$

(c) $4, \left(\frac{V}{5}, \frac{2V}{5}, \frac{3V}{5}, \frac{4V}{5}\right)$ (d) $4, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}, V\right)$

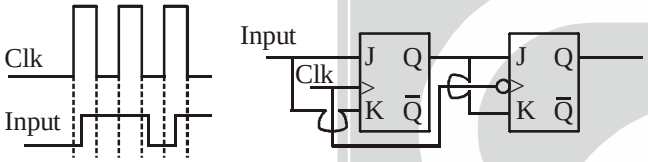
69. The following circuit (where $R_L >> R$) performs the operation of

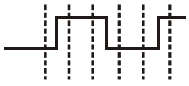


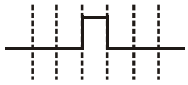
(a) OR gate for a negative logic system (b) NAND gate for a negative logic system

(c) AND gate for a positive logic system (d) AND gate for a negative logic system

70. In the T type master-slave JK flip flop is shown along with the clock and input waveforms. The Q_n output of flip flop was zero initially. Identify the correct output waveform.

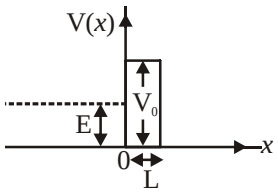


(a)  (b)

(c)  (d)

Common Data Questions

Common Data for Questions 71, 72 and 73: A beam of identical particles of mass *m* and energy *E* is incident from left on a potential barrier of width *L* (between $0 < x < L$) and height V_0 as shown in the figure $(E < V_0)$



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0, A_B and A_T denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

71. Throughout $0 < x < L$, the wave-function

(a) can be chosen to be real (b) is exponentially decaying

(c) is generally complex (d) is zero

72. Let the probability current associated with the incident wave be S_0. Let R be the reflection coefficient. Then

(a) the probability current vanishes in the classically forbidden region

(b) the probability current is TS_0 for $x > L$

(c) for, $x < 0$ the probability current is $S_0(1+R)$

(d) for $x > L$, the probability current is complex

73. The ratio of the reflected to the incident amplitude A_B / A_0 is

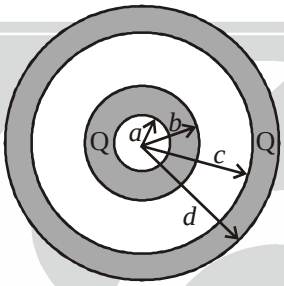
(a) $1 - A_T / A_0$

(b) $\sqrt{(1-T)}$ in magnitude

(c) a real negative number

(d) $\sqrt{\left(1 - \left|\frac{A_T}{A_0}\right|^2\right)\frac{E}{V_0 - E}}$

Common Data for Questions 74 and 75: Consider two concentric conducting spherical shells with inner and outer radii *a*, *b* and *c*, *d* as shown in the figure. Both the shells are given Q amount of positive charges.



74. The electric field in different regions are

(a) $\vec{E} = 0$ for $r < a; \vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a < r < b$, $\vec{E} = 0$ for $b < r < c; \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(b) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a; \vec{E} = 0$ for $a < r < b$, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c; \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(c) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a; \vec{E} = 0$ for $a < r < b$, $\vec{E} = 0$ for $b < r < c; \vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(d) $\vec{E} = 0$ for $r < a; \vec{E} = 0$ for $a < r < b$, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c; \vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

75. In order to have equal surface charge densities on the outer surfaces of both the shells, the following conditions should be satisfied

(a) $d = 4b$ and $c = 2a$

(b) $d = 2b$ and $c = \sqrt{2}a$

(c) $d = \sqrt{2}b$ and $c > a$

(d) $d > b$ and $c = \sqrt{2}a$

CAREER ENDEAVOUR

Linked Answer Questions : Q.76 to Q.85 carry two marks each.

Statement for Linked Answer Questions 76 and 77:

Consider the β-decay of a free neutron at rest in the laboratory.

76. Which of the following configurations of the decay products corresponds to the largest energy of the anti-neutrino $\bar{\nu}$? (rest mass of electron m_e = 0.51 MeV/c^2, rest mass of proton m_p = 938.27 MeV/c^2 and rest mass of neutron m_n = 939.57 MeV/c^2)

(a) In the laboratory, proton is produced at rest

(b) In the laboratory, momenta of proton, electron and the anti-neutrino all have the same magnitude

(c) In the laboratory, proton and electron fly-off with (nearly) equal and opposite momenta

(d) In the laboratory, electron is produced at rest

77. Using the result of the above problem, answer the following. Which of the following represents approximately the maximum allowed energy of the anti-neutrino $\bar{\nu}$?

(a) 1.3 MeV (b) 0.8 MeV (c) 0.5 MeV (d) 2.0 MeV

Statement for Linked Answer Questions 78 and 79:

Consider a two dimensional electron gas of N electrons of mass *m* each in a system of size L×L.

78. The density of states between energy ε and $\varepsilon + d\varepsilon$ is

(a) $\frac{4\pi L^2 m}{h^2} d\varepsilon$ (b) $\frac{4\pi L^2 m}{h^2} \frac{1}{\sqrt{\varepsilon}} d\varepsilon$

(c) $\frac{4\pi L^2 m}{h^2} \sqrt{\varepsilon} d\varepsilon$ (d) $\frac{4\pi L^2 m}{h^2} \varepsilon d\varepsilon$

79. The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electrons N is given by

(a) $\frac{1}{3} NE_F$ (b) $\frac{1}{2} NE_F$ (c) $\frac{2}{3} NE_F$ (d) $\frac{3}{5} NE_F$

Statement for Linked Answer Questions 80 and 81:

The rate of a clock in a spaceship "Suryashakti" is observed from each to be 3/5 of the rate of the clocks on earth.

80. The speed of the spaceship "Suryashakti" relative to earth is

(a) $\frac{4}{5}c$ (b) $\frac{3}{5}c$ (c) $\frac{9}{10}c$ (d) $\frac{2}{5}c$

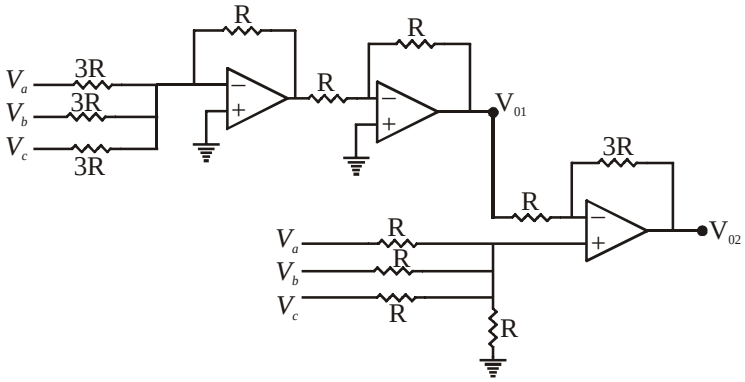
81. The rate of a clock in a spaceship "Aakashganga" is observed from earth to be 5/13 of the rate of the clocks on earth. If both Aakashganga and Suryashakti are moving in the same direction relative to someone on earth, then the speed of Aakashganga relative to Suryashakti is

(a) $\frac{12}{13}c$ (b) $\frac{4}{5}c$ (c) $\frac{8}{17}c$ (d) $\frac{5}{6}c$

CAREER ENDEAVOUR

Statement for Linked Answer Questions 82 and 83:

The following circuit contains three operational amplifiers and resistors



82. The output voltage at the end of second operational amplifier V_{01} is

(a) $V_{01} = 3(V_a + V_b + V_c)$ (b) $V_{01} = -\frac{1}{3}(V_a + V_b + V_c)$

(c) $V_{01} = \frac{1}{3}(V_a + V_b + V_c)$ (d) $V_{01} = \frac{4}{3}(V_a + V_b + V_c)$

83. The output V_{02} (at the end of third op amp) of the above circuit is

(a) $V_{02} = 2(V_a + V_b + V_c)$ (b) $V_{02} = 3(V_a + V_b + V_c)$

(c) $V_{02} = -\frac{1}{2}(V_a + V_b + V_c)$ (d) Zero

Statement for Linked Answer Questions 84 and 85:

The set V of all polynomials of a real variable x of degree two or less and with real coefficients, constitutes a real linear vector space $V \equiv \{c_0 + c_1 x + c_2 x^2 : c_0, c_1, c_2 \in R\}$.

84. For $f(x) = a_0 + a_1 x + a_2 x^2 \in V$ and $g(x) = b_0 + b_1 x + b_2 x^2 \in V$, which one of the following constitutes an acceptable scalar product?

(a) $(f, g) = a_0^2 b_0 + a_1^2 b_1 + a_2^2 b_2$ (b) $(f, g) = a_0^2 b_0^2 + a_1^2 b_1^2 + a_2^2 b_2^2$

(c) $(f, g) = a_0 b_0 - a_1 b_1 + a_2 b_2$ (d) $(f, g) = a_0 b_0 + \frac{a_1 b_1}{2} + \frac{a_2 b_2}{3}$

85. Using the scalar product obtained in the above question, identify the subspace of V that is orthogonal to $(1 + x)$:

(a) $\{f(x) : b(1 - x) + cx^2 ; b, c \in R\}$ (b) $\{f(x) = b(1 - 2x) + cx^2 ; b, c \in R\}$

(c) $\{f(x) : b + cx^2 ; b, c \in R\}$ (d) $\{f(x) : bx + cx^2 ; b, c \in R\}$

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.20 : Carry ONE mark each.

1. The value of the contour integral, $\left|\int_C \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with center at the origin is

(a) $2\pi r$ (b) $\frac{r^2}{2}$ (c) πr^2 (d) r

2. An electrostatic field $\vec{E}$ exists in a given region R. Choose the WRONG statement.
(a) Circulation of $\vec{E}$ is zero
(b) $\vec{E}$ can always be expressed as the gradient of a scalar field
(c) The potential difference between any two arbitrary points in the region R is zero
(d) The work done in a closed path lying entirely in R is zero

3. The Lagrangian of a free particle in spherical polar co-ordinates is given by $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2 \sin^2\theta\right)$. The quantity that is conserved is

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$ (c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}} + \dot{r}\dot{\theta}$

4. A conducting loop L of surface area S is moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}(\vec{r}, t) = \vec{B}_0 t^2$, B_0 is a positive constant of suitable dimensions. The emf induced, V_{emf}, in the loop is given by

(a) $-\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$ (b) $\oint_L \left(\vec{v} \times \vec{B}\right) \cdot d\vec{L}$

(c) $-\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} - \oint_L \left(\vec{v} \times \vec{B}\right) \cdot d\vec{L}$ (d) $-\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint_L \left(\vec{v} \times \vec{B}\right) \cdot d\vec{L}$

5. The eigenvalues of the matrix $A = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ are

(a) real and distinct (b) complex and distinct
(c) complex and coinciding (d) real and coinciding

6. $\sigma_i (i = 1, 2, 3)$ represent the Pauli spin matrices. Which one of the following is NOT true ?

(a) $\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij}$ (b) $Tr(\sigma_i) = 0$
(c) The eigenvalues of σ_i are ± 1 (d) det $(\sigma_i) = 1$

7. Which one of the functions given below represents the bound state eigenfunction of the operator $-\frac{d^2}{dx^2}$ in the region, $0 \le x < \infty$, with the eigenvalue -4 ?

(a) $A_0 e^{2x}$ (b) $A_0 \cosh 2x$ (c) $A_0 e^{-2x}$ (d) $A_0 \sinh 2x$

8. Pick the wrong statement
(a) the nuclear force is independent of electric charge
(b) the Yukawa potential is proportinoal to $r^{-1} \exp\left(\frac{m}{h} r\right)$. Where, r is the seperation between two nucleons.
(c) The range of nuclear force is of the order of $10^{-15} - 10^{-14}$ m
(d) the nucleons interact among each other by the exchange of mesons.

CAREER ENDEAVOUR

9. If p and q are the position and momentum variables, which one of the following is NOT a canonical transformation ?

(a) $Q = \alpha q$ and $P = \dfrac{1}{\alpha} p$, for $\alpha \neq 0$

(b) $Q = \alpha q + \beta p$ and $P = \beta q + \alpha p$ for α, β real and $\alpha^2 - \beta^2 = 1$

(c) $Q = p$ and $P = q$

(d) $Q = p$ and $P = -q$

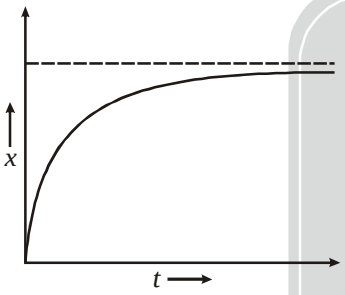
10. The Common Mode Rejection Ratio (CMRR) of a differential amplifier using an operational amplifier is 100 dB. The output voltage for a differential input of 200 μV is 2 V. The common mode gain is

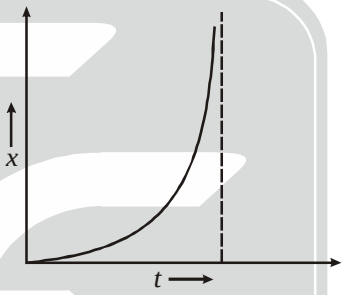
(a) 10 (b) 0.1 (c) 30 dB (d) 10 dB

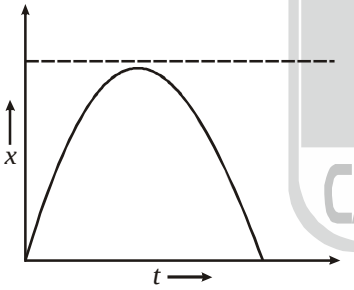
11. In an insulating solid which one of the following physical phenomena is a consequence of Pauli's exclusion principle ?

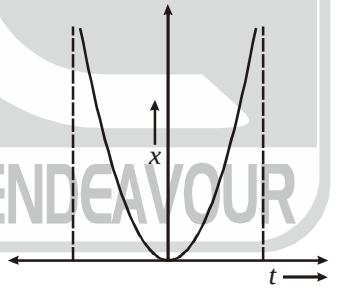
(a) Ionic conductivity (b) Ferromagnetism (c) Paramagnetism (d) Ferroelectricity

12. Which one of the following curves gives the solution of the differential equation $k_1 \dfrac{dx}{dt} + k_2 x = k_3$, where k_1, k_2 and k_3 are positive constant with initial conditions $x = 0$ at $t = 0$?

(a)


(b)


(c)


(d)


13. Identify which one is a first order phase transition ?

(a) A liquid to gas transition at its critical temperature

(b) A liquid to gas transition close to its triple point

(c) A paramagnetic to ferromagnetic transition in the absence of a magnetic field

(d) A metal to superconductor transition in the absence of a magnetic field

14. **Group- I** lists some physical phenomena while **Group-II** gives some physical parameters. Match the phenomena with the corresponding parameter.

Group I	**Group II**
P. Doppler Broadening	1. Moment of inertia
Q. Natural Broadening	2. Refractive index
R. Rotational spectrum	3. Lifetime of the energy level
S. Total internal reflection	4. Pressure

(a) P-4, Q-3, R-1, S-2 (b) P-3, Q-2, R-1, S-4

(c) P-2, Q-3, R-4, S-1 (d) P-1, Q-4, r-2, S-3

CAREER ENDEAVOUR

15. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant, B is :

(a) 2B (b) 4B (c) 6B (d) 12B

16. A superconducting ring is cooled in the presence of a magnetic field below its critical temperature (T_C). The total magnetic flux that passes through the ring is

(a) zero (b) $n\frac{h}{2e}$ (c) $\frac{nh}{4\pi e}$ (d) $\frac{ne^2}{hc}$

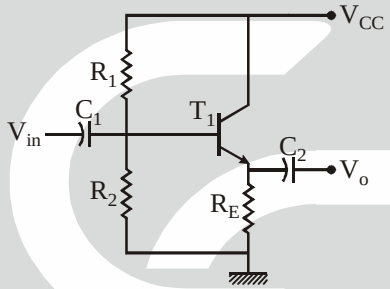
17. In a cubic crystal, atoms of mass M_1 lie on one set of planes and atoms of mass M_2 lie on planes interleaved between those of the first set. If C is the force constant between nearest neighbour planes, the frequency of lattice vibrations for the optical phonon branch with wavevector $\mathbf{k} = 0$ is

(a) $\sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (b) $\sqrt{C\left(\frac{1}{2M_1}+\frac{1}{M_2}\right)}$ (c) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{2M_2}\right)}$ (d) zero

18. In the quark model which one of the following represents a proton?

(a) udd (b) uud (c) $u\bar{b}$ (d) $c\bar{c}$

19. The circuit shown below:



(a) is a common-emitter amplifier (b) uses a *pnp* transistor
(c) is an oscillator (d) has a voltage gain less than one

20. Consider a nucleus with N neutrons and Z protons. If m_p, m_n and BE represents the mass of the proton, the mass of the neutron and the binding energy of the nucleus respectively and c is the velocity of light in free space, the mass of the nucleus is given by

(a) $Nm_n + Zm_p$ (b) $Nm_p + Zm_n$ (c) $Nm_n + Zm_p - \frac{BE}{c^2}$ (d) $Nm_p + Zm_n + \frac{BE}{c^2}$

CAREER ENDEAVOUR

Q.21 – Q.60 : Carry TWO marks each.

21. The magnetic field $(\text{in } Am^{-1})$ inside a long solid cylindrical conductor of radius $a = 0.1\,m$, is

$\vec{H} = \frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(\alpha r)\right]\hat{\phi}$, where $\alpha = \frac{\pi}{2a}$. What is the total current (in A) in the conductor ?

(a) $\frac{\pi}{2a}$ (b) $\frac{800}{\pi}$ (c) $\frac{400}{\pi}$ (d) $\frac{300}{\pi}$

22. Which one of the following current densities, $\vec{J}$, can generate the magnetic vector potential $\vec{A} = \left(y^2\hat{i} + x^2\hat{j}\right)$?

(a) $\frac{2}{\mu_0}\left(x\hat{i} + y\hat{j}\right)$ (b) $-\frac{2}{\mu_0}\left(\hat{i} + \hat{j}\right)$ (c) $\frac{2}{\mu_0}\left(\hat{i} - \hat{j}\right)$ (d) $\frac{2}{\mu_0}\left(x\hat{i} - y\hat{j}\right)$

23. The value of the integral $\int_C \frac{e^z}{z^2-3z+2}dz$, where the contour C is the circle $|z|=\frac{3}{2}$ is

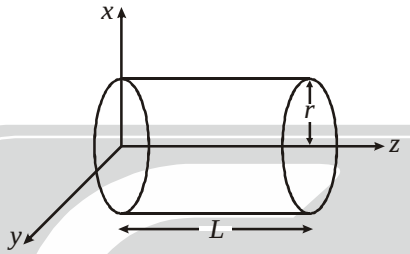
(a) $2\pi ie$ (b) πie (c) $-2\pi ie$ (d) $-\pi ie$

24. In a non-conducting medium characterized by $\varepsilon=\varepsilon_0$, $\mu=\mu_0$ and conductivity $\sigma=0$, the electric field (in $V\,m^{-1}$) is given by $\vec{E}=20\sin[10^8t-kz]\hat{j}$. The magnetic field, $\vec{H}$ (in Am^{-1}), is given by

(a) $20k\cos\left[10^8t-kz\right]\hat{i}$ (b) $\frac{20k}{10^8\mu_0}\sin\left[10^8t-kz\right]\hat{j}$

(c) $-\frac{20k}{10^8\mu_0}\sin\left[10^8t-kz\right]\hat{i}$ (d) $-20k\cos\left[10^8t-kz\right]\hat{j}$

25. A cylindrical rod of length L and radius r, made of an inhomogeneous dielectric, is placed with its axis along the z-direction with one end at the origin as shown below.



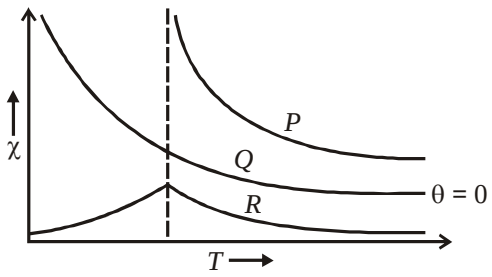
If the rod carries a polarization, $\vec{P}=(5z^2+7)\hat{k}$, the volume bound charge inside the dielectric is

(a) zero (b) $10\pi r^2L$ (c) $-5\pi r^2L$ (d) $-5\pi r^2L^2$

26. Let $T_{ij}=\sum_k \varepsilon_{ijk}a_k$ and $\beta_k=\sum_{i,j}\varepsilon_{ijk}T_{ij}$, where ε_{ijk} is the Levi-Civita density, defined to be zero if two of the indices coincide and +1 and –1 depending on whether *ijk* is even or odd permutation of 1, 2, 3. Then β_3 is equal to

(a) $2a_3$ (b) $-2a_3$ (c) a_3 (d) $-a_3$

27. The dependence of the magnetic susceptibility (χ) of a material with temperature (T) can be represented by $\chi\propto\frac{1}{T-\theta}$, where θ is the Curie-Weiss temperature. The plot of magnetic susceptibility versus temperature is sketched in the figure, as curves P, Q and R with curve Q having $\theta=0$. Which one of the following statements is correct ?



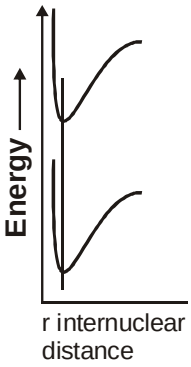
(a) Curve R represents a paramagnet and Q a ferromagnet
(b) Curve Q represents a ferromagnet and P an antiferromagnet
(c) Curve R represents an antiferromagnet and Q a paramagnetic
(d) Curve R represents an antiferromagnet and Q a ferromagnet

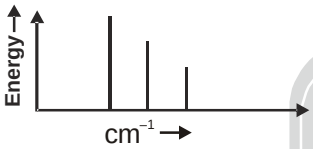
CAREER ENDEAVOUR

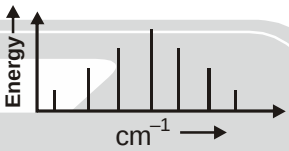
28. The dielectric constant of a material at optical frequencies is mainly due to
(a) ionic polarizability (b) electronic polarizability
(c) dipolar polarizability (d) ionic and dipolar polarizability

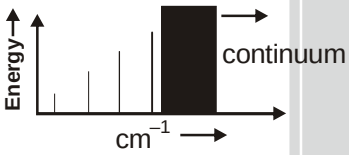
29. An electron of wavevector $\vec{k}_e$, velocity $\vec{v}_e$ and effective mass m_e is removed from a filled energy band. The resulting hole has wavevector $\vec{k}_h$, velocity $\vec{v}_h$, and effective mass m_h. Which one of the following statements is correct ?
(a) $\vec{k}_h = -\vec{k}_e\ ;\ \vec{v}_h = -\vec{v}_e\ ;\ m_h = -m_e$ (b) $\vec{k}_h = \vec{k}_e\ ;\ \vec{v}_h = \vec{v}_e\ ;\ m_h = m_e$
(c) $\vec{k}_h = \vec{k}_e\ ;\ \vec{v}_h = -\vec{v}_e\ ;\ m_h = -m_e$ (d) $\vec{k}_h = -\vec{k}_e\ ;\ \vec{v}_h = \vec{v}_e\ ;\ m_h = -m_e$

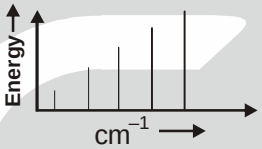
30. In a diatomic molecule, the internuclear separation of the ground and first excited electronic state are the same as shown in the figure. If the molecule is initially in the lowest vibrational state of the ground state, then the absorption spectrum will appear as



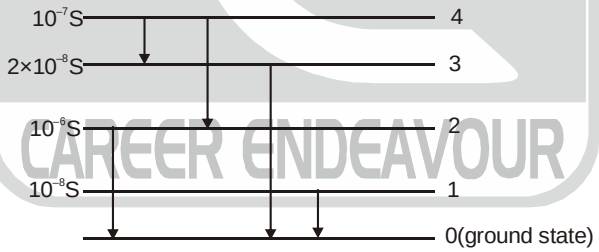
(a)


(b)


(c)


(d)


31. Five energy levels of a system including the ground state are shown below. Their lifetimes and the allowed electric dipole transitions are also marked.



Which one of the following transitions is the most suitable for a continuous wave (CW) laser?
(a) $1 \to 0$ (b) $2 \to 0$ (c) $4 \to 2$ (d) $4 \to 3$

32. Assuming the mean life time of a muon (in its rest frame) to be 2×10^{-6} s, its life time in the laboratory frame, when it is moving with a velocity $0.95\,c$ is
(a) 6.4×10^{-6} s (b) 0.62×10^{-6} s (c) 2.16×10^{-6} s (d) 0.19×10^{-6} s

33. Cesium has a nuclear spin of 7/2. The hyperfine spectrum of the D lines of the Cesium atom will consist of
(a) 10 lines (b) 4 lines (c) 6 lines (d) 14 lines

34. The probability that an energy level ε at a temperature T is unoccupied by a fermion of chemical potential μ is given by
(a) $\dfrac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$ (b) $\dfrac{1}{e^{(\varepsilon-\mu)/k_BT}-1}$ (c) $\dfrac{1}{e^{(\mu-\varepsilon)/k_BT}+1}$ (d) $\dfrac{1}{e^{(\mu-\varepsilon)/k_BT}-1}$

CAREER ENDEAVOUR

35. Consider the following expression for the mass of a nucleus with Z protons and A nucleons.

$$M(A,Z)=\frac{1}{c^2}\left[f(A)+yZ+zZ^2\right]$$

Here, $f(A)$ is a function of A.

$$y=-4a_A$$

$$z=a_cA^{-1/3}+4a_AA^{-1}$$

a_A and a_c are constants of suitable dimensions. For a fixed A, the expression of Z for the most stable nucleus is

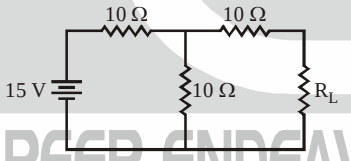
(a) $Z=\dfrac{A/2}{\left[1+\left(\dfrac{a_c}{a_A}\right)A^{2/3}\right]}$ (b) $Z=\dfrac{A/2}{\left[1-\left(\dfrac{a_c}{4a_A}\right)A^{2/3}\right]}$

(c) $Z=\dfrac{A/2}{\left[1+\left(\dfrac{a_c}{4a_A}\right)A^{2/3}\right]}$ (d) $Z=\dfrac{A}{\left(1+A^{2/3}\right)}$

36. The de-Broglie wavelength of particles of mass m with average momentum p at a temperature T in three dimension is given by

(a) $\lambda=\dfrac{h}{\sqrt{2mk_BT}}$ (b) $\lambda=\dfrac{h}{\sqrt{3mk_BT}}$ (c) $\lambda=\dfrac{h}{\sqrt{2k_BT}}$ (d) $\lambda=\dfrac{h}{\sqrt{3m}}$

37. Assuming an ideal voltage source, Thevenin's resistance and Thevenin's voltage respectively for the below circuit are



(a) 15 Ω and 7.5 V (b) 20 Ω and 5 V (c) 10 Ω and 10 V (d) 30 Ω and 15 V

38. Let $|n\rangle$ and $|p\rangle$ denote the isospin state with $I=\frac{1}{2}$, $I_3=\frac{1}{2}$ and $I=\frac{1}{2}$, $I_3=-\frac{1}{2}$ of a nuclear respectively.

Which one of the following two nuclear state has $I=0, I_3=0$?

(a) $\frac{1}{\sqrt{2}}\left(|nn\rangle-|pp\rangle\right)$ (b) $\frac{1}{\sqrt{2}}\left(|nn\rangle+|pp\rangle\right)$

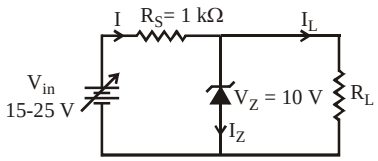
(c) $\frac{1}{\sqrt{2}}\left(|np\rangle-|pn\rangle\right)$ (d) $\frac{1}{\sqrt{2}}\left(|np\rangle+|pn\rangle\right)$

CAREER ENDEAVOUR

39. An amplifier of gain 1000 is made into a feedback amplifier by feeding 9.9 % of its output voltage in series with the input opposing. If $f_L = 20$ Hz and $f_H = 200$ kHz for the amplifier without feedback, then due to the feedback

(a) the gain decreases by 10 times
(b) the output resistance increases by 10 times
(c) the f_H increases by 100 times
(d) the input resistance decreases by 100 times

40. Pick the correct statement based on the below circuit.



(a) The maximum zener current, $I_{Z(\max)}$, when $R_L = 10$ kΩ is 15 mA

(b) The minimum zener current, $I_{Z(\min)}$, when $R_L = 10$ kΩ is 5 mA

(c) With $V_{in} = 20$ V, $I_L = I_Z$, when $R_L = 2$ kΩ

(d) The power dissipated across the zener when $R_L = 10$ kΩ and $V_{in} = 20$ V is 100 mW

41. The disintegration energy is defined to be the difference in the rest energy between the initial and final states. Consider the following process;

$$_{94}\mathrm{Pu}^{240} \longrightarrow {}_{92}\mathrm{U}^{236} + {}_{2}\mathrm{He}^{4}$$

The emitted α -particle has a kinetic energy 5.17 MeV. The value of disintegration energy is

(a) 5.26 MeV (b) 5.17 MeV (c) 5.08 MeV (d) 2.59 MeV

42. A classical particle is moving in an external potential field $V(x, y, z)$ which is invariant under the following infinitesimal transformations

$$x \to x' = x + \delta x,$$
$$y \to y' = y + \delta y,$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x' \\ y' \end{pmatrix} = R_Z \begin{pmatrix} x \\ y \end{pmatrix},$$

where R_Z is the matrix corresponding to rotation about the z-axis. The conserved quantities are (the symbols have their usual meaning)

(a) p_x, p_z, L_z (b) p_x, p_y, L_z, E (c) p_y, L_z, E (d) p_y, p_z, L_x, E

43. The spin function of a free particle in the basis in which S_z is diagonal, can be written as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively. In the given basis, the normalized eigenfunction of S_y with eigenvalue $-\frac{\hbar}{2}$.

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$

CAREER ENDEAVOUR

44. $\hat{A}$ and $\hat{B}$ represent two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A}\hat{B}$ to be Hermitian, it is sufficient that

(a) $\hat{B}$ is Hermitian (b) $\hat{B}$ is anti-Hermitian

(a) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute (d) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ anti-commute

45. Consider the set of vectors in three-dimensional real vector space $\mathbb{R}^3$, $S = \{(1,1,1), (1,-1,1), (1,1,-1)\}$. Which one of the following statements is true ?

(a) S is not a linearly independent set (b) S is a basis for $\mathbb{R}^3$

(c) The vectors in S are orthogonal (d) An orthogonal set of vectors cannot be generated from S

46. For a Fermi gas of N particles in three dimensions at T = 0 K, the Fermi energy, E_F is proportional to

(a) $N^{2/3}$ (b) $N^{3/2}$ (c) N^3 (d) N^2

47. The Lagrangian of a diatomic molecule is given by $L = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{k}{2}x_1x_2$, where m is the mass of each of the atoms and x_1 and x_2 are the displacements of atoms measured from the equilibrium position and $k > 0$. The normal frequencies are

(a) $\pm\left(\frac{k}{m}\right)^{1/2}$ (b) $\pm\left(\frac{k}{m}\right)^{1/4}$ (c) $\pm\left(\frac{k}{2m}\right)^{1/4}$ (d) $\pm\left(\frac{k}{2m}\right)^{1/2}$

48. A particle is in the normalized state $|\psi\rangle$ which is a superposition of the energy eigenstates $|E_0 = 10\text{ eV}\rangle$ and $|E_1 = 30\text{ eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV.

(a) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle + \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0 = 10\text{ eV}\rangle + \sqrt{\frac{2}{3}}|E_1 = 30\text{ eV}\rangle$

(c) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle - \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0 = 10\text{ eV}\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\text{ eV}\rangle$

49. The Lagrangian of a particle of mass m moving in one dimension is $L = \exp(\alpha t)\left[\frac{m\dot{x}^2}{2} - \frac{kx^2}{2}\right]$, where α and k are positive constant. The equation of motion of the particle is

(a) $\ddot{x} + \alpha\dot{x} = 0$ (b) $\ddot{x} + \frac{k}{m}x = 0$ (c) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$ (d) $\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$

50. Two monochromatic waves having frequencies ω and $\omega + \Delta\omega\left(\Delta\omega << \omega\right)$ and corresonding wavelength λ and $\lambda - \Delta\lambda\left(\Delta\lambda << \lambda\right)$ of same polarization, travelling along x-axis are superimposed on each other. The phase velocity and group velocity of the resultant wave are respectively given by

(a) $\frac{\omega\lambda}{2\pi}, \frac{\Delta\omega\lambda^2}{2\pi\Delta\lambda}$ (b) $\omega\lambda, \frac{\Delta\omega\lambda^2}{\Delta\lambda}$ (c) $\frac{\omega\Delta\lambda}{2\pi}, \frac{\Delta\omega\Delta\lambda}{2\pi}$ (d) $\omega\Delta\lambda, \omega\Delta\lambda$

CAREER ENDEAVOUR

Common Data for Questions 51 and 52 :

Consider a two level quantum system with energies $\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$.

51. The Helmholtz free energy of the system is given by

(a) $-k_B T \ln\left(1+e^{-\varepsilon/k_B T}\right)$ (b) $k_B T \ln\left(1+e^{-\varepsilon/k_B T}\right)$

(c) $\frac{3}{2}k_B T$ (d) $\varepsilon - k_B T$

52. The specific heat of the system is given by

(a) $\frac{\varepsilon}{k_B T}\frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$ (b) $\frac{\varepsilon^2}{k_B T^2}\frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)}$ (c) $-\frac{\varepsilon^2 e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$ (d) $\frac{\varepsilon^2}{k_B T^2}\frac{e^{-\varepsilon/k_B T}}{\left(1+e^{-\varepsilon/k_B T}\right)^2}$

Common Data for Questions 53 and 54 :

A free particle of mass *m* moves along the *x*-direction. At $t = 0$, the normalized wave function of the particle is given by $\psi(x, 0) = \frac{1}{(2\pi\alpha)^{1/4}}\exp\left[-\frac{x^2}{4\alpha^2} + ix\right]$, where α is a real constant.

53. The expectation value of the momentum in this state is

(a) $\hbar\alpha$ (b) $\hbar\sqrt{\alpha}$ (c) α (d) $\frac{\hbar}{\sqrt{\alpha}}$

54. The expectation value of the particle energy is

(a) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{3/2}}$ (b) $\frac{\hbar^2}{2m}\alpha^2$ (c) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{3/2}}$ (d) $\frac{\hbar^2}{8m\alpha^{3/2}}$

Common Data for Q.55 and Q.56:

Consider the Zeeman splitting of a single electron system for the $3d \rightarrow 3p$ electric dipole transition.

55. The Zeeman spectrum is :

(a) Randomly polarized (b) Only π polarized

(c) Only σ polarized (d) Both π and σ polarized

56. The fine structure line having the longest wavelength will split into

(a) 17 components (b) 10 components (c) 8 components (d) 4 components

Statement for Linked Answer Questions 57 and 58 :

The primitive translation vectors of the face centered cubic (fcc) lattice are

$\vec{a}_1 = \frac{a}{2}\left(\hat{j}+\hat{k}\right),\ \vec{a}_2 = \frac{a}{2}\left(\hat{i}+\hat{k}\right),\ \vec{a}_3 = \frac{a}{2}\left(\hat{i}+\hat{j}\right)$

57. The primitive translation vectors of the fcc reciprocal lattice are

(a) $\vec{b}_1 = \frac{2\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right);\ \vec{b}_2 = \frac{2\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right);\ \vec{b}_3 = \frac{2\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(b) $\vec{b}_1 = \frac{\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right);\ \vec{b}_2 = \frac{\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right);\ \vec{b}_3 = \frac{\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(c) $\vec{b}_1 = \frac{\pi}{2a}\left(-\hat{i}+\hat{j}+\hat{k}\right);\ \vec{b}_2 = \frac{\pi}{2a}\left(\hat{i}-\hat{j}+\hat{k}\right);\ \vec{b}_3 = \frac{\pi}{2a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(d) $\vec{b}_1 = \frac{3\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right);\ \vec{b}_2 = \frac{3\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right);\ \vec{b}_3 = \frac{3\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

CAREER ENDEAVOUR

58. The volume of primitive cell of the fcc reciprocal lattice is

(a) $4\left(\frac{2\pi}{a}\right)^3$ (b) $4\left(\frac{\pi}{a}\right)^3$ (c) $4\left(\frac{\pi}{2a}\right)^3$ (d) $4\left(\frac{3\pi}{a}\right)^3$

Statement for Linked Answer Questions 59 and 60 :

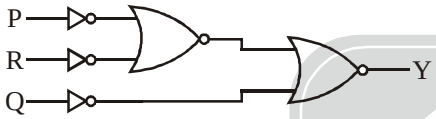
The Karnaugh map of a logic circuit is shown below:

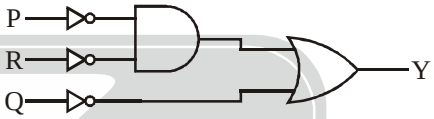
	$\bar{R}$	R
$\bar{P}\bar{Q}$	1	1
$\bar{P}Q$	1	
PQ		
$P\bar{Q}$	1	1

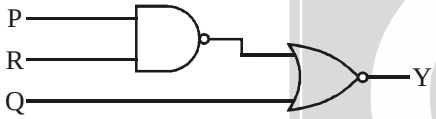
59. The minimized logic expression for the above map is

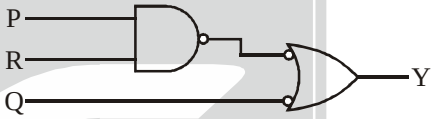
(a) $Y = \bar{P}\bar{R} + \bar{Q}$ (b) $Y = \bar{Q} \cdot PR$ (c) $Y = \bar{Q} + PR$ (d) $Y = Q \cdot \overline{PR}$

60. The corresponding logic implementation using gates is given as :

(a) 

(b) 

(c) 

(d) 

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Consider an anti-symmetric tensor P_0 with the indices i and j running from 1 to 5. The number of independent components of the tensor is
(a) 3 (b) 10 (c) 9 (d) 6

2. The value of the integral $\oint_C \frac{e^z \sin(z)}{z^2} dz$, where the contour C is the unit circle $|z-2|=1$, is
(a) $2\pi i$ (b) $4\pi i$ (c) πi (d) 0

3. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are
(a) 5, 2, –2 (b) –5, –1, –1 (c) 5, 1, –1 (d) –5, 1, 1

4. If $f(x) = \begin{cases} 0 & \text{for } x < 3 \\ x-3 & \text{for } x \geq 3 \end{cases}$, then the Laplace transform of $f(x)$ is
(a) $s^{-2}\, e^{3s}$ (b) $s^2\, e^{-3s}$ (c) s^{-2} (d) $s^{-2}\, e^{-3s}$

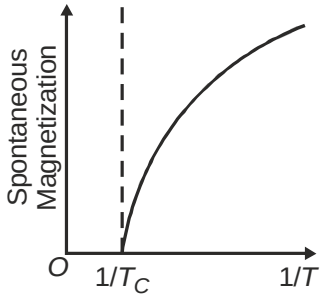
5. The valence electrons do not directly determine the following property of a metal.
(a) Electrical conductivity (b) Thermal conductivity
(c) Shear modulus (d) Metallic lustre

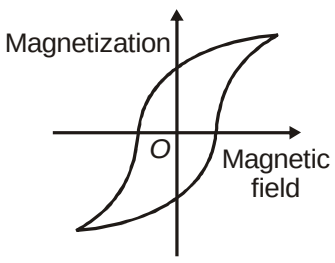
6. Consider X-ray diffraction from a crystal with a face-centered-cubic (fcc) lattice. The lattice plane for which there is NO diffraction peak is
(a) (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

7. The Hall coefficient, R_H of sodium depends on
(b) The effective charge carrier mass and carrier density
(b) The charge carrier density and relaxation time
(c) The charge carrier density only
(d) The effective charge carrier mass

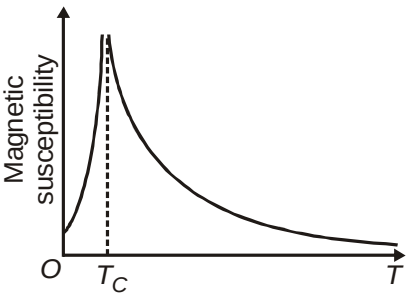
8. The Bloch theorem states that within a crystal, the wave function $\psi(\vec{r})$, of an electron has the form
(a) $\psi(\vec{r}) = u(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$, where $u(\vec{r})$ is an arbitrary function and $\vec{k}$ is an arbitrary vector.
(b) $\psi(\vec{r}) = u(\vec{r})\, e^{i\vec{G}\cdot\vec{r}}$, where $u(\vec{r})$ is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector.
(c) $\psi(\vec{r}) = u(\vec{r})\, e^{i\vec{G}\cdot\vec{r}}$, where $u(\vec{r}) = u(\vec{r} + \vec{A})$, $\vec{A}$ is a lattice vector and $\vec{G}$ is a reciprocal lattice vector.
(d) $\psi(\vec{r}) = u(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$, where $u(\vec{r}) = u(\vec{r} + \vec{A})$, $\vec{A}$ is a lattice vector and $\vec{k}$ is an arbitrary vector.

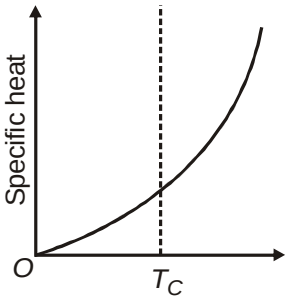
9. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represent the property of the medium ?
(T_C is the Curie temperature)

(a)


(b)


CAREER ENDEAVOUR

(c) 

(d) 

10. The thermal conductivity of a given material reduces, when it undergoes a transition from its normal state to the superconducting state. The reason is
(a) The cooper pairs cannot transfer energy to the lattice.
(b) Upon the formation of Cooper pairs, the lattice becomes less efficient in heat transfer.
(c) The electrons in the normal state loose their ability to transfer heat because of their coupling to the Cooper pairs.
(d) The heat capacity increases on transition to the superconducting state leading to a reduction in thermal conductivity.

11. The basic process underlying the neutron β -decay is

(a) $d \to u + e^- + \overline{\nu}_e$ (b) $d \to u + e^-$ (c) $s \to u + e^- + \overline{\nu}_e$ (d) $u \to d + e^- + \overline{\nu}_e$

12. In the nuclear shell model the spin parity of ^{15}N is given by

(a) $\frac{1}{2}^-$ (b) $\frac{1}{2}^+$ (c) $\frac{3}{2}^-$ (d) $\frac{3}{2}^+$

13. Match the reactions on the left with the associated interactions on the right

(1) $\pi^+ \to \mu^+ + \nu_\mu$ (i) Strong

(2) $\pi^0 \to \gamma + \gamma$ (ii) Electromagnetic

(3) $\pi^0 + n \to \pi^- + p$ (iii) Weak

(a) 1-iii, 2-ii, 3-i (b) 1-i, 2-ii, 3-iii (c) 1-ii, 2-i, 3-iii (d) 1-iii, 2-i, 3-ii

14. To detect trace amounts of a gaseous species in a mixture of gases, the preferred probing tools is
(a) Ionization spectroscopy with X-rays (b) NMR spectroscopy
(c) ESR spectroscopy (d) Laser spectroscopy

15. A collection of *N* atoms is exposed to a strong resonant electromagnetic radiation with N_g atoms in the ground state and N_e atoms in the excited state, such that $N_g + N_e = N$. This collection of two-level atoms will have the following population distribution:

(a) $N_g \ll N_e$ (b) $N_g \gg N_e$ (c) $N_g \approx N_e \approx \frac{N}{2}$ (d) $N_g - N_e \approx \frac{N}{2}$

16. Two states of an atom have definite parities. An electric dipole transition between these states is
(a) Allowed if both the states have even parity
(b) Allowed if both the states have odd parity
(c) Allowed if both the states have opposite parities
(d) Not allowed unless a static electric field is applied

17. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the
(a) Visible range of frequencies (b) Infrared range of frequencies
(c) Ultraviolet range of frequencies (d) Microwave range of frequencies

CAREER ENDEAVOUR

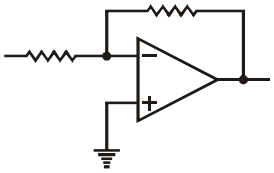
18. An insulating sphere of radius *a* carries a charge density $\rho(\vec{r}) = \rho_0(a^2 - r^2)\cos\theta\ ;\ r < a$. The leading order term for the electric field at a distance *d*, far away from the charge distribution, is proportional to

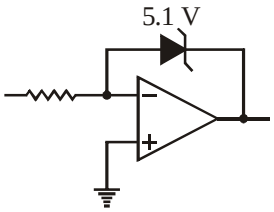
(a) d^{-1} (b) d^{-2} (c) d^{-3} (d) d^{-4}

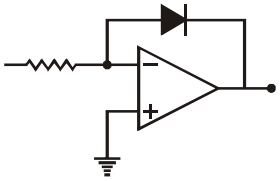
19. The voltage resolution of a 12-bit digital to analog converter (DAC), whose output varies from –10 V to +10 V is, approximately

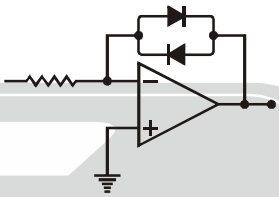
(a) 1 mV (b) 5 mV (c) 20 mV (d) 100 mV

20. In the of the following circuits, negative feedback does not operate for a negative input. Which one is it ? The opamps are running from ±15 V supplies.

(a) 

(b) 

(c) 

(d) 

21. A system of *N* non-interacting classical point particles is constrained to move on the two-dimensional surface of a sphere. The internal energy of the system is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$ (c) Nk_BT (d) $\frac{5}{2}Nk_BT$

22. Which of the following atoms cannot exhibit Bose-Einstein condensation, even in principle ?

(a) ${}^{1}H_{1}$ (b) ${}^{4}He_{2}$ (c) ${}^{23}Na_{11}$ (d) ${}^{40}K_{19}$

23. For the set of all Lorentz transformation with velocities along the *x*-axis, consider the two statements given below:

P : If *L* is a Lorentz transformation then, L^{-1} is also a Lorentz transformation

Q : If L_1 and L_2 are Lorentz transformation then, L_1L_2 is necessarily a Lorentz transformation

Choose the correct option.

(a) P is true and Q is false (b) Both P and Q are true
(c) Both P and Q are false (d) P is false and Q is true

24. Which of the following is an allowed wavefunction for a particle in a bound state ? *N* is a constant and $\alpha, \beta > 0$.

(a) $\psi = N\frac{e^{-\alpha r}}{r^3}$ (b) $\psi = N(1 - e^{-\alpha r})$

(c) $\psi = Ne^{-\alpha x}e^{-\beta(x^2+y^2+z^2)}$ (d) $\psi = \begin{cases} \text{non-zero constant} & \text{if} \quad r < R \\ 0 & \text{if} \quad r > R \end{cases}$

25. A particle is confined within a spherical region of radius one femtometer (10^{-15} m). Its momentum can be expected to be about

(a) $20\frac{keV}{c}$ (b) $200\frac{keV}{c}$ (c) $200\frac{MeV}{c}$ (d) $2\frac{GeV}{c}$

CAREER ENDEAVOUR

Q.26 – Q.55 : Carry TWO marks each.

26. For the complex function, $f(z)=\dfrac{e^{\sqrt{z}}-e^{-\sqrt{z}}}{\sin\left(\sqrt{z}\right)}$, which of the following statements is correct ?

(a) $z=0$ is a branch point
(b) $z=0$ is a pole of order one
(c) $z=0$ is a removable singularity
(d) $z=0$ is an essential singularity

27. The solution of the differential equation for $y(t):\dfrac{d^2y}{dt^2}-y=2\cosh(t)$, subject to the initial conditions $y(0)=0$ and $\left.\dfrac{dy}{dt}\right|_{t=0}=0$, is

(a) $\dfrac{1}{2}\cosh(t)+t\sinh(t)$
(b) $-\sinh(t)+t\cosh(t)$
(c) $t\cosh(t)$
(d) $t\sinh(t)$

28. Given the recurrence relation for the Legendre polynomials
$(2n+1)\,x\,P_n(x)=(n+1)\,P_{n+1}(x)+n\,P_{n-1}(x)$,
which of the following integrals has a non-zero value ?

(a) $\int_{-1}^{+1}x^2P_n(x)P_{n+1}(x)\,dx$
(b) $\int_{-1}^{+1}x\,P_n(x)P_{n+2}(x)\,dx$
(c) $\int_{-1}^{+1}x\left[P_n(x)\right]^2dx$
(d) $\int_{-1}^{+1}x^2\,P_n(x)P_{n+2}(x)\,dx$

29. For a two-dimensional free electron gas, the electronic density *n*, and the Fermi energy E_F, are related by

(a) $n=\dfrac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$
(b) $n=\dfrac{mE_F}{\pi\hbar^2}$
(c) $n=\dfrac{mE_F}{2\pi\hbar^2}$
(d) $n=\dfrac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

30. Far away from any of the resonance frequencies of a medium the real part of the dielectric permittivity is
(a) Always independent of frequency
(b) Monotonically decreasing with frequency
(c) Monotonically increasing with frequency
(d) A non-monotonic function of frequency

31. The ground state wavefunction of deuteron is in a superposition of *s* and *d* states. Which of the following is NOT true as a consequence ?
(a) It has a non-zero quadrupole moment
(b) The neutron-proton potential is non-central
(c) The orbital wavefunction is not spherically symmetric
(d) The Hamiltonian does not conserve the total angular momentum

32. The first three energy levels of $^{228}Th_{90}$ are shown below:

4^+ ———— 187 keV
2^+ ———— 57.5 keV
0^+ ———— 0 keV

The expected spin-parity and energy of the next level are given by

(a) (6^+ ; 400 keV)
(b) (6^+ ; 300 keV)
(c) (2^+ ; 400 keV)
(d) (4^+ ; 300 keV)

CAREER ENDEAVOUR

33. The quark content of Σ^+, K^-, π^- and p is indicated:

$|\Sigma^+\rangle = |uus\rangle; |K^-\rangle = |s\bar{u}\rangle; |\pi^-\rangle = |\bar{u}d\rangle; |p\rangle = |uud\rangle.$

In the process, $\pi^- + p \to K^- + \Sigma^+$, considering strong interactions only, which of the following statements is true ?

(a) The process is allowed because $\Delta S = 0$

(b) The process is allowed because $\Delta I_3 = 0$

(c) The process is not allowed because $\Delta S \neq 0$ and $\Delta I_3 \neq 0$

(d) The process is not allowed because the baryon number is violated

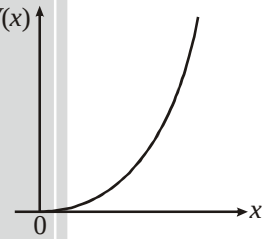
34. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are

(a) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z} - \frac{1}{I}\right)$ (b) $\frac{\hbar^2}{2I} l(l+1)$

(c) $\frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z} - \frac{1}{I}\right)$ (d) $\frac{\hbar^2}{2I} l(l+1) + \frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z} + \frac{1}{I}\right)$

35. A particle of mass *m* is confined in the potential

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for} \quad x > 0 \\ \infty & \text{for} \quad x \leq 0 \end{cases}$$



Let the wavefunction of the particle be given by $\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1$, where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is

(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$ (c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

36. Match the typical spectra of stable molecules with the corresponding wave-number range

1. Electronic spectra	i. 10^6 cm^{-1} and above
2. Rotational spectra	ii. $10^5 - 10^6$ cm^{-1}
3. Molecular dissociation	iii. $10^0 - 10^2$ cm^{-1}

(a) 1-ii, 2-i, 3-iii (b) 1-ii, 2-iii, 3-i (c) 1-iii, 2-ii, 3-i (d) 1-i, 2-ii, 3-iii

37. Consider the operations $P: \vec{r} \to -\vec{r}$ (parity) and $T: t \to -t$ (time-reversal). For the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which of the following set of transformations is correct ?

(a) $P: \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}$; $T: \vec{E} \to \vec{E}, \vec{B} \to -\vec{B}$

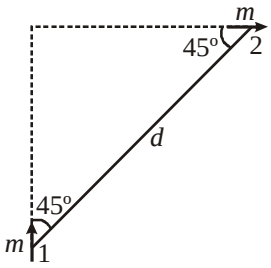
(b) $P: \vec{E} \to \vec{E}, \vec{B} \to \vec{B}$; $T: \vec{E} \to \vec{E}, \vec{B} \to \vec{B}$

(c) $P: \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}$; $T: \vec{E} \to -\vec{E}, \vec{B} \to -\vec{B}$

(d) $P: \vec{E} \to \vec{E}, \vec{B} \to -\vec{B}$; $T: \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}$

CAREER ENDEAVOUR

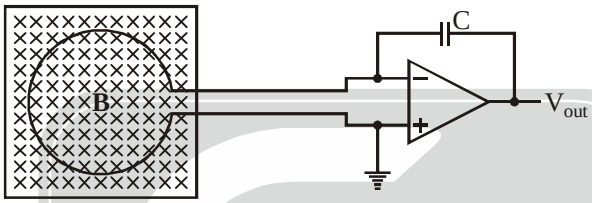
38. Two magnetic dipoles of magnitude m each are placed in a plane as shown.



The energy of interaction is given by

(a) zero (b) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$ (c) $\frac{3\mu_0}{2\pi}\frac{m^2}{d^3}$ (d) $-\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$

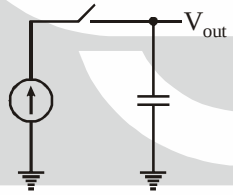
39. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux ϕ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged.



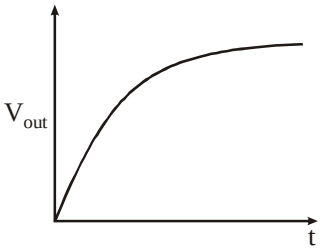
If the loop is pulled out of the region of the magnetic field at a constant speed u, the final output voltage V_{out} is independent of

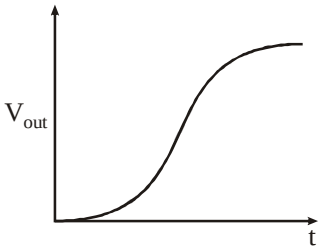
(a) ϕ_0 (b) u (c) R (d) C

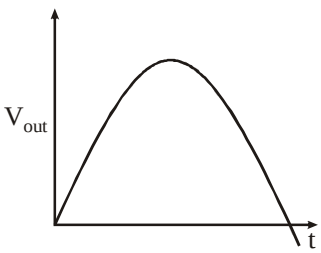
40. The figure shows a constant current source charging a capacitor that is initially uncharged.

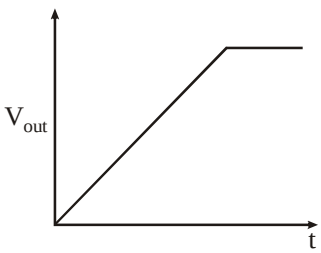


If the switch is closed at t = 0, which of the following plots depicts correctly the output voltage of the circuit as a function of time ?

(a)


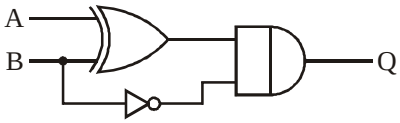
(b)


(c)


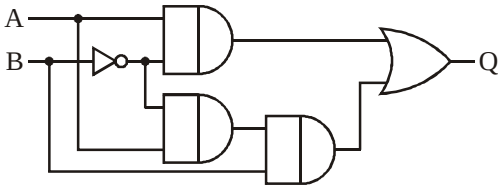
(d)


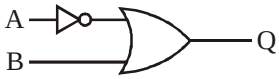
CAREER ENDEAVOUR

41. For any set of inputs A and B, the following circuits give the same output Q, except one. Which one is it ?

(a) 

(b)

(c) 

(d) 

42. CO_2 molecule has the first few energy levels uniformly separated by approximately 2.5 meV. At a temperature of 300 K, the ratio of the number of molecules in the 4th excited state to the number in the 2nd excited state is about

(a) 0.5 (b) 0.6 (c) 0.8 (d) 0.9

43. Which among the following sets of Maxwell relations is correct ? (U-internal energy, H-enthalpy, A-Helmholtz free energy and G-Gibb's free energy)

(a) $T = \left(\frac{\partial U}{\partial V}\right)_S$ and $P = \left(\frac{\partial U}{\partial S}\right)_V$ (b) $V = \left(\frac{\partial H}{\partial P}\right)_S$ and $T = \left(\frac{\partial H}{\partial S}\right)_P$

(c) $P = -\left(\frac{\partial G}{\partial V}\right)_T$ and $V = \left(\frac{\partial G}{\partial P}\right)_S$ (d) $P = -\left(\frac{\partial A}{\partial S}\right)_T$ and $S = -\left(\frac{\partial A}{\partial P}\right)_V$

44. For a spin-*s* particle, in the eigen basis of $\vec{S}^2$, S_z, the expectation value $\langle s\,m|S_x^2|s\,m\rangle$ is

(a) $\frac{\hbar^2\{s(s+1)-m^2\}}{2}$ (b) $\hbar^2\{s(s+1)-2m^2\}$

(c) $\hbar^2\{s(s+1)-m^2\}$ (d) $\hbar^2 m^2$

45. A particle is placed in a region with the potential $V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$, where $k, \lambda > 0$. Then,

(a) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable equilibrium

(b) $x = 0$ is a point of stable equilibrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium

(c) $x = 0$ and $x = \frac{k}{\lambda}$ are points of unstable equilibrium

(d) There are no points of stable or unstable equilibrium

46. A π^0 meson at rest decays into two photons, which move along the *x*-axis. They are both detected simultaneously after a time, $t = 10\,s$. In an inertial frame moving with a velocity $v = 0.6c$ in the direction of one of the photons, the time interval between the two detections is

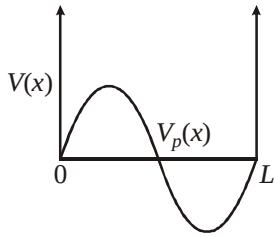
(a) 15 s (b) 0 s (c) 10 s (d) 20 s

47. A particle of mass *m* is confined in an infinite potential well :

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L \\ \infty & \text{otherwise} \end{cases}$$

CAREER ENDEAVOUR

It is subjected to a perturbing potential $V_p(x) = V_0 \sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in V_0, respectively. Which of the following are true ?



(a) $E^{(1)} = 0$; $E^{(2)} < 0$ (b) $E^{(1)} > 0$; $E^{(2)} = 0$

(c) $E^{(1)} = 0$; $E^{(2)}$ depends on the sign of V_0 (d) $E^{(1)} < 0$; $E^{(2)} < 0$

Common Data for Questions 48 and 49 :

In the presence of a weak magnetic field, atomic hydrogen undergoes the transition:

$^2P_{1/2} \rightarrow {}^1S_{1/2}$ by emission of radiation.

48. The number of distinct spectral lines that are observed in the resultant Zeeman spectrum is
(a) 2 (b) 3 (c) 4 (d) 6

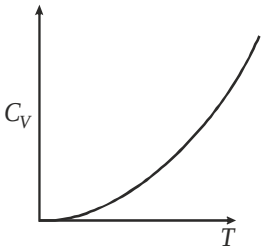
49. The spectral line corresponding to the transition $^2P_{1/2}\left(m_j = +\frac{1}{2}\right) \rightarrow {}^1S_{1/2}\left(m_j = -\frac{1}{2}\right)$ is observed along the direction of the applied magnetic field. The emitted electromagnetic field is
(a) Circularly polarized (b) Linearly polarized
(c) Unpolarized (d) Not emitted along the magnetic field direction

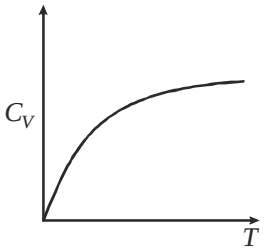
Common Data for Questions 50 and 51 :

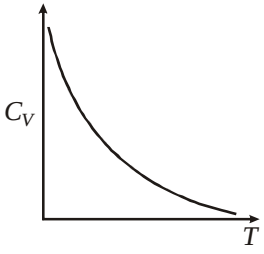
The partition function for a gas of photons is given by

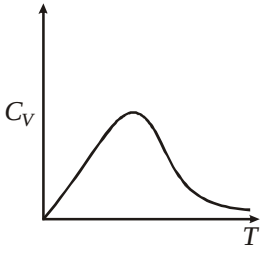
$$\ln Z = \frac{\pi^2}{45} \frac{V(k_B T)^3}{\hbar^3 C^3}$$

50. The specific heat of the photon gas varies with temperature as

(a) 

(b) 

(c) 

(d) 

CAREER ENDEAVOUR

51. The pressure of the photon gas is

(a) $\frac{\pi^2}{15}\frac{(k_BT)^3}{\hbar^3C^3}$ (b) $\frac{\pi^2}{8}\frac{(k_BT)^4}{\hbar^3C^3}$ (c) $\frac{\pi^2}{45}\frac{(k_BT)^4}{\hbar^3C^3}$ (d) $\frac{\pi}{45}\frac{(k_BT)^{3/2}}{\hbar^3C^3}$

Statement for Linked Answer Questions 52 and 53:

Consider the propagation of electromagnetic waves in a linear, homogeneous and isotropic material medium with electric permittivity ε, and magnetic permittivity μ.

52. For a plane wave of angular frequency ω and propagation vector $\hat{k}$ propagating in the medium Maxwell's equations reduce to

(a) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=\omega\varepsilon\vec{H}$; $\vec{k}\times\vec{H}=-\omega\mu\vec{E}$

(b) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=-\omega\varepsilon\vec{H}$; $\vec{k}\times\vec{H}=\omega\mu\vec{E}$

(c) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=-\omega\mu\vec{H}$; $\vec{k}\times\vec{H}=\omega\varepsilon\vec{E}$

(d) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=\omega\mu\vec{H}$; $\vec{k}\times\vec{H}=-\omega\varepsilon\vec{E}$

53. If ε and μ assume negative values in a certain frequency range, then the directions of the propagation vector $\hat{k}$ and the Poynting vector $\vec{S}$ in that frequency range are related as

(a) $\vec{k}$ and $\vec{S}$ are parallel

(b) $\vec{k}$ and $\vec{S}$ are anti-parallel

(c) $\vec{k}$ and $\vec{S}$ are perpendicular to each other

(d) $\vec{k}$ and $\vec{S}$ make an angle that depends on the magnitude of $|\varepsilon|$ and $|\mu|$

Statement for Linked Answer Questions 54 and 55 :

The Lagrangian for a simple pendulum is given by :

$$L=\frac{1}{2}ml^2\dot{\theta}^2-mgl(1-\cos\theta)$$

54. Hamilton's equations are then given by

(a) $\dot{p}_\theta=-mgl\sin\theta$; $\dot{\theta}=\frac{p_\theta}{ml^2}$ (b) $\dot{p}_\theta=mgl\sin\theta$; $\dot{\theta}=\frac{p_\theta}{ml^2}$

(c) $\dot{p}_\theta=-m\ddot{\theta}$; $\dot{\theta}=\frac{p_\theta}{m}$ (d) $\dot{p}_\theta=-\left(\frac{g}{l}\right)\theta$; $\dot{\theta}=\frac{p_\theta}{ml}$

55. The Poisson bracket between θ and $\dot{\theta}$ is

(a) $\{\theta,\dot{\theta}\}=1$ (b) $\{\theta,\dot{\theta}\}=\frac{1}{ml^2}$ (c) $\{\theta,\dot{\theta}\}=\frac{1}{m}$ (d) $\{\theta,\dot{\theta}\}=\frac{g}{l}$

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is **NOT TRUE** ?
   (a) Det A = Det B (b) Trace of A = Trace of B
   (c) A and B have the same eigenvectors (d) A and B have the same eigenvalues

2. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that
   (a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla} \cdot \vec{F} = 0$ (c) $\vec{\nabla} V = 0$ (d) $\nabla^2 V = 0$

3. The quantum mechanical operator for the momentum of a particle moving in one dimension is given by
   (a) $i\hbar \dfrac{d}{dx}$ (b) $-i\hbar \dfrac{d}{dx}$ (c) $i\hbar \dfrac{\partial}{\partial t}$ (d) $-\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2}$

4. A Carnot cycle operates on a working substance between two reservoirs at temperatures T_1 and T_2, with $T_1 > T_2$. During each cycle, an amout of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is **INCORRECT** ?
   (a) work done in one cycle is $Q_1 - Q_2$
   (b) $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$
   (c) entropy of the hotter reservoir decreases
   (d) entropy of the universe (consisting of the working substance and the two reservoirs) increases

5. In a first order phase transition, at the transition temperature, specfic heat of the system
   (a) diverges and its entropy remains the same
   (b) diverges and its entropy has finite discontinuity
   (c) remains unchanged and its entropy has finite discontinuity
   (d) has finite discontinuity and its entropy diverges

6. The semi-empirical mass formula for the binding energy of nucleus contains a surface correction term. This term depends on the mass number A of the nucleus as
   (a) $A^{-1/3}$ (b) $A^{1/3}$ (c) $A^{2/3}$ (d) A

7. The population inversion in a two level laser material cannot be achieved by optical pumping because
   (a) the rate of upward transitions is equal to the rate of downward transitions.
   (b) the upward transitions are forbidden but downward transitions are allowed.
   (c) the upward transitions are allowed but downward transitions are forbidden.
   (d) the spontaneous decay rate of the higher level is very low.

8. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a Curie temperature (T_c) is given by
   (a) $\dfrac{C}{T - T_c}$, for $T < T_c$ (b) $\dfrac{C}{T - T_c}$, for $T > T_c$ (c) $\dfrac{C}{T + T_c}$, for $T > T_c$ (d) $\dfrac{C}{T + T_c}$, for all temperatures

9. The order of magnitude of the energy gap of a typical superconductor is
   (a) 1 MeV (b) 1 KeV (c) 1 eV (d) 1 meV

CAREER ENDEAVOUR

10. Which of the following statements is **CORRECT** for a common emitter amplifier circuit ?
(a) The output is taken from the emitter
(b) There is 180° phase shift between input and output voltages
(c) There is no phase shift between input and output voltages
(d) Both *p-n* junctions are forward biased

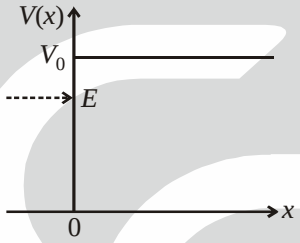
11. A 3 × 3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is
(a) 18 (b) 12 (c) 9 (d) 6

12. A heavy symmetrical top is rotating about its own axis of symmetry (the z-axis). If I_1, I_2 and I_3 are the principal moments of inertia along x, y and z respectively, then
(a) $I_2 = I_3 ; I_1 \neq I_2$ (b) $I_1 = I_3 ; I_1 \neq I_2$ (c) $I_1 = I_2 ; I_1 \neq I_3$ (d) $I_1 \neq I_2 \neq I_3$

13. An electron with energy E is incident from left on a potential barrier, given by

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$$

as shown in the figure.



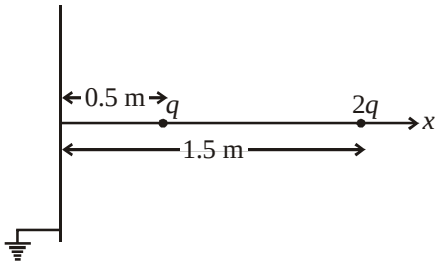
For $E < V_0$, the space part of the wavefunction for $x > 0$ is of the form
(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$ (c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

14. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L, the commutator $\left[L_x L_y, L_z\right]$ is equal to
(a) $i\hbar(L_x^2 + L_y^2)$ (b) $2i\hbar L_z$ (c) $i\hbar(L_x^2 - L_y^2)$ (d) 0

15. The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\langle 1/r^2 \rangle$ is
(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

CAREER ENDEAVOUR

16. Two charges q and $2q$ are placed along the x-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$ (c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

17. A uniform surface current is flowing in the positive y-direction over an infinite sheet lying in xy-plane. The direction of the magnetic field is

(a) along $\hat{i}$ for $z > 0$ and along $-\hat{i}$ for $z < 0$ (b) along $\hat{k}$ for $z > 0$ and along $-\hat{k}$ for $z < 0$

(c) along $-\hat{i}$ for $z > 0$ and along $\hat{i}$ for $z < 0$ (d) along $-\hat{k}$ for $z > 0$ and along $\hat{k}$ for $z < 0$

18. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform magnetic field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is

(a) $\vec{r} \times (m \times \vec{B})$ (b) $\vec{r} \times \vec{\nabla}(m \cdot \vec{B})$ (c) $\vec{m} \times \vec{B}$ (d) $\vec{m} \times \vec{B} + \vec{r} \times \vec{\nabla}(\vec{m} \cdot \vec{B})$

19. Which of the following expressions for a vector potential $\vec{A}$ **DOES NOT** represent a uniform magnetic field of magnitude B_0 along the z-direction?

(a) $\vec{A} = (0, B_0 x, 0)$ (b) $\vec{A} = (-B_0 y, 0, 0)$

(c) $\vec{A} = \left(\frac{B_0 x}{2}, \frac{B_0 y}{2}, 0\right)$ (d) $\vec{A} = \left(-\frac{B_0 y}{2}, \frac{B_0 x}{2}, 0\right)$

20. A neutron passing through a detector is detected because of

(a) the ionization it produces

(b) the scintillation light it produces

(c) the electron-hole pair it produces

(d) the secondary particles produced in a nuclear reaction in the detector medium.

21. An atom with one outer electron having orbital angular momentum ℓ is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is

(a) $2\ell + 2$ (b) $2\ell + 1$ (c) 2ℓ (d) $2\ell - 1$

22. For a multi electron atom ℓ, L and S specify the one electron orbital angular momentum, total angular momentum and total spin angular momentum respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by ℓ, L and S are

(a) $\Delta L = 0, \pm 1;\ \Delta S = 0;\ \Delta \ell = 0, \pm 1$ (b) $\Delta L = 0, \pm 1;\ \Delta S = 0;\ \Delta \ell = \pm 1$

(c) $\Delta L = 0, \pm 1;\ \Delta S = \pm 1;\ \Delta \ell = 0, \pm 1$ (d) $\Delta L = 0, \pm 1;\ \Delta S = \pm 1;\ \Delta \ell = \pm 1$

CAREER ENDEAVOUR

23. For a three-dimensional crystal having N primitive unit cells with a basis of p atoms, the number of optical branches is

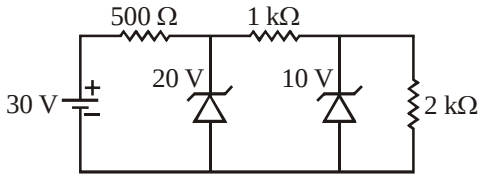
(a) 3 (b) $3p$ (c) $3p-3$ (d) $3N-3p$

24. For an intrinsic semiconductor, m_e^* and m_h^* are respectively the effective masses of electrons and holes near the corresponding band edge. At a finite temperature, the position of the Fermi level

(a) depends on m_e^* but not on m_h^* (b) depends on m_h^* but not on m_e^*

(c) depends on both m_e^* and m_h^* (d) depends neither on m_e^* nor m_h^*

25. In the following circuit, the voltage across and the current through the 2 kΩ resistance are



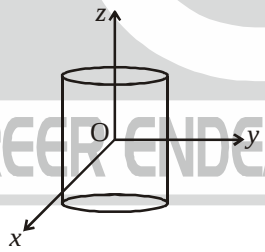
(a) 20 V, 10 mA (b) 20 V, 5 mA (c) 10 V, 10 mA (d) 10 V, 5 mA

Q.26 – Q.55 : Carry TWO marks each.

26. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1, 1, 1)$ is

(a) $\dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\dfrac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$ (c) $\dfrac{\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}$ (d) $\dfrac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

27. Consider a cylinder of height h and a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}\cdot\hat{n}\,ds$ over the closed surface of the cylinder is



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) zero

28. The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

(a) circles with different radii (b) circles with different centers

(c) straight lines with different slopes (d) straight lines with different intercepts on the y-axis

29. A particle is moving under the action of a generalized potential

$$V(q, \dot{q}) = \frac{(1+\dot{q})}{q^2}$$

The magnitude of the generalized force is

(a) $\frac{2(1+\dot{q})}{q^3}$ (b) $\frac{2(1-\dot{q})}{q^3}$ (c) $\frac{2}{q^3}$ (d) $\frac{\dot{q}}{q^3}$

30. Two bodies of mass m and $2m$ are connected by a spring constant k. The frequency of the normal mode is

(a) $\sqrt{\frac{3k}{2m}}$ (b) $\sqrt{\frac{k}{m}}$ (c) $\sqrt{\frac{2k}{3m}}$ (d) $\sqrt{\frac{k}{2m}}$

31. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$$Q = q^{\alpha}\cos(\beta p)$$
$$P = q^{\alpha}\sin(\beta p)$$

is canonical for

(a) $\alpha = 2,\ \beta = 1/2$ (b) $\alpha = 2,\ \beta = 2$ (c) $\alpha = 1,\ \beta = 1$ (d) $\alpha = 1/2,\ \beta = 2$

32. Two particles, each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is

(a) $\frac{5m}{4}$ (b) $2m$ (c) $\frac{5m}{2}$ (d) $\frac{25m}{8}$

33. The normalized eigenstates of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & ; \text{ if } 0 \le x \le a \\ \infty & ; \text{ otherwise} \end{cases}$$

are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \text{ where } n = 1, 2, 3, ...$$

The particle is subjected to a perturbation

$$V'(x) = V_0\cos\left(\frac{\pi x}{a}\right) \text{ for } 0 \le x \le \frac{a}{2}$$
$$= 0 \qquad \text{otherwise}$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is

(a) $\frac{2V_0}{3\pi}$ (b) $\frac{V_0}{3\pi}$ (c) $-\frac{V_0}{3\pi}$ (d) $-\frac{2V_0}{3\pi}$

34. If the isothermal compressibility of a solid is $\kappa_T = 10^{-10}\ (\text{Pa})^{-1}$, the pressure required to increase its density by 1 % is approximately

(a) 10^4 Pa (b) 10^6 Pa (c) 10^8 Pa (d) 10^{10} Pa

35. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

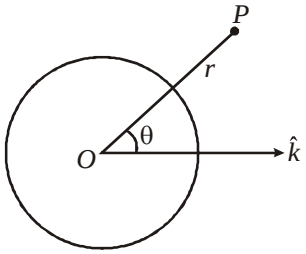
(a) $2k_B \ln N$ (b) $3k_B \ln N$ (c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

36. A system has two energy levels with energies ε and 2ε. The lower level is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is

(a) $\frac{1}{1+e^{-\varepsilon/k_BT}}$ (b) $\frac{1}{1+2e^{\varepsilon/k_BT}}$ (c) $\frac{1}{2e^{\varepsilon/k_BT}+4e^{2\varepsilon/k_BT}}$ (d) $\frac{1}{2e^{\varepsilon/k_BT}-4e^{2\varepsilon/k_BT}}$

CAREER ENDEAVOUR

37. A spherical conductor of radius a is placed in a electric field $\vec{E} = E_0\hat{k}$. The potential at a point $P(r, \theta)$ for $r > a$, is given by $\varphi(r,\theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$, where r is the distance of P from the center O of the sphere and θ is the angle OP makes with the z-axis.



The charge density on the sphere at $\theta = 30°$ is

(a) $3\sqrt{3}\varepsilon_0 E_0/2$ (b) $3\varepsilon_0 E_0/2$ (c) $\sqrt{3}\varepsilon_0 E_0/2$ (d) $\varepsilon_0 E_0/2$

38. According to the single particle nuclear shell model, the spin parity of the ground state of ${}^{17}_{8}O$ is

(a) $\frac{1}{2}^-$ (b) $\frac{3}{2}^-$ (c) $\frac{3}{2}^+$ (d) $\frac{5}{2}^+$

39. In the β- decay of neutron $n \to p + e^- + \bar{\nu}_e$, the anti-neutrino $\bar{\nu}_e$ escapes detection. Its existence is inferred from the measurement of

(a) energy distribution of electrons (b) angular distribution of electrons
(c) helicity distribution of electrons (d) forward-backward asymmetry of electrons

40. The isospin and the strangeness of Ω^- baryon are

(a) 1, –3 (b) 0, –3 (c) 1, 3 (d) 0, 3

41. The life time of an atomic state is 1 nanosecond. The natural line width of the spectral line in the emission spectrum of this state is of the order of

(a) $10^{-10}\,eV$ (b) $10^{-9}\,eV$ (c) $10^{-6}\,eV$ (d) $10^{-4}\,eV$

42. The degeneracy of an excited state of nitrogen atom having electronic configuration $1s^2\ 2s^2\ 2p^2\ 3d^1$ is

(a) 6 (b) 10 (c) 15 (d) 150

43. The far infrared rotational absorption spectrum of a diatomic molecule shows equidistant lines with spacing 20 cm^{-1}. The position of the first Stokes line in the rotational Raman spectrum of this molecule is

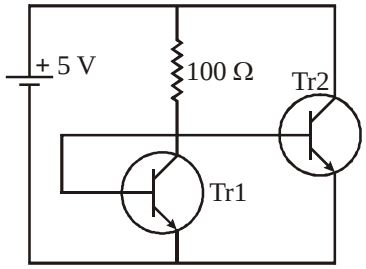
(a) 20 cm^{-1} (b) 40 cm^{-1} (c) 60 cm^{-1} (d) 120 cm^{-1}

44. A metal with body centered cubic (bcc) structure shows the first (i.e. smallest angle) diffraction peak at a Bragg angle of $\theta = 30°$. The wavelength of X-ray used is 2.1 Å. The volume of the PRIMITIVE unit cell of the metal is

(a) 26.2 $(Å)^3$ (b) 13.1 $(Å)^3$ (c) 9.3 $(Å)^3$ (d) 4.6 $(Å)^3$

CAREER ENDEAVOUR

45. In the following circuit, Tr1 and Tr2 are identical transistors having $V_{BE} = 0.7$ V. The current passing through the transistor Tr2 is



(a) 57 mA (b) 50 mA (c) 48 mA (d) 43 mA

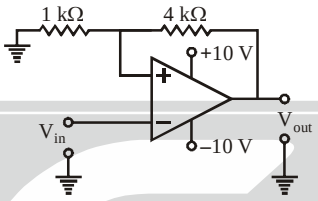
46. The following Boolean expression

$Y = A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot B \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot C \cdot D + \overline{A} \cdot B \cdot C \cdot D + A \cdot \overline{B} \cdot \overline{C} \cdot D$

can be simplified to

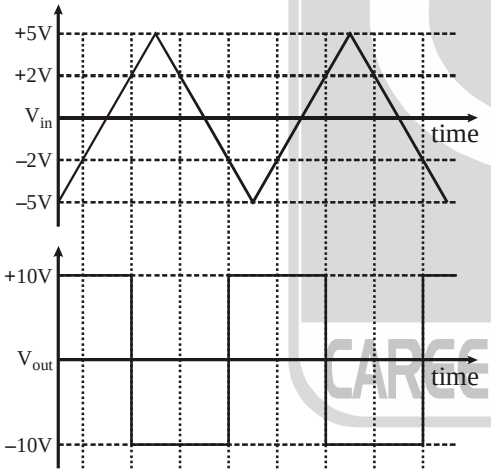
(a) $\overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{D}$ (b) $\overline{A} \cdot B \cdot \overline{C} + A \cdot \overline{D}$ (c) $A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot D$ (d) $A \cdot \overline{B} \cdot C + \overline{A} \cdot D$

47. Consider the following circuit.

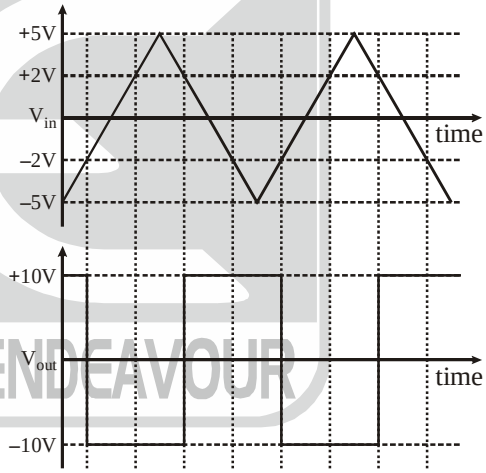


Which of the following correctly represents the output V_{out} corresponding to the input V_{in} ?

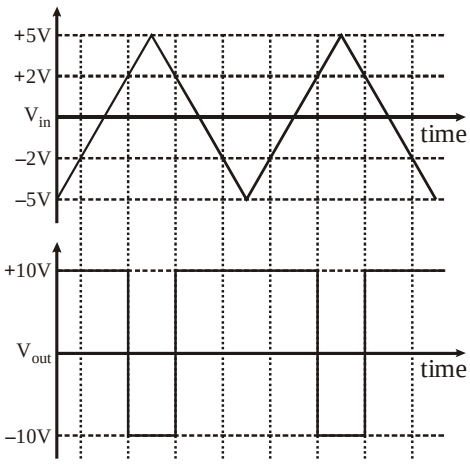
(a)



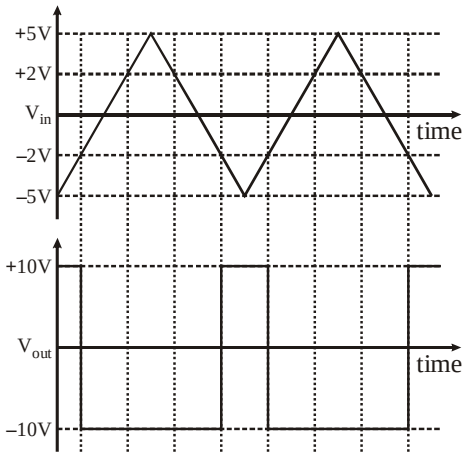
(b)



(c)



(d)



CAREER ENDEAVOUR

Common Data for Questions 48 and 49 :

Consider a function $f(z) = \dfrac{z \sin z}{(z-\pi)^2}$ of a complex variable z.

48. Which of the following statements is **TRUE** for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z = \pi$

(c) $f(z)$ has a pole of order 2 at $z = \pi$

(d) $f(z)$ has a simple pole at $z = \pi$

49. Consider a counterclockwise circular contour $|z| = 1$ about the origin. The integral $\oint f(z)\,dz$ over this contour is

(a) $-i\pi$ (b) zero (c) $i\pi$ (d) $2i\pi$

Common Data for Questions 50 and 51 :

The tight binding energy dispersion (*E*-*k*) relation for electrons in a one-dimensional array of atoms having lattice constant *a* and total length *L* is $E = E_0 - \beta - 2\gamma \cos(ka)$, where E_0, β and γ are constant and *k* is the wave-vector.

50. The density of states of electrons (including spin degeneracy) in the band is given by

(a) $\dfrac{L}{\pi\gamma a \sin(ka)}$ (b) $\dfrac{L}{2\pi\gamma a \sin(ka)}$ (c) $\dfrac{L}{2\pi\gamma a \cos(ka)}$ (d) $\dfrac{L}{\pi\gamma a \cos(ka)}$

51. The effective mass of electrons in the band is given by

(a) $\dfrac{\hbar^2}{\gamma a^2 \cos(ka)}$ (b) $\dfrac{\hbar^2}{2\gamma a^2 \cos(ka)}$ (c) $\dfrac{\hbar^2}{\gamma a^2 \sin(ka)}$ (d) $\dfrac{\hbar^2}{2\gamma a^2 \sin(ka)}$

Statement for Linked Answer Question 52 and 53 :

In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by

$$\psi_1 = \varphi_0 - 2\varphi_1 + 3\varphi_2$$

$$\psi_2 = \varphi_0 - \varphi_1 + \alpha\varphi_2$$

where α is a constant.

52. The value of α for which ψ_2 is orthogonal to ψ_1 is

(a) 2 (b) 1 (c) –1 (d) –2

53. For the value of α determined in Q. 52, the expression value of energy of the oscillator in the state ψ_2 is

(a) $\hbar\omega$ (b) $\dfrac{3\hbar\omega}{2}$ (c) $3\hbar\omega$ (d) $\dfrac{9\hbar\omega}{2}$

CAREER ENDEAVOUR

Statement for Linked Answer Question 54 and 55 :

A plane electromagnetic wave has the magnetic field given by

$$\vec{B}(x, y, z, t) = B_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{k}$$

where k is the wave number and $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the Cartesian unit vectors in x, y and z direction, respectively.

54. The electric field $\vec{E}(x, y, z, t)$ corresponding to the above wave is given by

(a) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(c) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{i}$ (d) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{j}$

55. The average Poynting vector is given by

(a) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (c) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (d) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$

   (a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

   (b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

   (c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

   (d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

2. Two uniform thin rods of equal length L, and masses M_1 and M_2 are joined together along the length. The moment of inertia of the combined rod of length 2L about an axis passing through the mid-point perpendicular to the length of the rod is,

   (a) $(M_1 + M_2)\frac{L^2}{12}$ (b) $(M_1 + M_2)\frac{L^2}{6}$ (c) $(M_1 + M_2)\frac{L^2}{3}$ (d) $(M_1 + M_2)\frac{L^2}{2}$

3. The space-time dependence of the electric field of a linearly polarized light in free space is given by $\hat{x}E_0\cos(\omega t - kz)$, where E_0, ω and k are the amplitude, the angular frequency and the wavevector, respectively. The time averaged energy density associated with the electric field is

   (a) $\frac{1}{4}\varepsilon_0 E_0^2$ (b) $\frac{1}{2}\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2$ (d) $2\varepsilon_0 E_0^2$

4. If the peak output voltage of a full wave rectifier is 10 V, its d.c. voltage is

   (a) 10.0V (b) 7.07V (c) 6.36 V (d) 3.18V

5. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x, y)$ is given by

   $$V(x, y) = 0 \text{ for } -a < x < a \text{ and } -a < y < a$$
   $$= \infty \text{ else where}$$

   The energy of the first excited state for this particle is given by,

   (a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

6. The isothermal compressibility, κ of an ideal gas at temperature T_0 and volume V_0, is given by

   (a) $-\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (c) $-V_0\frac{\partial P}{\partial V}\Big|_{T_0}$ (d) $V_0\frac{\partial P}{\partial P}\Big|_{T_0}$

7. The ground state of sodium atom (^{11}Na) is a $^2S_{1/2}$ state. The difference in energy levels arising in the presence of a weak external magnetic field B, given in terms of Bohr magneton, μ_B is

   (a) $\mu_B B$ (b) $2\mu_B B$ (c) $4\mu_B B$ (d) $6\mu_B B$

8. For an ideal Fermi gas in three dimensions, the electron velocity V_F at the Fermi surface is related to electron concentration n as,

   (a) $V_F \propto n^{2/3}$ (b) $V_F \propto n$ (c) $V_F \propto n^{1/2}$ (d) $V_F \propto n^{1/3}$

9. Which one of the following sets corresponds to fundamental particles ?

   (a) proton, electron and neutron (b) proton, electron and photon

   (c) electron, photon and neutrino (d) quark, electron and meson

CAREER ENDEAVOUR

10. In case of a Geiger-Muller (GM) counter, which one of the following statements is CORRECT?

(a) Multiplication factor of the detector is of the order of 10^{10}.
(b) Type of the particles detected can be identified.
(c) Energy of the particles detected can be distinguished.
(d) Operating voltage of the detector is few tens of Volts.

11. A plane electromagnetic wave traveling in free space is incident normally on a glass plate of refractive index 3/2. If there is no absorption by the glass, Its reflectivity is

(a) 4% (b) 16 % (c) 20% (d) 50%

12. A Ge semiconductor is doped with acceptor impurity concentration of 10^{15} atoms /cm^3. For the given hole mobility of 1800 cm^2 / V – s, the resistivity of this material is:

(a) 0.288 Ω cm (b) 0.694 Ω cm (c) 3.472 Ω cm (d) 6.944 Ω cm

13. A classical gas of molecules each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $(v_x + v_y)^2$ is:

(a) $\frac{k_B T}{m}$ (b) $\frac{3}{2}\frac{k_B T}{m}$ (c) $\frac{1}{2}\frac{k_B T}{m}$ (d) $\frac{2k_B T}{m}$

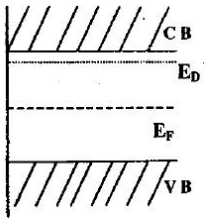
14. In a central force field the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by

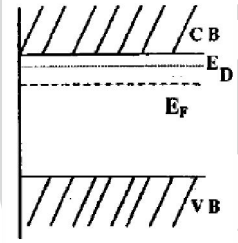
$$\frac{1}{r} = \frac{m}{L^2}(1 + \varepsilon \cos\theta)$$

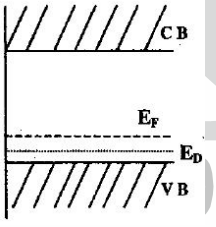
Where, ε is the eccentricity of the particle's motion. Which one of the following choices for ε gives rise to a parabolic trajectory ?

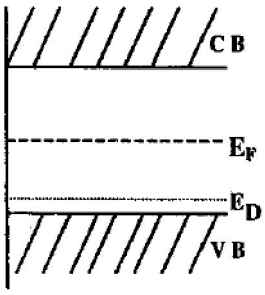
(a) $\varepsilon = 0$ (b) $\varepsilon = 1$ (c) $0 < \varepsilon < 1$ (d) $\varepsilon > 1$

15. Identify the CORRECT energy band diagram for silicon doped with Arsenic. Here CB, VB, E_D and E_F conduction band, valence band, impurity level and Fermi level, respectively.

(a) 

(b) 

(c) 

(d) 

16. The first stokes line of a rotational Raman spectrum is observed at 12.96 cm^{-1}. Considering the rigid rotor approximation, the rotational constant is given by

(a) 6.48 cm^{-1} (b) 3.24 cm^{-1} (c) 2.16 cm^{-1} (d) 1.62 cm^{-1}

17. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the CORRECT equation of state (P being the pressure),

(a) $PV = \frac{1}{3}E$ (b) $PV = \frac{2}{3}E$ (c) $PV = E$ (d) $PV = \frac{5}{3}E$

18. Consider the wavefunction $\psi = \psi(\vec{r}_1, \vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}\left[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2) + \phi_2(\vec{r}_1)\phi_1(\vec{r}_2)\right]$$

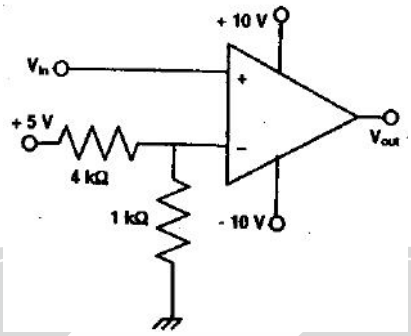
Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta+\beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta-\beta\alpha)$ (c) $\alpha\alpha$ (d) $\beta\beta$

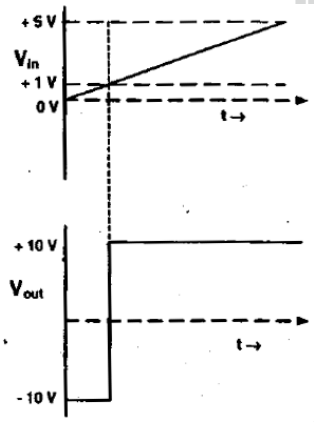
19. The electric and the magnetic fields $\vec{E}(z,t)$ and $\vec{B}(z,t)$, respectively corresponding to the scalar potential $\phi(z,t)=0$ and vector potential $\vec{A}(z,t)=\hat{i}tz$ are

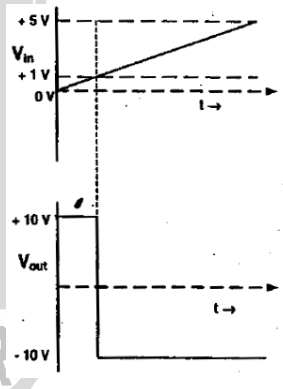
(a) $\vec{E}=\hat{i}z$ and $\vec{B}=-\hat{j}t$ (b) $\vec{E}=\hat{i}z$ and $\vec{B}=\hat{j}t$ (c) $\vec{E}=-\hat{i}z$ and $\vec{B}=-\hat{j}t$ (d) $\vec{E}=-\hat{i}z$ and $\vec{B}=\hat{j}t$

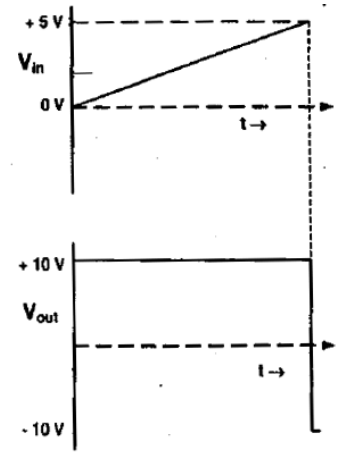
20. Consider the following OP-AMP circuit.

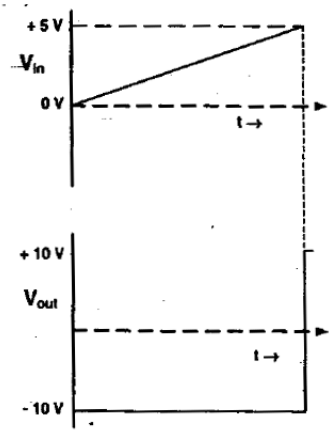


Which one of the following correctly represents the output V_{out} corresponding to the input V_{in} ?

(a)


(b)


(c)


(d)


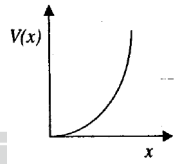
CAREER ENDEAVOUR

21. Deuteron has only one bound state with spin parity 1^+, isospin 0 and electric quadrupole moment 0.286 efm^2. These data suggest that the nuclear forces are having
(a) only spin and isospin dependence (b) no spin dependence and no tensor components
(c) spin dependence but no tensor components (d) spin dependence along with tensor components

22. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is
(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

23. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids ?
(a) Debye's T^3 law (b) Dulong Petit's law
(c) Optical branches in lattices (d) Thermal expansion

24. A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is CORRECT ?



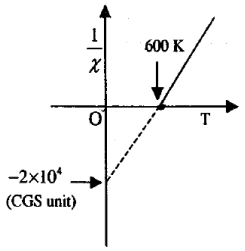
(a) The parity of the first excited state is even (b) The parity of the ground state is even
(c) The ground state energy is $\frac{1}{2}\hbar\omega$ (d) The first excited state energy is $\frac{7}{2}\hbar\omega$

25. The number of independent components of the symmetric tensor A_{ij} with indices i, j = 1, 2, 3 is
(a) 1 (b) 3 (c) 6 (d) 9

Q.26 – Q.55 : Carry TWO marks each.

26. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$, where $\delta << 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are
(a) 1 and $(1+2\delta)$ (b) $(1+\delta)$ and $(1-\delta)$ (c) $(1+2\delta)$ and $(1-2\delta)$ (d) $(1+\delta)$ and $(1-2\delta)$

27. Inverse susceptibility $(1/\chi)$ as a function of temperature, T for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. The values of the Curie constant, C and the Weiss molecular field constant, λ, in CGS units, are



(a) $C = 5\times10^{-5}, \lambda = 3\times10^{-2}$ (b) $C = 3\times10^{-2}, \lambda = 5\times10^{-5}$
(c) $C = 3\times10^{-2}, \lambda = 2\times10^{-4}$ (d) $C = 2\times10^{4}, \lambda = 3\times10^{-2}$

CAREER ENDEAVOUR

28. A plane polarized electromagnetic wave in free space at time t = 0 is given by $\vec{E}(x,z) = 10\hat{j}\exp\left[i(6x+8z)\right]$.

The magnetic field $\vec{B}(x,z,t)$ is given by

(a) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i(6x+8z-10ct)\right]$.

(b) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\exp\left[i(6x+8z-10ct)\right]$.

(c) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i(6x+8z-ct)\right]$.

(d) $\vec{B}(x,z,t) = \frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\exp\left[i(6x+8z+ct)\right]$.

29. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0,1,1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

30. Match the typical spectroscopic regions specified in List-I with the corresponding type of transitions in List-II and find the correct answer using the codes given below the list :

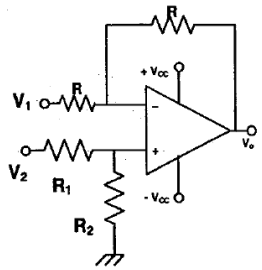
List-I	List-II
P. Infrared region	1. Electronic transitions involving valence electrons
Q. Ultraviolet visible region	2. Nuclear transitions
R. X-ray region	3. Vibrational transitions of molecules
S. γ - rays region	4. Transitions involving inner shell electrons

Codes :

	P	Q	R	S
(a)	1	3	2	4
(b)	2	4	1	3
(c)	3	1	4	2
(d)	4	1	2	3

CAREER ENDEAVOUR

31. In the following circuit, for the output voltage to $V_0 = (-V_1 + V_2/2)$ the ratio R_1/R_2 is



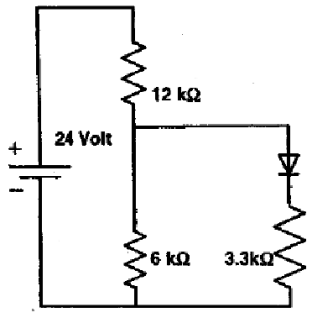
(a) 1/2 (b) 1 (c) 2 (d) 3

32. The terms $\left\{\hat{J}_1, \hat{J}_2\right\}_j$ arising from $2s^1\,3d^1$ electronic configuration in J – J scheme are

(a) $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$ (b) $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$

(c) $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$ (d) $\left\{\frac{3}{2}, \frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

33. In the following circuit, the voltage drop across the ideal diode in forward bias condition is 0.7 V.



The current passing through the diode is

(a) 0.5 mA (b) 1.0 mA (c) 1.5 mA (d) 2.0 mA

34. Choose the CORRECT statement from the following

(a) Neutron interacts through electromagnetic interaction

(b) Electron does not interact through weak interaction

(c) Neutrino interacts through weak and electromagnetic interaction

(d) Quark interacts through strong interaction but not through weak interaction

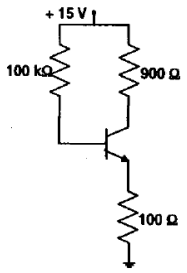
35. A rod of proper length ℓ_0 oriented parallel to the x-axis moves with speed 2c/3 along the x-axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

(a) $0.35\,\ell_0$ (b) $0.48\,\ell_0$ (c) $0.87\,\ell_0$ (d) $0.97\,\ell_0$

36. A simple cubic crystal with lattice parameter a_c undergoes transition into a tetragonal structure with lattice parameter $a_t = b_t = \sqrt{2}\,a_c$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacings of (101) planes for the cubic and the tetragonal structure is

(a) $\sqrt{\frac{1}{6}}$ (b) $\frac{1}{6}$ (c) $\sqrt{\frac{3}{8}}$ (d) $\frac{3}{8}$

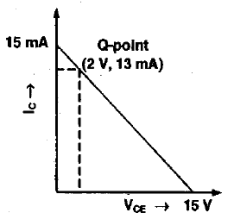
37. Consider the following circuit in which the current gain β_{dc} of the transistor is 100.



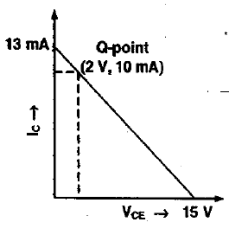
Which one of the following correctly represents the load line (collector current I_c with respect to collectr-emitter voltage V_{CE}) and Q-point of this circuit?

CAREER ENDEAVOUR

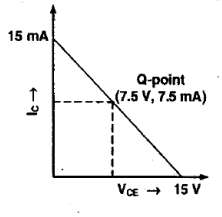
(a)



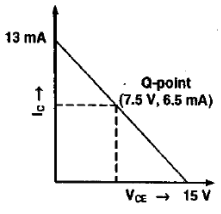
(b)



(c)



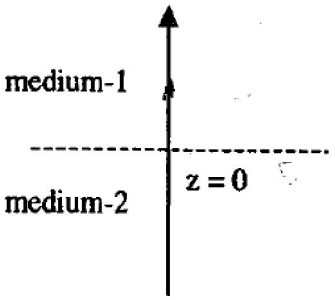
(d)



38. Consider a system whose three energy levels are given by 0, ε and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta = \frac{1}{k_B T}$ is given by

(a) $1+2e^{-\beta\epsilon}$ (b) $2e^{-\beta\epsilon}+e^{-2\beta\epsilon}$ (c) $\left(1+e^{-\beta\epsilon}\right)^2$ (d) $1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}$

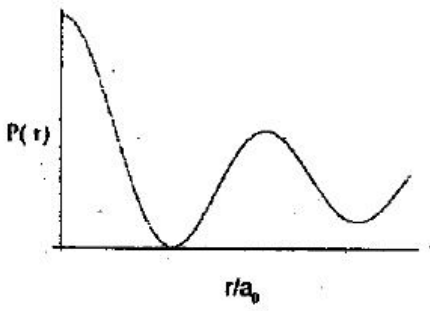
39. Two infinitely extended homogeneous isotropic dielectric media (medium -1 and medium – 2 with dielectric constants $\frac{\varepsilon_1}{\varepsilon_0}=2$ and $\frac{\varepsilon_2}{\varepsilon_0}=5$, respectively) meet at the z = 0 plane as shown in the figure. A uniform electric field exists everywhere. For $z \geq 0$, the electric field is given by $\vec{E}_1 = 2\hat{i}-3\hat{j}+5\hat{k}$. The interface separating the two media is charge free.
The electric displacement vector in the medium-2 is given by

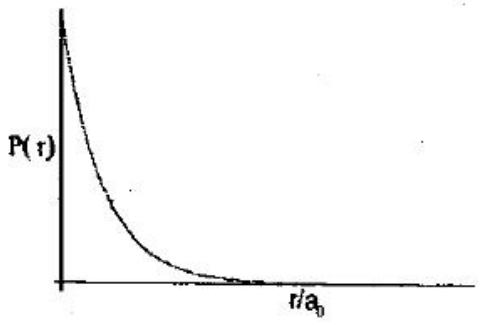


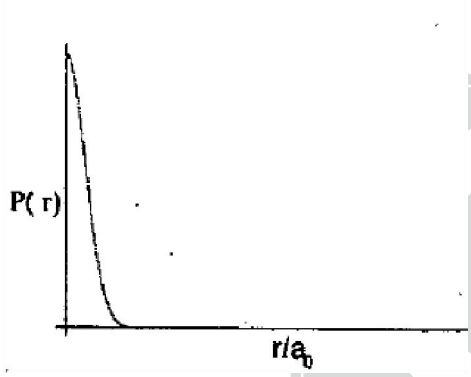
(a) $\overrightarrow{D_2}=\varepsilon_0\left[10\hat{i}+15\hat{j}+10\hat{k}\right]$ (b) $\overrightarrow{D_2}=\varepsilon_0\left[10\hat{i}-15\hat{j}+10\hat{k}\right]$

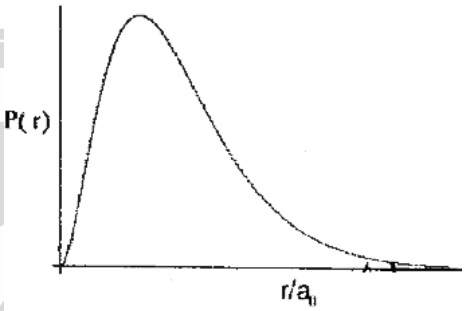
(c) $\overrightarrow{D_2}=\varepsilon_0\left[4\hat{i}-6\hat{j}+10\hat{k}\right]$ (d) $\overrightarrow{D_2}=\varepsilon_0\left[4\hat{i}+6\hat{j}+10\hat{k}\right]$

40. The ground state wavefunction for the hydrogen atom is given by $\psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where a_0 is the Bohr radius.
The plot of the radial probability density, P(r) for the hydrogen atom in the ground state is

(a) 

(b) 

(c) 

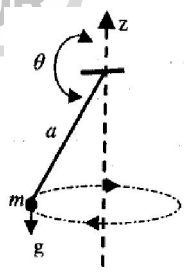
(d) 

41. Total binding energies of O^{15}, O^{16} and O^{17} are 111.96 MeV, 127.62 MeV and 131.76 MeV, respectively. The energy gap between $^1p_{1/2}$ and $^1d_{5/2}$ neutron shells for the nuclei whose mass number is close to 16, is:
(a) 4.1 MeV (b) 11.5 MeV (c) 15.7 MeV (d) 19.8 MeV

42. A particle of mass m is attached to a fixed point 'O' by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure.
The Lagrangian of the particle is

$$L(\theta,\phi) = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) - mga\cos\theta$$

where θ and ϕ are the polar angles.



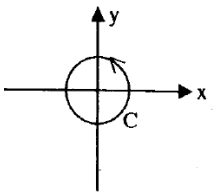
The Hamiltonian of the particle is:

(a) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) - mga\cos\theta$ (b) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

(c) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) - mga\cos\theta$ (d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) + mga\cos\theta$

CAREER ENDEAVOUR

43. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0\left(\hat{i} + \hat{j} + k\right)$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is,



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

44. The value of the integral $\oint_C e^{1/z} dz$, using the contour C of circle with unit radius $|z| = 1$ is:

(a) 0 (b) $1 - 2\pi i$ (c) $1 + 2\pi i$ (d) $2\pi i$

45. A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that N/2 spins are aligned parallel and the remaining N/2 spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is,

(a) $2Nk_B \ell n\ 2$ (b) $2\frac{N}{2}k_B\ \ell n\ 2$ (c) $\frac{3N}{2}k_B\ \ell n\ 2$ (d) $Nk_B\ \ell n\ 2$

46. The equilibrium vibration frequency for a oscillator is observed at 2990 cm^{-1}. The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96. Considering the oscillator to be anharmonic, the anharmonicity constant is

(a) 0.005 (b) 0.02 (c) 0.05 (d) 0.1

47. At a certain temperature T, the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are, respectively,

(a) 355 m/s, 434 m/s (b) 820 m/s, 917 m/s
(c) 152 m/s, 301 m/s (d) 422 m/s, 600 m/s

Common data for Q.48 and Q.49

The wavefunction of a particle moving in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$

48. The energy of the particle is

(a) $\frac{5\hbar^2k^2}{2m}$ (b) $\frac{3\hbar^2k^2}{4m}$ (c) $\frac{\hbar^2k^2}{2m}$ (d) $\frac{\hbar^2k^2}{m}$

49. The probability current density for the real part of the wavefunction is

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

Common data for Q.50 and Q. 51

The dispersion relation for a one dimensional monatomic crystal with lattice spacing a, which interacts via nearest neighbour harmonic potential is given by

$$\omega = A\left|\sin\frac{Ka}{2}\right|,$$

where A is a constant of a appropriate unit.

CAREER ENDEAVOUR

50. The group velocity at the boundary of the first Brillouin zone is

(a) 0 (b) 1 (c) $\sqrt{\frac{Aa^2}{2}}$ (d) $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

51. The force constant between the nearest neighbour of the lattice is (M is the mass of the atom)

(a) $\frac{MA^2}{4}$ (b) $\frac{MA^2}{2}$ (c) MA^2 (d) $2MA^2$

Statement for Linked Answer Q.52 and Q.53

In a hydrogen atom, consider that the electronic charge is uniformly distributed in a spherical volume of radius $a\left(=0.5\times10^{-10}\,m\right)$ around the proton. The atom is placed in a uniform electric field $E=30\times10^5\ V/m$. Assume that the spherical distribution of the negative charge remains undistorted under the electric field.

52. In the equilibrium condition, the separation between the positive and the negative charge centers is

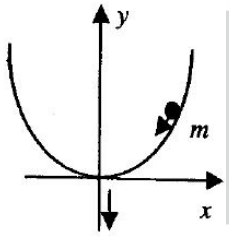
(a) $8.66\times10^{-16}\,m$ (b) $2.60\times10^{-15}\,m$ (c) $2.60\times10^{-16}\,m$ (d) $8.66\times10^{-15}\,m$

53. The polarizability of the hydrogen atom in unit of $\left(C^2m/N\right)$ is

(a) 2.0×10^{-40} (b) 1.4×10^{-41} (c) 1.4×10^{-40} (d) 2.0×10^{-39}

Statement for Linked Answer Q.54 and Q.55

A particle of mass m slides under the gravity without friction along parabolic path $y=ax^2$ as shown in the figure. Here a is a constant.



54. The Lagrangian for this particle is given by,

(a) $L=\frac{1}{2}m\dot{x}^2-mgax^2$ (b) $L=\frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2-mgax^2$

(c) $L=\frac{1}{2}m\dot{x}^2+mgax^2$ (d) $L=\frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2+mgax^2$

55. The Lagrange's equation of motion of the particle is

(a) $\ddot{x}=2gax$ (b) $m\left(1+4a^2x^2\right)\ddot{x}=-2mgax-4ma^2x\,\dot{x}^2$

(c) $m\left(1+4a^2x^2\right)\ddot{x}=2mgax+4ma^2x\,\dot{x}^2$ (d) $\ddot{x}=-2gax$

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. $f(x)$ is a symmetric periodic function of x i.e. $f(x) = f(-x)$. Then, in general, the Fourier series of the function $f(x)$ will be of the form

(a) $f(x) = \sum_{n=1}^{\infty} (a_n \cos(nkx) + b_n \sin(nkx))$ (b) $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nkx))$

(c) $f(x) = \sum_{n=1}^{\infty} (b_n \sin(nkx))$ (d) $f(x) = a_0 + \sum_{n=1}^{\infty} (b_n \sin(nkx))$

2. In the most general case, which one of the following quantities is NOT a second order tensor?
(a) Stress (b) Strain (c) Moment of inertia (d) Pressure

3. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is
(a) c (b) $-c$ (c) $0.15c$ (d) $-0.15c$

4. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be (k is the Boltzmann constant and T is non zero)

(a) Zero (b) Infinite (c) $\frac{3}{2}kT$ (d) kT

5. Across a first order phase transition, the free energy is
(a) proportional to the temperature
(b) a discontinuous function of the temperature
(c) a continuous function of the temperature but its derivative is discontinuous
(d) such that the first derivative with respect to temperature is continuous

6. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same
(a) pressure and temperature (b) volume and temperature
(c) pressure and volume (d) volume and energy

7. The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where a and E_0 are positive constants. The temperature of the system is
(a) negative for some energies (b) increases monotonically with energy
(c) decreases monotonically with energy (d) zero

8. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant)

(a) Zero (b) Nk (c) $\frac{1}{2}Nk$ (d) $Nk\ ln(2)$

9. The decay process $n \to p^+ + e^- + \bar{\nu}_e$ violates
(a) baryon number (b) lepton number (c) isospin (d) strangeness

10. The isospin (I) and baryon number (B) of the up quark is
(a) I = 1, B = 1 (b) I = 1, B = 1/3 (c) I = 1/2, B = 1 (d) I = 1/2, B = 1/3

11. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range r_0. Given that, $\cot(kr_0 + \delta) \approx -\frac{\gamma}{k}$ where δ is the phase shift, k the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

(a) $\delta \approx -\frac{k}{\gamma} - kr_0$ (b) $\delta \approx -\frac{\gamma}{k} - kr_0$ (c) $\delta \approx \frac{\pi}{2} - kr_0$ (d) $\delta \approx -\frac{\pi}{2} - kr_0$

CAREER ENDEAVOUR

12. In the β decay process, the transition $2^+ \rightarrow 3^+$, is
(a) allowed both by Fermi and Gamow-Teller selection rule
(b) allowed by Fermi and but not by Gamow-Teller selection rule
(c) not allowed by Fermi but allowed by Gamow-Teller selection rule
(d) not allowed both by Fermi and Gamow-Teller selection rule

13. At a surface current, which one of the magnetostatic boundary condition is NOT CORRECT?
(a) Normal component of the magnetic field is continuous.
(b) Normal component of the magnetic vector potential is continuous.
(c) Tangential component of the magnetic vector potential is continuous.
(d) Tangential component of the magnetic vector potential is not continuous.

14. Interference fringes are seen at an observation plane $z = 0$, by the superposition of two plane waves $A_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$ and $A_2 \exp[i(\vec{k}_2 \cdot \vec{r} - \omega t)]$, where A_1 and A_2 are real amplitudes. The condition for interference maximum is

(a) $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = (2m+1)\pi$ (b) $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = 2m\pi$ (c) $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = (2m+1)\pi$

(d) $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = 2m\pi$

15. For a scalar function φ satisfying the Laplace equation, $\nabla\varphi$ has
(a) zero curl and non-zero divergence (b) non-zero curl and zero divergence
(c) zero curl and zero divergence (d) non-zero curl and non-zero divergence

16. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is CORRECT?
(a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.
(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.
(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.
(d) There will be no reflected light and the transmitted light is circularly polarized.

17. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings.

(a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = i\hbar L_z$ (c) $[L_z, L_+] = \hbar L_+$ (d) $[L_z, L_-] = \hbar L_-$

18. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

(a) $p_q = 2\beta q$ and it is a conserved quantity. (b) $p_q = 2\beta q$ and it is not a conserved quantity.

(c) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity. (d) $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

19. What should be the clock frequency of a 6-bit A/D converter so that its maximum conversion time is $32\mu s$?
(a) 1 MHz (b) 2 MHz (c) 0.5 MHz (d) 4 MHz

20. A phosphorous doped silicon semiconductor (doping density: 10^{17} / cm^3) is heated from 100ºC to 200ºC. Which one of the following statements is CORRECT?
(a) Position of Fermi level moves towards conduction band
(b) Position of dopant level moves towards conduction band
(c) Position of Fermi level moves towards middle of energy gap
(d) Position of dopant level moves towards middle of energy gap

CAREER ENDEAVOUR

21. Considering the BCS theory of superconductors, which one of the following statements is NOT CORRECT? (h is the Planck's constant and e is the electronic charge)

(a) Presence of energy gap at temperatures below the critical temperature

(b) Different critical temperatures or isotopes

(c) Quantization of magnetic flux in superconducting ring in the unit of $\left(\frac{h}{e}\right)$

(d) Presence of Meissner effect

22. Group I contains elementary excitations in solids. Group II gives the associated fields with these excitations. MATCH the excitations with their associated field and select your answer as per codes given below.

Group-I	**Group-II**
(P) phonon	(i) photon + lattice vibration
(Q) plasmon	(ii) electron + elastic deformation
(R) polaron	(iii) collective electron oscillations
(S) polariton	(iv) elastic wave

Codes:

(a) (P–iv), (Q-iii), (R-i), (S-ii) (b) (P–iv), (Q-iii), (R-ii), (S-i)

(c) (P–i), (Q-iii), (R-ii), (S-iv) (d) (P–iii), (Q-iv), (R-ii), (S-i)

23. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is _____

24. A voltage regulator has ripple rejection of –50dB. If input ripple is 1mV, what is the output ripple voltage in μV? The answer should be up to two decimal places _____

25. The number of spectral lines allowed in the spectrum for the $3^2D \rightarrow 3^2P$ transition in sodium is _____

Q.26 – Q.55 : Carry TWO marks each.

26. Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(s)$ is NOT CORRECT?

(a) $F(t) = \delta(t),\ f(s) = 1,$ (Singularity at +0)

(b) $F(t) = 1,\ f(s) = \frac{1}{s}$ $(s > 0)$

(c) $F(t) = \sin kt,\ f(s) = \frac{s}{s^2 + k^2}, (s > 0)$

(d) $F(t) = te^{kt},\ f(s) = \frac{1}{(s-k)^2}, (s > k, s > 0)$

CAREER ENDEAVOUR

27. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\nabla(\vec{A}\cdot\vec{B}\times\vec{r})$ is

(a) $\vec{A}\cdot\vec{B}$ (b) $\vec{A}\times\vec{B}$ (c) $\vec{r}$ (d) zero

28. $\Gamma\left(n+\frac{1}{2}\right)$ is equal to [Given $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma(1/2) = \sqrt{\pi}$]

(a) $\frac{n!}{2^n}\sqrt{\pi}$ (b) $\frac{2n!}{n!2^n}\sqrt{\pi}$ (c) $\frac{2n!}{n!2^{2n}}\sqrt{\pi}$ (d) $\frac{n!}{2^{2n}}\sqrt{\pi}$

29. The relativistic form of Newton's second law of motion is

(a) $F = \frac{mc}{\sqrt{c^2 - v^2}}\frac{dv}{dt}$ (b) $F = \frac{m\sqrt{c^2 - v^2}}{c}\frac{dv}{dt}$ (c) $F = \frac{mc^2}{c^2 - v^2}\frac{dv}{dt}$ (d) $F = m\frac{c^2 - v^2}{c^2}\frac{dv}{dt}$

30. Consider a gas of atoms obeying Maxwell-Boltzmann statistics. The average value of $e^{i\vec{a}.\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is,

(a) One (b) Zero (c) $e^{-\frac{1}{2}a^2mkT}$ (d) $e^{-\frac{3}{2}a^2mkT}$

31. The electromagnetic form factor $F(q^2)$ of a nucleus is given by, $F(q^2) = \exp\left|-\frac{q^2}{2Q^2}\right|$

where Q is a constant. Given that $F(q^2) = \frac{4\pi}{q}\int_0^\infty rdr\ \rho(r)\sin qr \quad \int d^3r\,\rho(r) = 1$

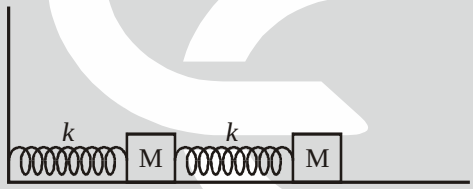
where $\rho(r)$ is the charge density, the root mean square radius of the nucleus is given by

(a) $1/Q$ (b) $\sqrt{2}/Q$ (c) $\sqrt{3}/Q$ (d) $\sqrt{6}/Q$

32. A uniform circular disk of radius R and mass M is rotating with angular speed ω about an axis, passing through its center and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is,

(a) $\frac{\sqrt{3}}{4}\omega MR^2$ (b) $\frac{\sqrt{3}}{8}\omega MR^2$ (c) $\frac{\sqrt{7}}{8}\omega MR^2$ (d) $\frac{\sqrt{7}}{4}\omega MR^2$

33. Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,



(a) $\sqrt{\frac{3+\sqrt{2}}{2}}\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{3+\sqrt{3}}{2}}\sqrt{\frac{k}{M}}$ (c) $\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{M}}$ (d) $\sqrt{\frac{3+\sqrt{6}}{2}}\sqrt{\frac{k}{M}}$

34. A charge distribution has the charge density given by $\rho = Q\{\delta(x-x_0) - \delta(x+x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

(a) $\frac{2Q\hat{x}}{9\pi\in_0 x_0^2}$ (b) $\frac{Q\hat{x}}{4\pi\in_0 x_0^3}$ (c) $\frac{Q\hat{x}}{4\pi\in_0 x_0^2}$ (d) $\frac{Q\hat{x}}{16\pi\in_0 x_0^2}$

35. A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If $\hat{k}_i$, $\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expression is correct?

(a) $(\hat{k}_i - \hat{k}_r)\times\hat{n} \neq 0$ (b) $(\hat{k}_i - \hat{k}_r)\cdot\hat{n} = 0$ (c) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r = 0$ (d) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r \neq 0$

36. In a normal Zeeman effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition $5^1D_2 \rightarrow 5^1P_1$ of cadmium atom is to be observed. The spectrometer has a resolution of 0.01 nm. The minimum magnetic field needed to observe this is (m_e = 9.1 × 10^{-31} *kg*, e = 1.6×10^{-19} *C*, c = 3 × 10^8 *m/s*)

(a) 0.26 T (b) 0.52 T (c) 2.6 T (d) 5.2 T

CAREER ENDEAVOUR

37. The spacing between vibrational energy levels in CO molecule is found to be $8.44\text{l} \times 10^{-2}$ eV. Given that the reduced mass of CO is 1.14×10^{-26} kg, Planck's constant is 6.626×10^{-34} Js and $1\text{eV} = 1.6 \times 10^{-19}$J. The force constant of the bond is CO molecule is

(a) 1.87 N/m (b) 18.7 N/m (c) 187 N/m (d) 1870 N/m

38. A lattice has the following primitive vectors (in A): $= \vec{a} = 2(\hat{j} + \hat{k}), \vec{b} = 2(\hat{k} + \hat{i}), \vec{c} = 2(\hat{i} + \hat{j})$. The reciprocal lattice corresponding to the above lattice is

(a) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$ (b) BCC lattice with cube edge of $(2\pi)A^{-1}$

(c) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$ (d) FCC lattice with cube edge of $(2\pi)A^{-1}$

39. The total energy of an ionic solid is given by an expression $E = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where α is Madelung constant, r is the distance between the nearest neighbours in the crystal and B is a constant. If r_0 is the equilibrium separation between the nearest neighbours then the value of B is

(a) $\frac{\alpha e^2 r_0^8}{36\pi \in_0}$ (b) $\frac{\alpha e^2 r_0^8}{4\pi \in_0}$ (c) $\frac{2\alpha e^2 r_0^{10}}{9\pi \in_0}$ (d) $\frac{\alpha e^2 r_0^{10}}{36\pi \in_0}$

40. A proton is confined to a cubic box, whose sides have length 10^{-12} *m*. What is the minimum kinetic energy of the proton? The mass of proton is 1.67×10^{-27} *kg* and Planck's constant is 6.63×10^{-34} *Js*.

(a) 1.1×10^{-17} *J* (b) 3.3×10^{-17} *J* (c) 9.9×10^{-17} *J* (d) 6.6×10^{-17} *J*

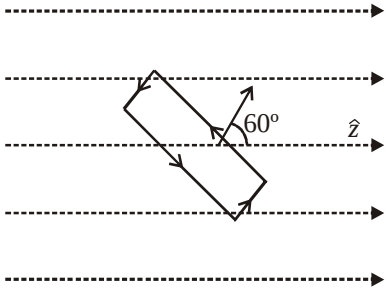
41. For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z = 1$ is (your answer should be an integer) _____.

42. The degenerate eigenvalue of the matrix

$$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$$ is (your answer should be an integer) _____

43. Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \to \mu^- + \bar{\nu}_\mu$ in the pion rest frame. $m_\pi = 139.6\,MeV/c^2$, $m_\mu = 105.7\,MeV/c^2, m_\nu \approx 0$. The energy (in MeV) of the emitted neutrino, to the nearest integer is _____

44. In a constant magnetic field of 0.6 Tesla along the *z* direction, find the value of the path integral $\oint \vec{A}.\vec{dl}$ in the units of (Tesla m^2) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60º to the z-axis, as shown in the figure. The answer should be up to two decimal places. ____



CAREER ENDEAVOUR

45. A spin-half particle is in a linear superposition $0.8\left|\uparrow\right\rangle + 0.6\left|\downarrow\right\rangle$ of its spin-up and spin-down states. If $\left|\uparrow\right\rangle$ and $\left|\downarrow\right\rangle$ are the eigen states of σ_z then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here, symbols have their usual meanings _____

46. Consider the wave function $Ae^{ikr}(r_0 / r)$, where *A* is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k / m)$, is _____.

47. An *n*-channel junction field effect transistor has 5 *m*A source to drain current at shorted gate (I_{DSS}) and 5V pinch off voltage (V_P). Calculate the drain current in *m*A for a gate-source voltage (V_{GS}) of –2.5V. The answer should be up to two decimal places _____

Common Data Questions

Common Data for Questions 48 and 49: There are four energy levels *E*, 2*E*, 3*E* and 4*E* (where $E > 0$). The canonical partition function of two particles is, if these particles are

48. Two identical fermions

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$ (b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(c) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$ (d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

49. Two distinguishable particles

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$ (b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(c) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$ (d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Common Data for Questions 50 and 51: To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

we add a small perturbation given by

$$\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

where ε is a small quantity.

50. The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$ (c) (0, 0, 1) (d) (1, 0, 0)

51. A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3 + 2\varepsilon, 7 + 2\varepsilon$ (b) $3 + 2\varepsilon, 2 + \varepsilon$ (c) $3, 7 + 2\varepsilon$ (d) $3, 2 + 2\varepsilon$

Linked Answer Questions

Statement for Linked Answer Q.52 and Q.53: In the Schmidt model of nuclear magnetic moments, We have,

$$\vec{\mu} = \frac{e\hbar}{2Mc}(g_\ell \vec{l} + g_s \vec{S})$$

where the symbols have their usual meaning

CAREER ENDEAVOUR

52. For the case $J = l + 1/2$, where J is the total angular momentum, the expectation value of $\vec{S} \cdot \vec{J}$ in the nuclear ground state is equal to,

(a) $(J - 1)/2$ (b) $(J + 1)/2$ (c) $J/2$ (d) $-J/2$

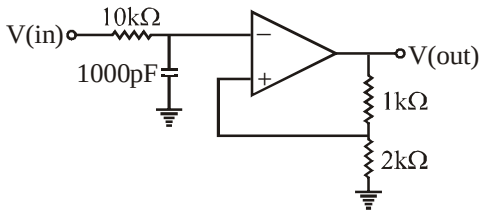
53. For the O^{17} nucleus (A = 17, Z = 8), the effective magnetic moment is given by,

$$\vec{\mu}_{eff} = \frac{e\hbar}{2Mc} g\vec{J}$$

where g is equal to, (g_s = 5.59 for proton and –3.83 for neutron)

(a) 1.12 (b) –0.77 (c) –1.28 (d) 1.28

Statement for Linked Answer Questions 54 and 55: Consider the following circuit



54. For this circuit the frequency above which the gain will decrease by 20 *dB* per decade is

(a) 15.9 kHz (b) 1.2 kHz (c) 5.6 kHz (d) 22.5 kHz

55. At 1.2 kHz the closed loop gain is

(a) 1 (b) 1.5 (c) 3 (d) 0.5

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

2. Which one of the following quantities is invariant under Lorentz transformation?

(a) Charge density (b) Charge (c) Current (d) Electric field

3. The number of normal Zeeman splitting components of ${}^1P \rightarrow {}^1D$ transition is

(a) 3 (b) 4 (c) 8 (d) 9

4. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8}s, then the proper half-life is _____$\times10^{-8}$ s. (c = 3×10^8 m/s)

5. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

(a) 0º (b) 45º (c) 90º (d) 120º

6. Two masses m and 3m are attached to the two ends of a massless spring with force constant K. If m = 100g and K = 0.3 N/m, then the natural angular frequency of oscillation is _____ Hz.

7. The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E} = \hat{x}10\cos\left(6\pi\times10^7\ t - 0.4\pi z\right)V/m$$

The phase velocity of the wave is _____$\times10^8$ m/s.

8. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

9. The recoil momemtum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

10. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to

(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

11. Which one of the following high energy processes is allowed by conservation laws?

(a) $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$ (b) $\pi + p \rightarrow \pi^0 + n$

(c) $n \rightarrow p + e^- + \nu_e$ (d) $\mu^+ \rightarrow e^- + \gamma$

12. The length element ds of an arc is given by, $(ds)^2 = 2\left(dx^1\right)^2 + \left(dx^2\right)^2 + \sqrt{3}\,dx^1dx^2$. The metric tensor g_{ij} is

(a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

CAREER ENDEAVOUR

13. The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)-\psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)+\psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)+\psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)-\psi_1(x_2)\psi_2(x_1)\right]$

14. If the vector potential,

$$\vec{A}=\alpha x\hat{x}+2y\hat{y}-3z\hat{z}$$

satisfies the Coulomb gauge, the value of the constant α is _____

15. At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillator is _____ k_BT.

16. Which one of the following is a fermion?

(a) α particle (b) ${}_4Be^7$ nucleus (c) hydrogen atom (d) deuteron

17. Which one of the following three-quark states (qqq), denoted by X, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript

(a) X^{++} (b) X^{+} (c) X^{-} (d) X^{--}

18. The Hamilton's canonical equations of motion in terms of Poisson Brackets are

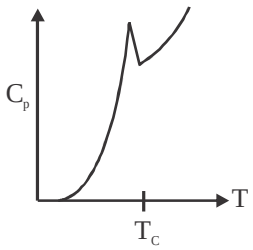
(a) $\dot{q}=\{q,H\};\ \dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\};\ \dot{p}=\{H,q\}$

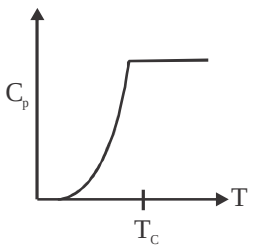
(c) $\dot{q}=\{H,p\};\ \dot{p}=\{H,q\}$ (d) $\dot{q}=\{p,H\};\ \dot{p}=\{q,H\}$

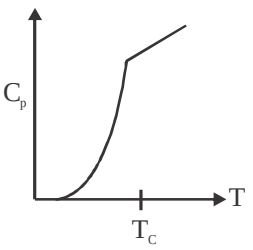
19. The Miller indices of a plane passing through the three points having coordinates (0, 0, 1), (1, 0, 0), $\left(\frac{1}{2},\frac{1}{2},\frac{1}{4}\right)$ are

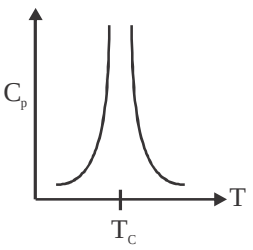
(a) (2 1 2) (b) (1 1 1) (c) (1 2 1) (d) (2 1 1)

20. The plot of specific heat versus temperature across the superconducting transition temperature (T_C) is most appropriately represented by

(a) 

(b) 

(c) 

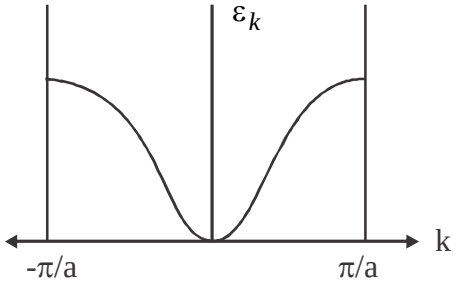
(d) 

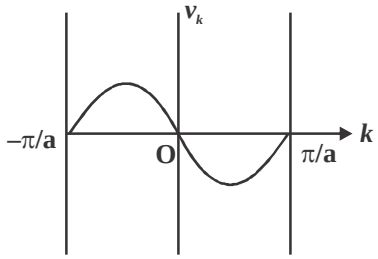
21. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does NOT commute with

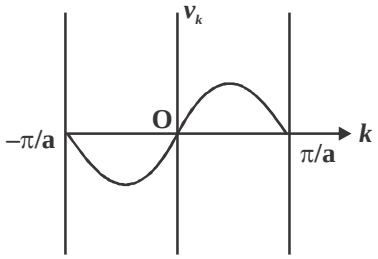
(a) S_z (b) L^2 (c) S^2 (d) $\left(\vec{L}+\vec{S}\right)^2$

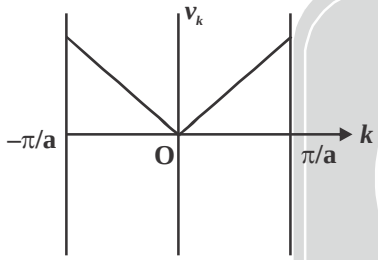
CAREER ENDEAVOUR

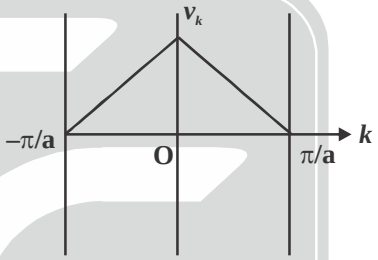
22. The energy, ε_k for band electrons as a function of the wave vector, k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as ('a' is lattice constant)



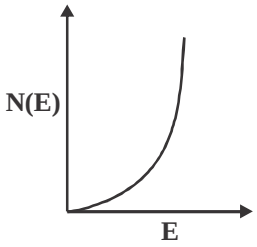
(a)


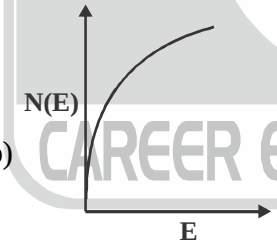
(b)


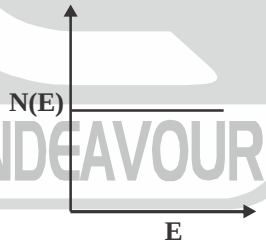
(c)


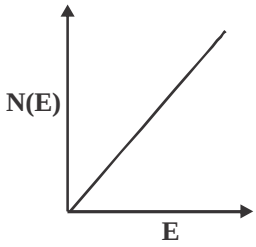
(d)


23. For a free electron gas in two dimensions, the variation of the density of states, N(E) as a function of energy E, is the best represented by

(a)


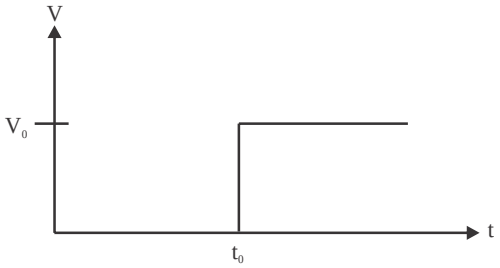
(b)


(c)


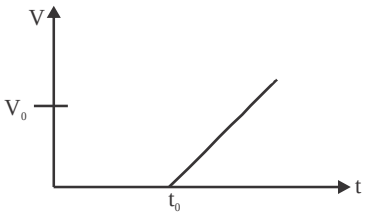
(d)


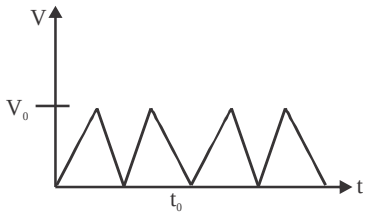
CAREER ENDEAVOUR

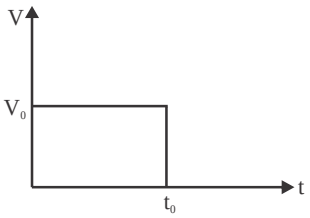
24. The input given to be an ideal OP-AMP integrator circuit is

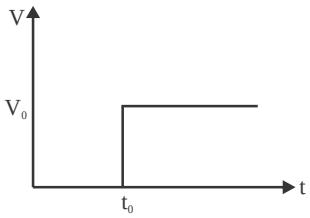


The correct output of the integrator circuit is

(a)


(b)


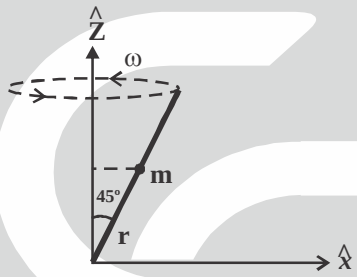
(c)


(d)


25. The minimum number of flip-flops required to construct a mod-75 counter is _____

Q.26 – Q.55 : Carry TWO marks each.

26. A bead of mass 'm' can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, *r* is the distance of the bead from the origin. The momentum conjugate to 'r' is



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

27. An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is:

(Hint: $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$)

(a) $-a_0^2/2$ (b) $-a_0^2$ (c) $-3a_0^2/2$ (d) $-2a_0^2$

28. For Nickel, the number density is 8×10^{23} atoms/cm^3 and electronic configuration is $1s^2\ 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic states is _____ $\times 10^9$ A/m.

(Given the value of Bohr magneton $\mu_B = 9.21\times10^{-21}\ Am^2$)

29. A particle of mass 'm' is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$$

CAREER ENDEAVOUR

where a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

30. The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is _____ meV.

$\left(k_BT = 25meV \text{ at } 300\text{K}\right)$

31. A particle of mass m is subjected to a potential

$$V(x,y) = \frac{1}{2}m\omega^2\left(x^2+y^2\right), \quad -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of 'g' is _____

32. A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$$

where n, ℓ, m in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is _____ $\hbar^2$.

33. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$ where k is a positive constant. The orbital angular momentum of the planet is

(a) $2r_0km$ (b) $\sqrt{2r_0km}$ (c) r_0km (d) $\sqrt{r_0km}$

34. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

35. The value of the integral

$$\oint_C \frac{z^2}{e^z+1}dz$$

where C is the circle $|z| = 4$, is

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

36. A ray of light insid Region 1 in the xy-plane is incident at the semicircle boundary that carries no free charges. The electric field at the point $P(r_0, \pi/4)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to x-axis. If ε_1 and ε_2 are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is _____

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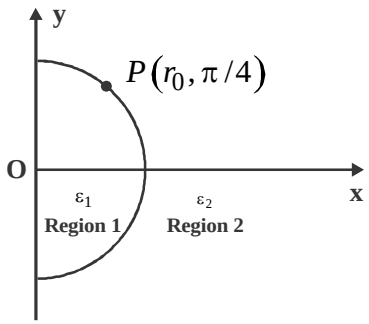


37. The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

38. Given that the linear transformation of a generalized coordinate 'q' and the corresponding momentum p,

$$Q = q + 4ap$$
$$p = q + 2p$$

is canonical, the value of the constant 'a' is _____

39. The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is _____ Tesla.

(Given: $m_p = 1.67 \times 10^{-27}\, kg$, $e = 1.6 \times 10^{-19}\, C$)

40. For a system of two bosons, each of which can occupy any of the two energy levels 0 and ε, the mean energy of the system at a temperature T with $\beta = \frac{1}{k_B T}$ is given by

(a) $\frac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (b) $\frac{1}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon\, e^{-\beta\varepsilon} + \varepsilon\, e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

41. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and I_0, respectively. The intensities of the individual waves are

(a) $3I_0$ and I_0 (b) $4I_0$ and I_0 (c) $5I_0$ and $4I_0$ (d) $9I_0$ and I_0

42. ψ_1 and ψ_2 are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

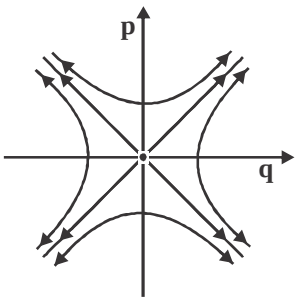
where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state ψ_2, its probability to be in spin-up state is _____

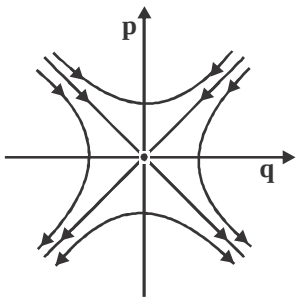
CAREER ENDEAVOUR

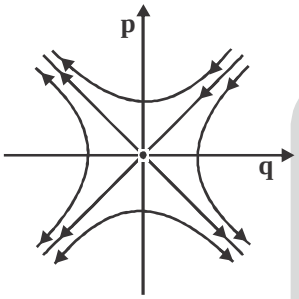
43. Neutrons moving with speed 10^3 m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30°, the interplanar spacing of the crystal is _____Å.

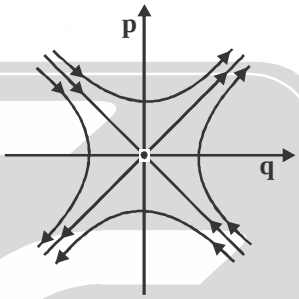
(Given: $m_n = 1.675\times10^{-27}\,kg,\ h = 6.626\times10^{-34}\ J.s$)

44. The Hamiltonian of a particle of mass 'm' is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which of the following figures describes the motion of the particle in phase space?

(a) 

(b) 

(c) 

(d) 

45. The intensity of a laser in free space is 150 mW/m^2. The corresponding amplitude of the electric field of the laser is _____ V/m. $\left(\varepsilon_0 = 8.854\times10^{-12}\ C^2/N.m^2\right)$

46. The emission wavelength for the transition $^1D_2 \rightarrow {}^1F_3$ is 3122 Å. The ratio of populations of the final to the initial states at a temperature 5000 K is $(h = 6.626\times10^{-34}\ J.s,\ c = 3\times10^8\ m/s,\ k_B = 1.380\times10^{-23}\ J/K)$

(a) 2.03×10^{-5} (b) 4.02×10^{-5} (c) 7.02×10^{-5} (d) 9.83×10^{-5}

47. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) $k_B \ln 2$ (b) $2k_B \ln 2$ (c) $2k_B \ln 2$ (d) $3k_B \ln 4$

48. A particle is confined to a one dimensional potential box with potential

$$V(x) = 0, \quad 0 < x < a$$
$$= \infty, \quad \text{otherwise}$$

If the particle is subjected to a perturbation, within the box, $W = \beta x$, where β is a small constant, the first order correction to the ground state energy is

(a) 0 (b) $a\beta/4$ (c) $a\beta/2$ (d) $a\beta$

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49. Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion (π) pairs at rest is _____ MeV. (Given : $m_\mu = 105\ MeV/c^2, m_\pi = 140\ MeV/c^2$)

50. A one dimensional harmonic oscillator is in the superposition of number states, $|n\rangle$, given by

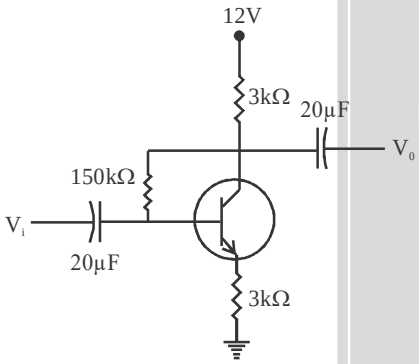
$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _____ $\hbar\omega$.

51. A nucleus X undergoes a first forbidden β -decay to a nucleus Y. If the angular momentum (I) and parity (P), denoted by I^P as $\frac{7^-}{2}$ for X, which of the following is a possible I^P value for Y?

(a) $\frac{1^+}{2}$ (b) $\frac{1^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{3^-}{2}$

52. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_C is _____ mA.



53. In order to measure a maximum of 1V with a resolution of 1mV using a n-bit A/D converter, working under the principle of ladder network, the minimum value of n is _____

54. If L_+ and L_- are the angular momentum ladder operators, then, the expectation value of $(L_+L_- + L_-L_+)$, in the state $|\ell = 1, m = 1\rangle$ of an atom is _____ $\hbar^2$

55. A low pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively, are
(a) 0.71 and 45° (b) 0.71 and –45° (c) 0.5 and –90° (d) 0.5 and 90°

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct ?

(a) $V = -2T; E = -T$ (b) $V = -T; E = 0$

(c) $V = \frac{-T}{2}; E = \frac{T}{2}$ (d) $V = \frac{-3T}{2}; E = \frac{-T}{2}$

2. The lattice parameters a, b, c of an orthorhombic crystal are related by $a = 2b = 3c$. In units of a, the interplanar separation between the (110) planes is_____ (upto three decimal places).

3. Consider $w = f(z) = u(x, y) + iv(x, y)$ to be an analytic function in a domain D. Which one of the following options is NOT correct ?

(a) $u(x, y)$ satisfies Laplace equation in D

(b) $v(x, y)$ satisfies Laplace equation in D

(c) $\int_{z_1}^{z_2} f(z)\,dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) $f(z)$ can be Taylor expanded in D

4. Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators, respectively, for a particle. The commutator $\left[L_x, p_y\right]$ gives

(a) $-i\hbar p_z$ (b) 0 (c) $i\hbar p_x$ (d) $i\hbar p_z$

5. The dispersion relation for photons in a one dimensional monatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by, $\omega(k) = \sqrt{\frac{2C}{M}\left[1 - \cos(ka)\right]}$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes $(\lambda \gg a)$, the ratio of the phase velocity to the group velocity is_____

6. For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because

(a) the chemical potential of the photons is zero (b) photons obey Pauli exclusion principle

(c) photons are spin-1 particles (d) the entropy of the photons is very large

7. Four forces are given below in Cartesian and spherical polar coordinates.

(i) $\vec{F}_1 = K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2 = K\left(x^3\hat{y} - y^3\hat{z}\right)$

(iii) $\vec{F}_3 = K\left(x^3\hat{x} + y^3\hat{y}\right)$ (iv) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.

(a) (iii) and (iv) are conservative but (i) and (ii) are not

(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

CAREER ENDEAVOUR

8. The value of $\int_0^3 t^2\delta(3t-6)\,dt$ is_____ (upto one decimal place)

9. The mean kinetic energy of a nucleon in a nucleus of atomic weight A varies as A^n, where n is_____ (upto two decimal places)

10. In Bose-Einstein condensates, the particles
(a) have strong interparticle attraction (b) condense in real space
(c) have overlapping wavefunctions (d) have large and positive chemical potential

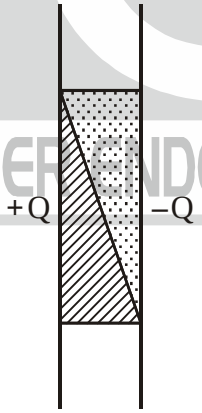
11. A beam of X-ray of intensity I_0 is incident normally on a metal sheet of thickness 2 mm. The intensity of the transmitted beam is $0.025I_0$. The linear absorption coefficient of the metal sheet (in m^{-1}) is _____ (upto one decimal place)

12. In a Hall effect experiment, the Hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)
(a) $n \approx p$ (b) $n > p$ (c) $\mu_e > \mu_h$ (d) $m_e^* > m_n^*$

13. The Pauli matrices for three spin-$\frac{1}{2}$ particles are $\vec{\sigma}_1$, $\vec{\sigma}_2$ and $\vec{\sigma}_3$, respectively. The dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is_____

14. The decay $\mu^+ \to e^+ + \gamma$ is forbidden, because it violates
(a) momentum and lepton number conservations
(b) baryon and lepton number conservations
(c) angular momentum conservation
(d) lepton number conservation

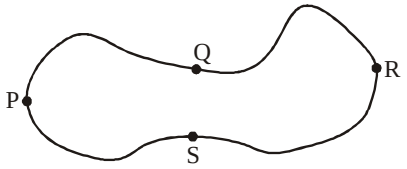
15. The space between two plates of a capacitor carrying charges +Q and –Q is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct ?



(a) $\vec{E}$ and $\vec{D}$ are continuous (b) $\vec{E}$ is continuous and $\vec{D}$ is discontinuous
(c) $\vec{D}$ is continuous and $\vec{E}$ is discontinuous (d) $\vec{E}$ and $\vec{D}$ are discontinuous

CAREER ENDEAVOUR

16. Given that magnetic flux through the closed loop PQRSP is ϕ. If $\int_P^R \vec{A}\cdot\vec{dl} = \phi_1$ along PQR, the value of $\int_P^R \vec{A}\cdot\vec{dl}$ along PSR is



(a) $\phi - \phi_1$ (b) $\phi_1 - \phi$ (c) $-\phi_1$ (d) ϕ_1

17. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is_____

18. Consider a complex function $f(z) = \dfrac{1}{z\left(z+\dfrac{1}{2}\right)\cos(z\pi)}$. Which one of the following statements is correct ?

(a) $f(z)$ has simple poles at $z = 0$ and $z = -\dfrac{1}{2}$

(b) $f(z)$ has a second order pole at $z = -\dfrac{1}{2}$

(c) $f(z)$ has infinite number of second order poles

(d) $f(z)$ has all simple poles

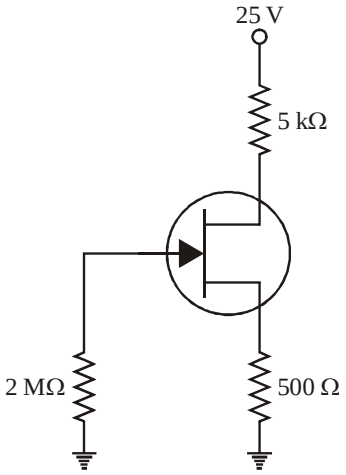
19. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is _____

20. In an inertial frame S, two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame S' moving with a velocity $0.6\,c\,\hat{y}$ with respect to S are given by

(a) $ct'_A = 0;\ ct'_B = -3/2$ (b) $ct'_A = 0;\ ct'_B = 0$

(c) $ct'_A = 0;\ ct'_B = 3/2$ (d) $ct'_A = 0;\ ct'_B = 1/2$

21. In the given circuit, the voltage across the source resistor is 1 V. The drain voltage (in V) is _____



CAREER ENDEAVOUR

22. If $f(x) = e^{-x^2}$ and $g(x) = |x|e^{-x^2}$, then
(a) f and g are differentiable everywhere
(b) f is differentiable everywhere but g is not
(c) g is differentiable everywhere but f is not
(d) g is discontinuous at $x = 0$

23. Consider a system of N non-interacting spin-$\frac{1}{2}$ particles, each having a magnetic moment μ, is in a magnetic field $\vec{B} = B\hat{z}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

(a) $\Omega = \dfrac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\ \frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$ (b) $\Omega = \dfrac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$

(c) $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\ \frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$ (d) $\Omega = \dfrac{N!}{\left(N + \frac{E}{\mu B}\right)!}$

24. Which one of the following DOES NOT represent an exclusive OR operation for inputs A and B?
(a) $(A + B)\overline{AB}$ (b) $A\bar{B} + B\bar{A}$ (c) $(A + B)(\bar{A} + \bar{B})$ (d) $(A + B)AB$

25. An operator for a spin-$\frac{1}{2}$ particle is given by $\hat{A} = \lambda\vec{\sigma}\cdot\vec{B}$, where $\vec{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{y})$, $\vec{\sigma}$ denotes Pauli matrices and λ is a constant. The eigenvalue of $\hat{A}$ are
(a) $\frac{\pm\lambda B}{\sqrt{2}}$ (b) $\pm\lambda B$ (c) $0, \lambda B$ (d) $0, -\lambda B$

Q.26 – Q.55 : Carry TWO marks each.

26. Match the phrases in **Group I** and **Group II** and identify the correct option.

Group I	**Group II**
(P) Electron spin resonance (ESR)	(i) radio frequency
(Q) Nuclear magnetic resonance (NMR)	(ii) visible range frequency
(R) Transition between vibrational states of a molecule	(iii) microwave frequency
(S) Electronic transition	(iv) far-infrared range

(a) (P-i), (Q-ii), (R-iii), (S-iv) (b) (P-ii), (Q-i), (R-iv), (S-iii)
(c) (P-iii), (Q-iv), (R-i), (S-ii) (d) (P-iii), (Q-i), (R-iv), (S-ii)

27. The entropy of a gas containing N particles enclosed in a volume V is given by $S = Nk_B\,\ell n\left(\dfrac{aVE^{3/2}}{N^{5/2}}\right)$, where E is the total energy, a is a constant and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by

(a) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{5/2}}\right) - \dfrac{5}{2}\right]$ (b) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{5/2}}\right) - \dfrac{3}{2}\right]$

(c) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{3/2}}\right) - \dfrac{5}{2}\right]$ (d) $\mu = -k_BT\left[\ell n\left(\dfrac{aVE^{3/2}}{N^{3/2}}\right) - \dfrac{3}{2}\right]$

CAREER ENDEAVOUR

28. The atomic masses of $^{152}_{63}Eu$, $^{152}_{62}Sm$, $^{1}_{1}H$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (*amu*), respectively. Using the above information, the *Q*-value of the reaction $^{152}_{63}Eu + n \rightarrow {}^{152}_{62}Sm + p$ is_____ $\times 10^{-3}$ *amu* (upto three decimal places)

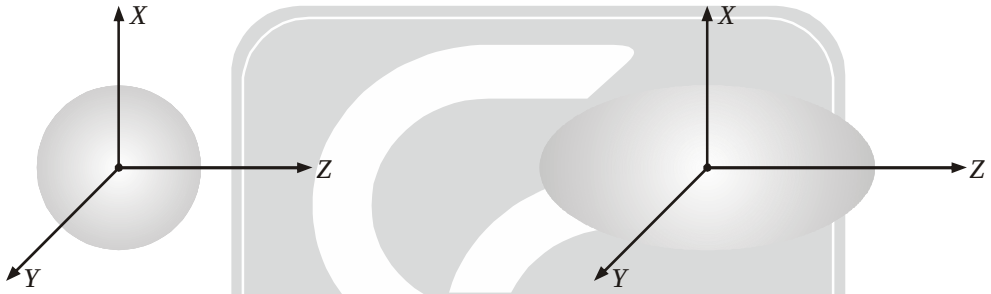
29. A particle with rest mass *M* is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the *z*-axis. Their velocities are given by

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z};\ \vec{v}_2 = (-0.8c)\hat{z}$

30. The band gap of an intrinsic semiconductor is $E_g = 0.72\ eV$ and $m_h^* = 6m_e^*$. At 300K, the Fermi level with respect to the edge of the valence band (in eV) is at _____ (upto three decimal places) $k_B = 1.38\times10^{-23}\ JK^{-1}$.

31. A charge $-q$ is distributed uniformly over a sphere, with a positive charge *q* at its center in (i). Also in (ii), a charge $-q$ is distributed uniformly over an ellipsoid with a positive charge *q* at its center. With respect to the origin of the coordinate system, which one of the following statements is correct ?

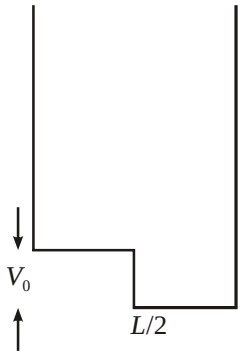


(i) (ii)

(a) The dipole moment is zero in both (i) and (ii)
(b) The dipole moment is non-zero in (i) but zero in (ii)
(c) The dipole moment is zero in (i) but non-zero in (ii)
(d) The dipole moment is non-zero in both (i) and (ii)

32. The number of permitted transitions from $^2P_{3/2} \rightarrow {}^2S_{1/2}$ in the presence of a weak magnetic field is_____

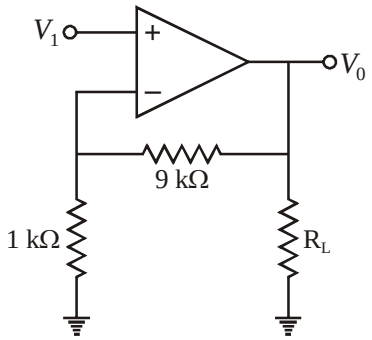
33. A particle is confined in a box of length *L* as shown below.



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

(a) $E = \frac{\hbar^2\pi^2}{2mL^2} + V_0$ (b) $E = \frac{\hbar^2\pi^2}{2mL^2} - \frac{V_0}{2}$ (c) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{4}$ (d) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{2}$

34. In the given circuit, if the open loop gain $A = 10^5$, the feedback configuration and the closed loop gain A_f are



(a) series-shunt, $A_f = 9$ (b) series-series, $A_f = 10$
(c) series-shunt, $A_f = 10$ (d) shunt-shunt, $A_f = 10$

35. A plane wave $(\hat{x} + i\hat{y})E_0 \exp[i(kz - \omega t)]$ after passing through an optical element emerges as $(\hat{x} - i\hat{y})E_0 \exp[i(kz - \omega t)]$, where k and ω are the wavevector and the angular frequency, respectively. The optical element is
(a) quarter wave plate (b) half wave plate
(c) polarizer (d) Faraday rotator

36. A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $v(0) = 10\, ms^{-1}$. If the air exerts a functional force of the form, $f = -kv$, then for $k = 0.05\, Nm^{-1}s$, the velocity (in ms^{-1}) at time $t = 0.2\ s$ is_____ (upto two decimal places)
(use $g = 10\, ms^{-2}$ and $e = 2.72$)

37. The Lagrangian for a particle of mass m at a position $\vec{r}$ moving with a velocity $\vec{v}$ is given by $L = \frac{m}{2}\vec{v}^2 + C\vec{r}\cdot\vec{v} - V(r)$, where $V(r)$ is a potential and C is a constant. If $\vec{p}_c$ is the canonical momentum, then its Hamiltonian is given by

(a) $\frac{1}{2m}\left(\vec{p}_c + C\vec{r}\right)^2 + V(r)$ (b) $\frac{1}{2m}\left(\vec{p}_c - C\vec{r}\right)^2 + V(r)$

(c) $\frac{p_c^2}{2m} + V(r)$ (d) $\frac{1}{2m}p_c^2 + C^2r^2 + V(r)$

CAREER ENDEAVOUR

38. A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance r from the axis of the solenoid is proportional to

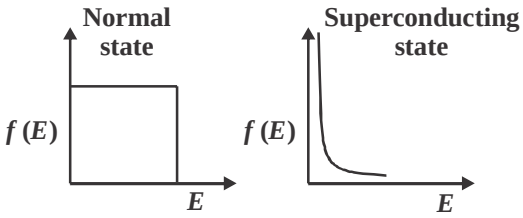
(a) r^2 inside the solenoid and $\frac{1}{r}$ outside (b) r inside the solenoid and $\frac{1}{r^2}$ outside

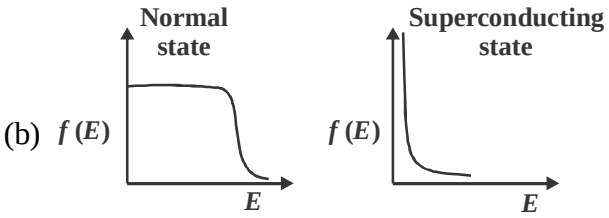
(c) r^2 inside the solenoid and $\frac{1}{r^2}$ outside (d) r inside the solenoid and $\frac{1}{r}$ outside

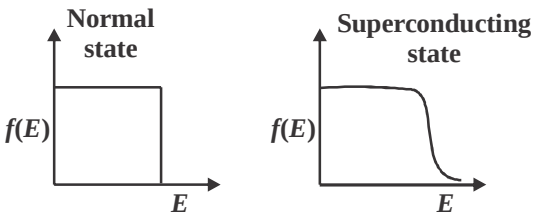
39. In the nuclear shell model, the potential is modeled as $V(r) = \frac{1}{2}m\omega^2r^2 - \lambda\vec{L}\cdot\vec{S},\ \lambda > 0$. The correct spin-parity and isospin assignments for the ground state of ^{13}C is

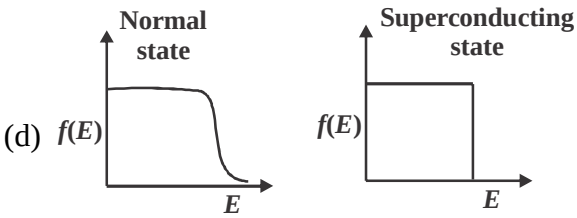
(a) $\frac{1^-}{2}; \frac{-1}{2}$ (b) $\frac{1^+}{2}; \frac{-1}{2}$ (c) $\frac{3^+}{2}; \frac{1}{2}$ (d) $\frac{3^-}{2}; \frac{-1}{2}$

40. Which one of the following represents the electron occupancy for a superconductor in its normal and superconducting states?

(a) 

(b) 

(c) 

(d) 

41. In a rigid-rotator of mass *M*, if the energy of the first excited state is 1 *meV*, then the fourth excited state energy (in *meV*) is_____

42. The binding energy per molecule of NaCl (lattice parameter is 0.563 nm) is 7.95 eV. The repulsive term of the potential is of the form $\frac{K}{r^9}$, where K is a constant. The value of the Madelung constant is _____(upto three decimal places)

(Electron charge $e = 1.6\times10^{-19} C; \varepsilon_0 = 8.854\times10^{-12} C^2 N^{-1} m^{-2}$)

43. The Hamiltonian for a system of two particles of masses m_1 and m_2 at $\vec{r}_1$ and $\vec{r}_2$ having velocities $\vec{v}_1$ and $\vec{v}_2$ is given by $H = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2}\hat{z}\cdot(\vec{r}_1 \times \vec{r}_2)$, where *C* is a constant. Which one of the following statements is correct ?

(a) The total energy and total momentum are conserved
(b) Only the total energy is conserved
(c) The total energy and the *z*-component of the total angular momentum are conserved
(d) The total energy and total angular momentum are conserved

44. Given that the Fermi energy of gold is 5.54 eV, the number density of electrons is _____$\times10^{28}$ m^{-3} (upto one decimal place).
(Mass of electron = 9.11×10^{-21} kg; h = 6.626×10^{-24}J.s; 1 eV = 1.6×10^{-19} J)

45. Suppose a linear harmonic oscillator of frequency ω and mass *m* is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{i\frac{\pi}{2}}|\psi_1\rangle\right]$ at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\psi|x|\psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is_____

CAREER ENDEAVOUR

46. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If $\vec{F}_c$ and $\vec{F}_{Co}$ denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

(a) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = \vec{F}_c$ (b) $\vec{F}_c$ is radially inward and $\vec{F}_{Co} = -2\vec{F}_c$

(c) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = -2\vec{F}_c$ (d) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = 2\vec{F}_c$

47. A function $y(z)$ satisfies the ordinary differential equation $y'' + \frac{1}{z}y' - \frac{m^2}{z^2}y = 0$, where $m = 0, 1, 2, 3, ...$

Consider the four statements *P*, *Q*, *R*, *S* as given below.

P: z^m and z^{-m} are linearly independent solutions for all values of *m*

Q: z^m and z^{-m} are linearly independent solutions for all values of $m > 0$

R: $\ell n\ z$ and 1 are linearly independent solutions for $m = 0$

S: z^m and $\ell n\ z$ are linearly independent solutions for all values of *m*

The correct option for the combination of valid statement is

(a) *P*, *R* and *S* only (b) *P* and *R* only (c) *Q* and *R* only (d) *R* and *S* only

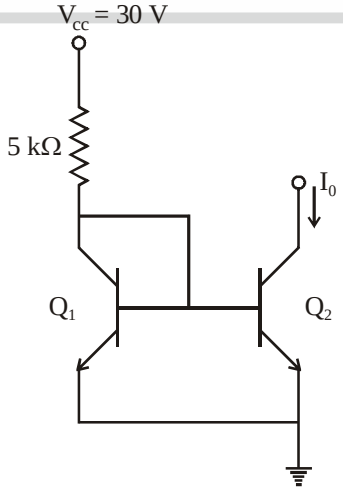
48. The average energy *U* of a one dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature *T* is given by

(a) $U = \frac{1}{2}\hbar\omega \coth\left(\frac{1}{2}\beta\hbar\omega\right)$ (b) $U = \frac{1}{2}\hbar\omega \sinh\left(\frac{1}{2}\beta\hbar\omega\right)$

(c) $U = \frac{1}{2}\hbar\omega \tanh\left(\frac{1}{2}\beta\hbar\omega\right)$ (d) $U = \frac{1}{2}\hbar\omega \cosh\left(\frac{1}{2}\beta\hbar\omega\right)$

49. Consider a system of eight non-interacting, identical quantum particles of spin-$\frac{3}{2}$ in a one dimensional box of length *L*. The minimum excitation energy of the system, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is_____

50. In the simple current source shown in the figure, Q_1 and Q_2 are identical transistors with current gain $\beta = 100$ and $V_{BE} = 0.7\,V$



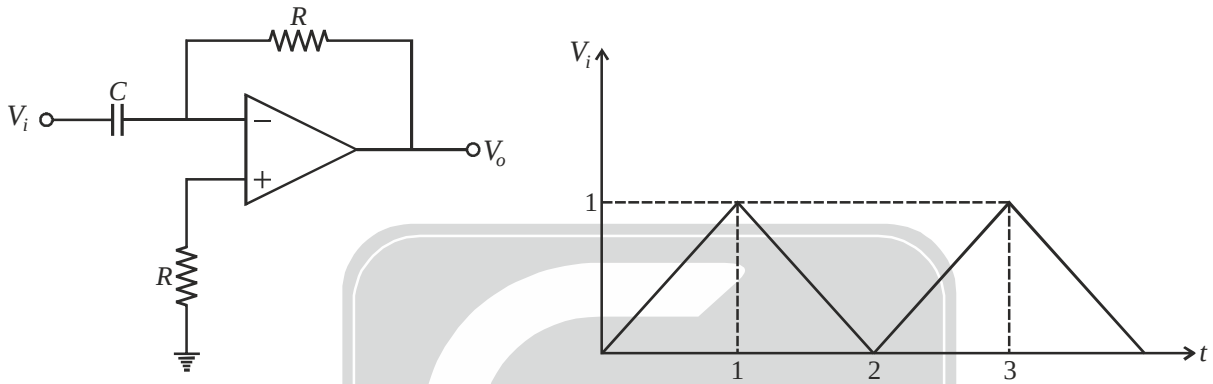
The current I_0 (in mA) is_____(upto two decimal places)

CAREER ENDEAVOUR

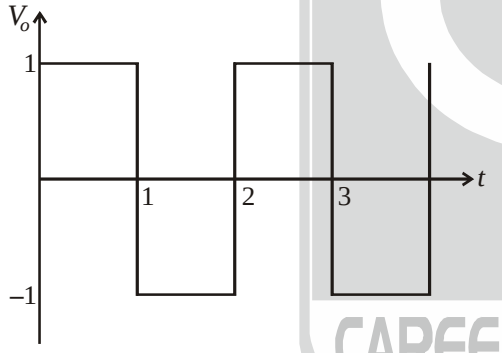
51. The Heaviside function is defined as $H(t)=\begin{cases}+1 & \text{for } t>0\\ -1 & \text{for } t<0\end{cases}$ and its Fourier transform is given by $-2i/\omega$. The Fourier transform of $\frac{1}{2}\left[H\left(t+\frac{1}{2}\right)-H\left(t-\frac{1}{2}\right)\right]$ is

(a) $\dfrac{\sin\left(\frac{\omega}{2}\right)}{\omega/2}$ (b) $\dfrac{\cos\left(\frac{\omega}{2}\right)}{\omega/2}$ (c) $\sin\left(\frac{\omega}{2}\right)$ (d) 0

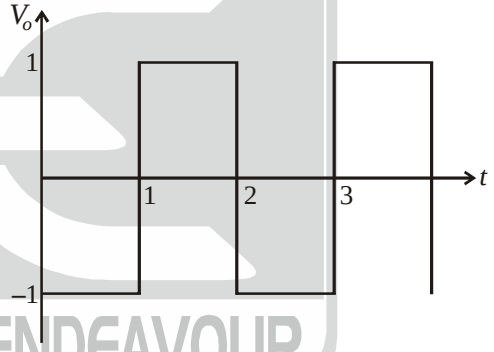
52. Consider the circuit shown in the figure, where $RC = 1$. For an input signal V_i shown below, choose the correct V_0 from the options:



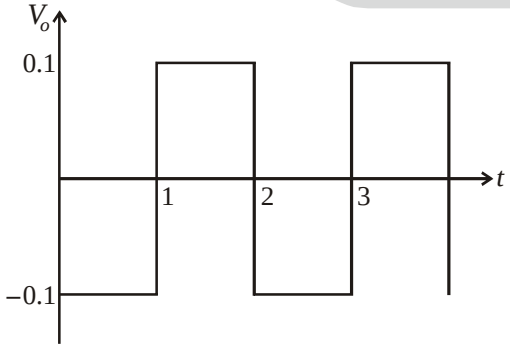
(a)



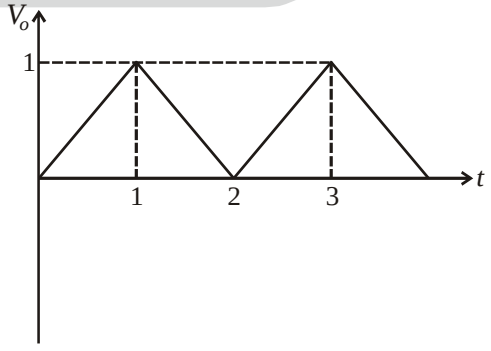
(b)



(c)



(d)



CAREER ENDEAVOUR

53. Let the Hamiltonian for two spin-$\frac{1}{2}$ particles of equal masses m, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be $H = \frac{1}{2m} p_1^2 + \frac{1}{2m} p_2^2 + \frac{1}{2} m\omega^2 \left(r_1^2 + r_2^2\right) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1eV$ and $k = 0.2eV$. If the ground state has net spin zero, then the energy (in eV) is_____

54. The excitation wavelength of laser in a Raman effect experiment is 546 nm. If the Stokes line is observed at 552 nm, then the wave number of the anti-Stokes line (in cm^{-1}) is_____

55. A monochromatic plane wave (wavelength = 600 nm) $E_0 \exp\left[i(kz - \omega t)\right]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1 \exp\left\lfloor i\left(\vec{k}_1 \cdot \vec{r} - \omega t\right)\right\rfloor$ in the first order of diffraction. Here $E_1 < E_0$ and $\vec{k}_1 = \left|\vec{k}_1\right|\left[\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}\right]$. The period (in μm) of the diffraction grating is_____(upto one decimal place)

CAREER ENDEAVOUR

PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Consider the linear differential equation $\frac{dy}{dx} = xy$. If $y = 2$ at $x = 0$, then the value of y at $x = 2$ is given by

(a) e^{-2} (b) $2e^{-2}$ (c) e^{2} (d) $2e^{2}$

2. Which of the following magnetic vector potentials gives rise to a uniform magnetic field $B_0\hat{k}$?

(a) $B_0 z\hat{k}$ (b) $-B_0 x\hat{j}$

(c) $\frac{B_0}{2}(-y\hat{i} + x\hat{j})$ (d) $\frac{B_0}{2}(y\hat{i} + x\hat{j})$

3. The molecule $^{17}O_2$ is

(a) Raman active but not NMR (nuclear magnetic resonance) active
(b) Infrared active and Raman active but not NMR active
(c) Raman active and NMR active
(d) Only NMR active

4. There are four electrons in the 3*d* shell of an isolated atom. The total magnetic moment of the atom in units of Bohr magneton is _____.

5. Which of the following transitions is NOT allowed in the case of an atom, according to the electric dipole radiation selection rule?

(a) $2s - 1s$ (b) $2p - 1s$ (c) $2p - 2s$ (d) $3d - 2p$

6. In the SU(3) quark model, the triplet of mesons $\left(\pi^+, \pi^0, \pi^-\right)$ has

(a) Isospin = 0, Strangeness = 0 (b) Isospin = 1, Strangeness = 0
(c) Isospin = 1/2, Strangeness = +1 (d) Isospin = 1/2, Strangeness = –1

7. The magnitude of the magnetic dipole moment associated with a square shaped loop carrying a steady current *I* is *m*. If this loop is changed to a circular shape with the same current *I* passing through it, the magnetic dipole moment becomes $\frac{pm}{\pi}$. The value of *p* is _____

8. The total power emitted by a spherical black body of radius *R* at a temperature *T* is P_1. Let P_2 be the total power emitted by another spherical black body of radius *R*/2 kept at temperature 2*T*. The ratio, P_1 / P_2 is _____. (Give your answer upto two decimal places).

9. The entropy *S* of a system of *N* spins, which may align either in the upward or in the downward direction, is given by $S = -k_B N[p \ln p + (1-p)\ln(1-p)]$. Here k_B is the Boltzmann constant. The probability of alignment in the upward direction is *p*. The value of *p*, at which the entropy is maximum, is _____. (Give your answer upto one decimal place).

10. For a system at constant temperature and volume, which of the following statements is correct at equilibrium?

(a) The Helmholtz free energy attains a local minimum
(b) The Helmholtz free energy attains a local maximum
(c) The Gibbs free energy attains a local minimum
(d) The Gibbs free energy attains a local maximum

CAREER ENDEAVOUR

11. N atoms of an ideal gas are enclosed in a container of volume V. The volume of the container is changed to $4V$, while keeping the total energy constant. The change in the entropy of the gas, in units of $Nk_B \ln 2$, is _____, where k_B is the Boltzmann constant.

12. Which of the following is an analytic function of z everywhere in the complex plane?

(a) z^2 (b) $(z^*)^2$ (c) $|z|^2$ (d) $\sqrt{z}$

13. In a Young's double slit experiment using light, the apparatus has two slits of unequal widths. When only slit-1 is open, the maximum observed intensity on the screen is $4I_0$. When only slit-2 is open, the maximum observed intensity is I_0. When both the slits are open, an interference pattern appears on the screen. The ratio of the intensity of the principal maximum to that of the nearest minimum is _____.

14. Consider a metal which obeys the Sommerfeld model exactly. If E_F is the Fermi energy of the metal at $T = 0$K and R_H is its Hall coefficient, which of the following statements is correct?

(a) $R_H \propto E_F^{3/2}$ (b) $R_H \propto E_F^{2/3}$

(c) $R_H \propto E_F^{-3/2}$ (d) R_H is independent of E_F

15. A one-dimensional linear chain of atoms contains two types of atoms of masses m_1 and m_2 (where $m_2 > m_1$), arranged alternately. The distance between successive atoms is the same. Assume that the harmonic approximation is valid. At the first Brillouin zone boundary, which of the following statements is correct?

(a) The atoms of mass m_2 are at rest in the optical mode, while they vibrate in the acoustical mode.

(b) The atoms of mass m_1 are at rest in the optical mode, while they vibrate in the acoustical mode.

(c) Both types of atoms vibrate with equal amplitudes in the optical as well as in the acoustical modes.

(d) Both types of atoms vibrate, but with unequal, non-zero amplitudes in the optical as well as in the acoustical modes.

16. Which of the following operators is Hermitian?

(a) $\frac{d}{dx}$ (b) $\frac{d^2}{dx^2}$ (c) $i\frac{d^2}{dx^2}$ (d) $\frac{d^3}{dx^3}$

17. The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of $m_0 c$, where c is the speed of light in vacuum, is _____. (Give your answer upto two decimal places)

18. The number density of electrons in the conduction band of a semiconductor at a given temperature is 2×10^{19} m^{-3}. Upon lightly doping this semiconductor with donor impurities, the number density of conduction electrons at the same temperature becomes 4×10^{20} m^{-3}. The ratio of majority to minority charge carrier concentration is _____.

19. Two blocks are connected by a spring of spring constant k. One block has mass m and the other block has mass $2m$. If the ratio $k/m = 4$s^{-2}, the angular frequency of vibration ω of the two block spring system in s^{-1} is _____. (Give your answer upto two decimal places).

20. A particle moving under the influence of a central force $\vec{F}(\vec{r}) = -k\vec{r}$ (where $\vec{r}$ is the position vector of the particle and k is a positive constant) has non-zero angular momentum. Which of the following curves is a possible orbit for this particle?

(a) A straight line segment passing through the origin (b) An ellipse with its center at the origin

(c) An ellipse with one of the foci at the origin (d) A parabola with its vertex at the origin

21. Consider the reaction ${}^{54}_{25}Mn + e^- \rightarrow {}^{54}_{24}Cr + X$. The particle X is

(a) γ (b) ν_e (c) n (d) π^0

CAREER ENDEAVOUR

22. The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresponding Born approximation is known as the first Born approximation. Such an approximation is valid for
(a) large incident energies and weak scattering potentials
(b) large incident energies and strong scattering potentials
(c) small incident energies and weak scattering potentials
(d) small incident energies and strong scattering potentials

23. Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90° and the magnitude of the incident wave vector is equal to $\sqrt{2\pi}$ fm^{-1} then the total scattering cross section in units of fm^2 is _____.

24. A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E} = E_0\hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent *n* is _____

25. A solid material is found to have a temperature independent magnetic susceptibility, $\chi = C$. Which of the following statements is correct?
(a) If *C* is positive, the material is a diamagnet
(b) If *C* is positive, the material is a ferromagnet
(c) If *C* is negative, the material could be a type I superconductor
(d) If *C* is positive, the material could be a type I superconductor

Q.26 – Q.55 : Carry TWO marks each.

26. An infinite, conducting slab kept in a horizontal plane carries a uniform charge density σ. Another infinite slab of thickness *t*, made of a linear dielectric material of dielectric constant *k*, is kept above the conducting slab. The bound charge density on the upper surface of the dielectric slab is

(a) $\frac{\sigma}{2k}$ (b) $\frac{\sigma}{k}$ (c) $\frac{\sigma(k-2)}{2k}$ (d) $\frac{\sigma(k-1)}{k}$

27. The number of spectroscopic terms resulting from the L.S. coupling of a 3*p* electron and a 3*d* electron is _____

28. Which of the following statements is NOT correct?
(a) A deuteron can be disintegrated by irradiating it with gamma rays of energy 4 MeV.
(b) A deuteron has no excited states
(c) A deuteron has no electric quadrupole moment
(d) The $^1S_0$ state of deuteron cannot be formed

29. If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle \vec{s}_1 \cdot \vec{s}_2 \rangle$ for the ground state is

(a) $-\frac{3}{2}\hbar^2$ (b) $-\frac{3}{4}\hbar^2$ (c) 0 (d) $\frac{1}{4}\hbar^2$

30. A two-dimensional square rigid box of side *L* contains six non-interacting electrons at $T = 0K$. The mass of the electron is *m*. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is _____

CAREER ENDEAVOUR

31. An alpha particle is accelerated in a cyclotron. It leaves the cyclotron with a kinetic energy of 16 MeV. The potential difference between the D electrodes is 50 kilovolts. The number of revolutions the alpha particle makes in its spiral path before it leaves the cyclotron is _____

32. Let V_i be the i^{th} component of a vector field $\vec{V}$, which has zero divergence. If $\partial_j \equiv \partial / \partial x_j$, the expression for $\in_{ijk} \in_{lmk} \partial_j \partial_l V_m$ is equal to

(a) $-\partial_j \partial_k V_i$ (b) $\partial_j \partial_k V_i$ (c) $\partial_j^2 V_i$ (d) $-\partial_j^2 V_i$

33. The direction of $\vec{\nabla} f$ for a scalar field $f(x, y, z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$ at the point $P(1, 1, 2)$ is

(a) $\frac{(-\hat{j} - 2\hat{k})}{\sqrt{5}}$ (b) $\frac{(-\hat{j} + 2\hat{k})}{\sqrt{5}}$ (c) $\frac{(\hat{j} - 2\hat{k})}{\sqrt{5}}$ (d) $\frac{(\hat{j} + 2\hat{k})}{\sqrt{5}}$

34. σ_x, σ_y and σ_z are the Pauli matrices. The expression $2\sigma_x\sigma_y + \sigma_y\sigma_x$ is equal to

(a) $-3i\sigma_z$ (b) $-i\sigma_z$ (c) $i\sigma_z$ (d) $3i\sigma_z$

35. A particle of mass $m = 0.1$ kg is initially at rest at origin. It starts moving with a uniform acceleration $\vec{a} = 10\hat{i}$ ms^{-2} at $t = 0$. The action S of the particle, in units of J-s, at $t = 2s$ is _____. (Give your answer upto two decimal places).

36. A periodic function $f(x)$ of period 2π is defined in the interval $(-\pi < x < \pi)$ as:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

The appropriate Fourier series expansion for $f(x)$ is

(a) $f(x) = (4/\pi)[\sin x + (\sin 3x)/3 + (\sin 5x)/5 + ...]$

(b) $f(x) = (4/\pi)[\sin x - (\sin 3x)/3 + (\sin 5x)/5 - ...]$

(c) $f(x) = (4/\pi)[\cos x + (\cos 3x)/3 + (\cos 5x)/5 + ...]$

(d) $f(x) = (4/\pi)[\cos x - (\cos 3x)/3 + (\cos 5x)/5 - ...]$

37. Atoms, which can be assumed to be hard spheres of radius R, are arranged in an *fcc* lattice with lattice constant a, such that each atom touches its nearest neighbours. Take the center of one of the atoms as the origin. Another atom of radius r (assumed to be hard sphere) is to be accommodated at a position $(0, a/2, 0)$ without distorting the lattice. The maximum value of r/R is _____. (Give your answer upto two decimal places).

38. In an inertial frame of reference S, an observer finds two events occurring at the same time at co-ordinates $x_1 = 0$ and $x_2 = d$. A different inertial frame S' moves with velocity v with respect to S along the positive x-axis. An observer in S' also notices these two events and finds them to occur at times t_1' and t_2' and at positions x_1' and x_2', respectively. If $\Delta t' = t_2' - t_1'$, $\Delta x' = x_2' - x_1'$ and $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, which of the following statements is true?

(a) $\Delta t' = 0, \Delta x' = \gamma d$ (b) $\Delta t' = 0, \Delta x' = d/\gamma$

(c) $\Delta t' = -\gamma v d/c^2, \Delta x' = \gamma d$ (d) $\Delta t' = -\gamma v d/c^2, \Delta x' = d/\gamma$

CAREER ENDEAVOUR

39. The energy vs. wave vector $(E-k)$ relationship near the bottom of a band for a solid can be approximated as $E = A(ka)^2 + B(ka)^4$, where the lattice constant $a = 2.1\text{Å}$. The values of A and B are 6.3×10^{-19}J and 3.2×10^{-20}J, respectively. At the bottom of the conduction band, the ratio of the effective mass of the electron to the mass of free electron is _____. (Give your answer upto two decimal places)
(Take $\hbar = 1.05 \times 10^{-34}$ J-s, mass of free electron = 9.1×10^{-31}kg)

40. The electric field component of a plane electromagnetic wave travelling in vacuum is given by $\vec{E}(z, t) = E_0 \cos(kz - \omega t)\hat{i}$. The Poynting vector for the wave is

(a) $(c\varepsilon_0/2)E_0^2 \cos^2(kz-\omega t)\hat{j}$ (b) $(c\varepsilon_0/2)E_0^2 \cos^2(kz-\omega t)\hat{k}$

(c) $c\varepsilon_0 E_0^2 \cos^2(kz-\omega t)\hat{j}$ (d) $c\varepsilon_0 E_0^2 \cos^2(kz-\omega t)\hat{k}$

41. Consider a system having three energy levels with energies 0, 2ε and 3ε, with respective degeneracies of 2, 2 and 3. Four bosons of spin zero have to be accommodated in these levels such that the total energy of the system is 10ε. The number of ways in which it can be done is _____.

42. The Lagrangian of a system is given by

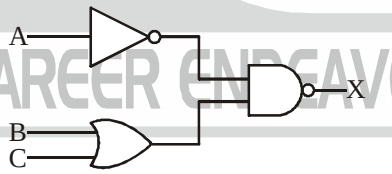
$L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta$, where m, l and g are constants.

Which of the following is conserved?

(a) $\dot{\varphi}\sin^2\theta$ (b) $\dot{\varphi}\sin\theta$ (c) $\frac{\dot{\varphi}}{\sin\theta}$ (d) $\frac{\dot{\varphi}}{\sin^2\theta}$

43. Protons and α-particles of equal initial momenta are scattered off a gold foil in a Rutherford scattering experiment. The scattering cross sections for proton on gold and α-particle on gold are σ_p and σ_α respectively. The ratio σ_α/σ_p is _____.

44. For the digital circuit given below, the output X is



(a) $\overline{\bar{A} + B\cdot C}$ (b) $\overline{\bar{A}\cdot(B+C)}$ (c) $\bar{A}\cdot(B+C)$ (d) $A + \overline{(B\cdot C)}$

45. The Fermi energies of two metals X and Y are 5eV and 7eV and their Debye temperatures are 170 K and 340 K, respectively. The molar specific heats of these metals at constant volume at low temperatures can be written as $(C_V)_X = \gamma_X T + A_X T^3$ and $(C_V)_Y = \gamma_Y T + A_Y T^3$, where γ and A are constants. Assuming that the thermal effective mass of the electrons in the two metals are same, which of the following is correct?

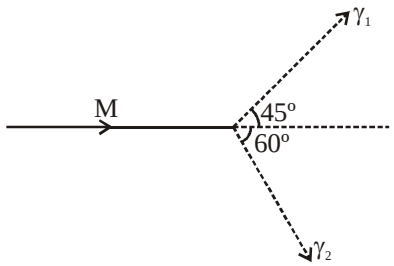
(a) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = 8$ (b) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = \frac{1}{8}$

(c) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = \frac{1}{8}$ (d) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = 8$

46. A two-level system has energies zero and E. The level with zero energy is non-degenerate, while the level with energy E is triply degenerate. The mean energy of a classical particle in this system at a temperature T is

(a) $\frac{Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$ (b) $\frac{Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$ (c) $\frac{3Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$ (d) $\frac{3Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$

47. A particle of rest mass M is moving along the positive *x*-direction. It decays into two photons γ_1 and γ_2 as shown in the figure. The energy of γ_1 is 1GeV and the energy of γ_2 is 0.82 GeV. The value of M (in units of GeV/c^2) is _____. (Give your answer upto two decimal places)



48. If *x* and *p* are the *x* components of the position and the momentum operators of a particle respectively, the commutator $[x^2,\ p^2]$ is

(a) $i\hbar(xp - px)$ (b) $2i\hbar(xp - px)$

(c) $i\hbar(xp + px)$ (d) $2i\hbar(xp + px)$

49. The *x*-*y* plane is the boundary between free space and a magnetic material with relative permeability μ_r. The magnetic field in the free space is $B_x\hat{i} + B_z\hat{k}$. The magnetic field in the magnetic material is

(a) $B_x\hat{i} + B_z\hat{k}$ (b) $B_x\hat{i} + \mu_r B_z\hat{k}$

(c) $\frac{1}{\mu_r}B_x\hat{i} + B_z\hat{k}$ (d) $\mu_r B_x\hat{i} + B_z\hat{k}$

CAREER ENDEAVOUR

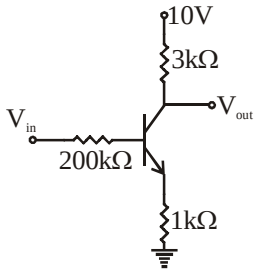
50. Let $|l,\ m\rangle$ be the simultaneous eigenstates of L^2 and L_z. Here $\vec{L}$ is the angular momentum operator with Cartesian components $(L_x,\ L_y,\ L_z)$, l is the angular momentum quantum number and *m* is the azimuthal quantum number. The value of $\langle 1,\ 0\,|\,(L_x + iL_y)\,|\,1,\ -1\rangle$ is

(a) 0 (b) $\hbar$ (c) $\sqrt{2}\,\hbar$ (d) $\sqrt{3}\,\hbar$

51. For the parity operator P, which of the following statements is NOT true?

(a) $P^\dagger = P$ (b) $P^2 = -P$ (c) $P^2 = I$ (d) $P^\dagger = P^{-1}$

52. For the transistor shown in the figure, assume $V_{BE} = 0.7$V and $\beta_{dc} = 100$. If $V_{in} = 5$V, V_{out} (in Volts) is _____. (Give your answer upto one decimal place).



53. The state of a system is given by

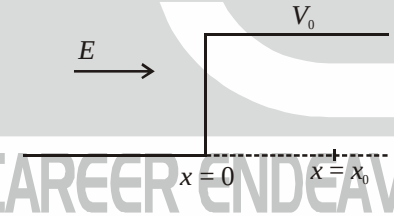
$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$

where $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is _____. (Give your answer upto two decimal places)

54. According to the nuclear shell model, the respective ground state spin-parity values of ${}^{15}_{8}O$ and ${}^{17}_{8}O$ nuclei are

(a) $\frac{1}{2}^{+}, \frac{1}{2}^{-}$ (b) $\frac{1}{2}^{-}, \frac{5}{2}^{+}$ (c) $\frac{3}{2}^{-}, \frac{5}{2}^{+}$ (d) $\frac{3}{2}^{-}, \frac{1}{2}^{-}$

55. A particle of mass *m* and energy *E*, moving in the positive *x* direction, is incident on a step potential at $x = 0$, as indicated in the figure. The height of the potential is V_0, where $V_0 > E$. At $x = x_0$, where $x_0 > 0$, the probability of finding the electron is $1/e$ times the probability of finding it at $x = 0$. If $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$, the value of x_0 is



(a) $\frac{2}{\alpha}$ (b) $\frac{1}{\alpha}$ (c) $\frac{1}{2\alpha}$ (d) $\frac{1}{4\alpha}$

CAREER ENDEAVOUR

PHYSICS-PH

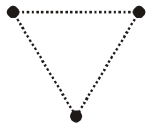
Q.1 – Q.25 : Carry ONE mark each.

1. In the nuclear reaction $^{13}C_6 + \nu_e \rightarrow {}^{13}N_7 + X$, the particle X is

(a) an electron (b) an anti-electron (c) a muon (d) a pion

2. Two identical masses of 10 gm each are connected by a massless spring of spring constant 1 N/m. The non-zero angular eigen frequency of the system is _____ rad/s. (up to two decimal places).

3. Consider a triatomic molecule of the shape shown in the figure below in three dimensions. The heat capacity of this molecule at high temperature (temperature much higher than the vibrational and rotational energy scales of the molecule but lower than its bond dissociation energies) is:



(a) $\frac{3}{2}k_B$ (b) $3k_B$ (c) $\frac{9}{2}k_B$ (d) $6k_B$

4. For the Hamiltonian $H = a_0 I + \vec{b}.\vec{\sigma}$ where $a_0 \in R$, $\vec{b}$ is a real vector, I is the 2×2 identity matrix and $\vec{\sigma}$ are the Pauli matrices, the ground state energy is

(a) $|\mathrm{b}|$ (b) $2a_0 - |\mathrm{b}|$ (c) $a_0 - |\mathrm{b}|$ (d) a_0

5. The Poisson bracket $\left[x, xp_y + yp_x\right]$ is equal to

(a) $-x$ (b) y (c) $2p_x$ (d) p_y

6. The wavefunction of which orbital is spherically symmetric:

(a) p_x (b) p_y (c) s (d) d_{xy}

7. A monochromatic plane wave in free space with electric field amplitude of 1 V/m is normally incident on a fully reflecting mirror. The pressure exerted on the mirror is _____$\times 10^{-12}$ Pa. (up to two decimal places) $(\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m})$.

8. The electronic ground state energy of the Hydrogen atom is –13.6 eV. The highest possible electronic energy eigenstate has an energy equal to

(a) 0 (b) 1 eV (c) +13.6 eV (d) ∞

9. Consider a one-dimensional lattice with a weak periodic potential $U(x) = U_0 \cos\left(\frac{2\pi x}{a}\right)$. The gap at the edge of the Brillouin zone $\left(k = \frac{\pi}{a}\right)$ is:

(a) U_0 (b) $\frac{U_0}{2}$ (c) $2U_0$ (d) $\frac{U_0}{4}$

10. Identical charge q are placed at five vertices of a regular hexagon of side a. The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively

(a) 0, 0 (b) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{q}{4\pi\varepsilon_0 a}$ (c) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{5q}{4\pi\varepsilon_0 a}$ (d) $\frac{\sqrt{5}q}{4\pi\varepsilon_0 a^2}, \frac{\sqrt{5}q}{4\pi\varepsilon_0 a}$

CAREER ENDEAVOUR

11. A reversible Carnot engine is operated between temperatures T_1 and $T_2\,(T_2 > T_1)$ with a photon gas as the working substance. The efficiency of the engine is

(a) $1-\frac{3T_1}{4T_2}$ (b) $1-\frac{T_1}{T_2}$ (c) $1-\left(\frac{T_1}{T_2}\right)^{3/4}$ (d) $1-\left(\frac{T_1}{T_2}\right)^{4/3}$

12. The best resolution that a 7 bit A/D converter with 5V full scale can achieve is _____ mV. (up to two decimal places).

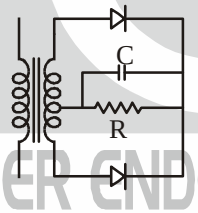
13. If the Lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2$ is modified to $L = L_0 + aq\left(\frac{dq}{dt}\right)$, which one of the following is TRUE?

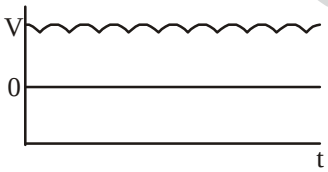
(a) Both the canonical momentum and equation of motion do not change
(b) Canonical momentum changes, equation of motion does not change
(c) Canonical momentum does not change, equation of motion changes
(d) Both the canonical momentum and equation of motion change

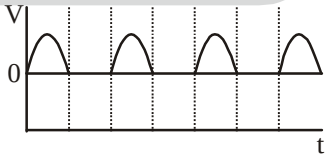
14. A parallel plate capacitor with square plates of side 1 m separated by 1 micro meter is filled with a medium of dielectric constant of 10. If the charges on the two plates are 1 C and -1 C, the voltage across the capacitor is _____kV. (up to two decimal places). ($\varepsilon_0 = 8.854\times10^{-12}$ F/m)

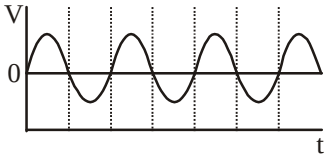
15. The contour integral $\oint \frac{dz}{1+z^2}$ evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and closed in the lower half-plane by a half circle is equal to _____. (up to two decimal places).

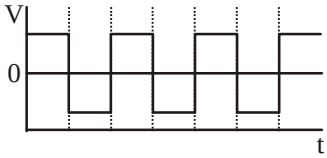
16. In the figure given below, the input to the primary of the transformer is a voltage varying sinusoidally with time. The resistor R is connected to the centre tap of the secondary. Which one of the following plots represents the voltage across the resistor R as a function of time?



(a)


(b)


(c)


(d)


17. Light is incident from a medium of refractive index $n = 1.5$ onto vacuum. The smallest angle of incidence for which the light is not transmitted into vacuum is _____ degrees. (up to two decimal places).

18. Electromagnetic interactions are :

(a) C conserving
(b) C non-conserving but CP conserving
(c) CP non-conserving but CPT conserving
(d) CPT non-conserving

19. The Compton wavelength of a proton is _____ fm. (up to two decimal places).
$(m_p = 1.67\times10^{-27}\,\text{kg},\ h = 6.626\times10^{-34}\,\text{Js},\ e = 1.602\times10^{-19}\,\text{C},\ c = 3\times10^{8}\,\text{ms}^{-1})$

20. Which one of the following conservation laws is violated in the decay $\tau^+ \to \mu^+ + \mu^+ + \mu^-$
(a) Angular momentum (b) Total Lepton number
(c) Electric charge (d) Tau number

21. The coefficient of e^{ikx} in the Fourier expansion of $u(x) = A\sin^2(ax)$ for $k = -2a$ is
(a) $A/4$ (b) $-A/4$ (c) $A/2$ (d) $-A/2$

22. The phase space trajectory of a free particle bouncing between two hard walls elastically in one dimension is a
(a) straight line (b) parabola (c) rectangle (d) circle

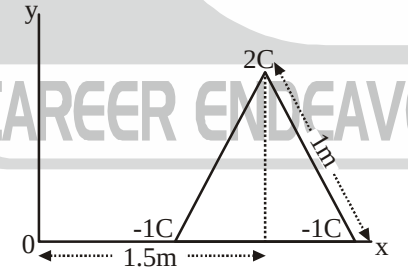
23. The atomic mass and mass density of Sodium are 23 and $0.968\ \text{g cm}^{-3}$, respectively. The number density of valence electrons is _____$\times10^{22}\ \text{cm}^{-3}$. (Up to two decimal places.)
(Avogadro number, $N_A = 6.022\times10^{23}$).

24. The degeneracy of the third energy level of a 3-dimensional isotropic quantum harmonic oscillator is
(a) 6 (b) 12 (c) 8 (d) 10

25. A one dimensional simple harmonic oscillator with Hamiltonian $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$ is subjected to a small perturbation, $H_1 = \alpha x + \beta x^3 + \gamma x^4$. The first order correction to the ground state energy is dependent on
(a) only β (b) α and γ (c) α and β (d) only γ

Q.26 – Q.55 : Carry TWO marks each.

26. Three charges (2 C, –1 C, –1 C) are placed at the vertices of an equilateral triangle of side 1m as shown in the figure. The component of the electric dipole moment about the marked origin along the $\hat{y}$ direction is ___ cm.



27. An object travels along the x-direction with velocity $c/2$ in a frame O. An observer in a frame O' sees the same object travelling with velocity $c/4$. The relative velocity of O' with respect to O in units of c is _____. (up to two decimal places).

28. The energy density and pressure of a photon gas are given by $u = aT^4$ and $P = u/3$, where T is the temperature and a is the radiation constant. The entropy per unit volume is given by αaT^3. The value of α is _____. (up to two decimal places).

29. A person weighs w_p at Earth's north pole and w_e at the equator. Treating the Earth as a perfect sphere of radius 6400 km, the value $100\times(w_p - w_e)/w_p$ is _____. (up to two decimal places).
(Take $g = 10\ \text{ms}^{-2}$).

CAREER ENDEAVOUR

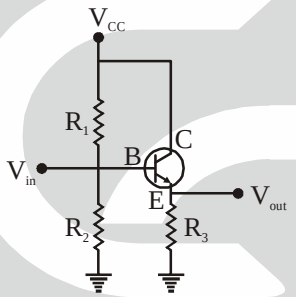
30. The minimum number of NAND gates required to construct an OR gate is:
(a) 2 (b) 4 (c) 5 (d) 3

31. The total energy of an inert-gas crystal is given by $E(R) = \frac{0.5}{R^{12}} - \frac{1}{R^6}$ (in eV), where R is the inter-atomic spacing in Angstroms. The equilibrium separation between the atoms is _____ Angstroms. (up to two decimal places).

32. The imaginary part of an analytic complex function is $v(x, y) = 2xy + 3y$. The real part of the function is zero at the origin. The value of the real part of the function at $1+i$ is _____. (up to two decimal places).

33. Consider N non-interacting, distinguishable particles in a two-level system at temperature T. The energies of the levels are 0 and ε, where $\varepsilon > 0$. In the high temperature limit $(k_B T >> \varepsilon)$, what is the population of particles in the level with energy ε?
(a) $\frac{N}{2}$ (b) N (c) $\frac{N}{4}$ (d) $\frac{3N}{4}$

34. For the transistor amplifier circuit shown below with $R_1 = 10\ k\Omega$, $R_2 = 10\ k\Omega$, $R_3 = 1\ k\Omega$, and $\beta = 99$. Neglecting the emitter diode resistance, the input impedance of the amplifier looking into the base for small ac signal is _____ $k\Omega$. (up to two decimal places).



35. Which one of the following gases of diatomic molecules is Raman, infrared, and NMR active?
(a) $^{1}H - ^{1}H$ (b) $^{12}C - ^{16}O$ (c) $^{1}H - ^{35}Cl$ (d) $^{16}O - ^{16}O$

36. Let X be a column vector of dimension $n > 1$ with at least one non-zero entry. The number of non-zero eigenvalues of the matrix $M = XX^T$ is
(a) 0 (b) n (c) 1 (d) $n - 1$

37. A free electron of energy 1 eV is incident upon a one-dimensional finite potential step of height 0.75 eV. The probability of its reflection from the barrier is _____ (up to two decimal places).

38. An infinite solenoid carries a time varying current $I(t) = At^2$, with $A \neq 0$. The axis of the solenoid is along the $\hat{z}$ direction. $\hat{r}$ and $\hat{\theta}$ are the usual radial and polar directions in cylindrical polar coordinates. $\vec{B} = B_r \hat{r} + B_\theta \hat{\theta} + B_z \hat{z}$ is the magnetic field at a point outside the solenoid. Which one of the following statements is true?
(a) $B_r = 0, B_\theta = 0, B_z = 0$ (b) $B_r \neq 0, B_\theta \neq 0, B_z = 0$
(c) $B_r \neq 0, B_\theta \neq 0, B_z \neq 0$ (d) $B_r = 0, B_\theta = 0, B_z \neq 0$

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39. Consider two particles and two non-degenerate quantum levels 1 and 2. Level 1 always contains a particle. Hence, what is the probability that level 2 also contains a particle for each of the two cases:
(i) when the two particles are distinguishable and (ii) when the two particles are bosons?
(a) (i) 1/2 and (ii) 1/3 (b) (i) 1/2 and (ii) 1/2 (c) (i) 2/3 and (ii) 1/2 (d) (i) 1 and (ii) 0

40. The real space primitive lattice vectors are $\vec{a}_1 = a\hat{x}$ and $\vec{a}_1 = \frac{a}{2}\left(\hat{x} + \sqrt{3}\hat{y}\right)$. The reciprocal space unit vectors $\vec{b}_1$ and $\vec{b}_2$ for this lattice are, respectively

(a) $\frac{2\pi}{a}\left(\hat{x} - \frac{\hat{y}}{\sqrt{3}}\right)$ and $\frac{4\pi}{a\sqrt{3}}\hat{y}$ (b) $\frac{2\pi}{a}\left(\hat{x} + \frac{\hat{y}}{\sqrt{3}}\right)$ and $\frac{4\pi}{a\sqrt{3}}\hat{y}$

(c) $\frac{2\pi}{a\sqrt{3}}\hat{x}$ and $\frac{4\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}} + \hat{y}\right)$ (d) $\frac{2\pi}{a\sqrt{3}}\hat{x}$ and $\frac{4\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}} - \hat{y}\right)$

41. The geometric cross-section of two colliding protons at large energies is very well estimated by the product of the effective sizes of each particle. This is closest to
(a) 10 b (b) 10 mb (c) 10 μb (d) 10 pb

42. A uniform volume charge density is placed inside a conductor (with resistivity $10^{-2}\,\Omega\text{m}$). The charge density becomes $1/(2.718)$ of its original value after time _____ femto seconds. (up to two decimal places) $(\varepsilon_0 = 8.854\times10^{-12}\,\text{F/m})$

43. Water freezes at 0°C at atmospheric pressure $(1.01\times10^5\ \text{Pa})$. The densities of water and ice at this temperature and pressure are 1000 kg/m^3 and 934 kg/m^3 respectively. The latent heat of fusion is 3.34×10^5 J/kg. The pressure required for depressing the melting temperature of ice by 10°C is ____ GPa. (up to two decimal places)

44. The integral $\int_0^\infty x^2 e^{-x^2} dx$ is equal to _____. (up to two decimal places).

45. J^P for the ground state of the $^{13}C_6$ nucleus is
(a) 1^+ (b) $\frac{3^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{1^-}{2}$

46. A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10 gm and 1 cm respectively, the final linear velocity of the cylinder is _____ m/s. (up to two decimal places)

47. Consider a one-dimensional potential well of width 3 nm. Using the uncertainty principle $\left(\Delta x.\Delta p \geq \hbar/2\right)$, an estimate of the minimum depth of the well such that it has at least one bound state for an electron is $(m_e = 9.31\times10^{-31}kg,\ h = 6.626\times10^{-34}\,J\,s,\ e = 1.602\times10^{-19}C)$:
(a) 1 μeV (b) 1 meV (c) 1 eV (d) 1 MeV

48. Consider an ideal operational amplifier as shown in the figure below with $R_1 = 5\ \text{k}\Omega$, $R_2 = 1\ \text{k}\Omega$, $R_L = 100\ \text{k}\Omega$. For an applied input voltage $V = 10$ mV, the current passing through R_2 is _____ μA. (up to two decimal places).

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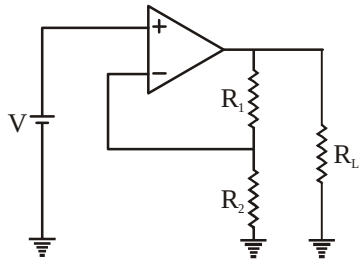

49. The π^+ decays at rest to μ^+ and ν_π. Assuming the neutrino to be massless, the momentum of the neutrino is _____ MeV/c. (up to two decimal places)
$\left(m_x = 139\text{MeV}/c^2,\ m_\mu = 105\ \text{MeV}/c^2\right)$.

50. Consider the differential equation $dy/dx + y\tan(x) = \cos(x)$. If $y(0) = 0$, $y(\pi/3)$ is _____. (up to two decimal places).

51. Consider a metal with free electron density of $6\times10^{22}\ \text{cm}^{-3}$. The lowest frequency electromagnetic radiation to which this metal is transparent is 1.38×10^{16} Hz. If this metal had a free electron density of $1.8\times10^{23}\ \text{cm}^{-3}$ instead, the lowest frequency electromagnetic radiation to which it would be transparent is _____$\times10^{16}$ Hz. (up to two decimal places).

52. Using Hund's rule, the total angular momentum quantum number J for the electronic ground state of the nitrogen atom is
(a) 1/2 (b) 3/2 (c) 0 (d) 1

53. Consider a 2-dimensional electron gas with a density of $10^{19}\ \text{m}^{-2}$. The Fermi energy of the system is _____eV. (up to two decimal places).
$\left(m_e = 9.31\times10^{-31}\text{kg},\ h = 6.626\times10^{-34}\text{Js},\ e = 1.602\times10^{-19}\text{C}\right)$

54. Which one of the following operators is Hermitian?
(a) $i\dfrac{\left(p_x x^2 - x^2 p_x\right)}{2}$ (b) $i\dfrac{\left(p_x x^2 + x^2 p_x\right)}{2}$ (c) $e^{ip_x a}$ (d) $e^{-ip_x a}$

55. Positronium is an atom made of an electron and a positron. Given that the Bohr radius for the ground state of the Hydrogen atom to be 0.53 Angstroms, the Bohr radius for the ground state of positronium is _____ Angstroms. (up to two decimal places).

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SOLUTION

2001 – 2017

OBJECTIVE QUESTION

1.1. $\iint_S \vec{r}\cdot\hat{n}dS = \iiint_V \left(\vec{\nabla}\cdot\vec{r}\right)dV$ (using Divergence Theorem)

$$= 3\iiint_V dV \qquad \left[as\ \vec{\nabla}\cdot\vec{r} = \vec{\nabla}\cdot\left(x\hat{i}+y\hat{j}+z\hat{k}\right) = 3\right]$$

$$= 3V$$

Correct option is (d)

1.2. We know that,

If A is Hermitian then it is equal to its complex conjugate transpose i.e. $M^{\dagger} = M$

If A is anti-Hermitian then it is equal to its complex conjugate transpose with negative sign i.e. $M^{\dagger} = -M$

If A is unitary matrix then the conjugate transpose is equal to its inverse i.e. $M^{\dagger} = M^{-1}$

If A is orthogonal then the transpose of the matrix is equal to its inverse i.e. $M^{T} = M^{-1}$

$$\left[i\left(A^{\dagger}-A\right)\right]^{\dagger} = (-i)^{\dagger}\left[A-A^{\dagger}\right] = i\left(A^{\dagger}-A\right)$$

Correct option is (a)

1.3. $f(z) = z^{10}$ is analytic everywhere in the complex argand plane.

So, according to Cauchy integral theorem,

$$\int_C z^{10}dz = 0$$

Correct option is (a)

1.4. $\vec{A} = \frac{1}{\sqrt{2}}(1,1,0);\ \vec{B} = \frac{1}{\sqrt{2}}(0,1,1);\ \vec{C} = \frac{1}{\sqrt{2}}(1,0,1)$

Now, $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} = \frac{1}{\sqrt{2}}\left(\frac{1}{2}-0\right) - 0\left(\frac{1}{2}-0\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{2}-0\right) = \frac{1}{\sqrt{2}} \neq 0$

So, $\vec{A}, \vec{B}, \vec{C}$ are linearly independent.

Correct option is (b)

1.5. We have Lagrangian,

$$L = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu}{r}$$

Canonical momentum corresponding to coordinates θ, $P_{\theta} = \frac{\partial L}{\partial \dot{\theta}} = r^2\dot{\theta}$

Linear momentum, $P_r = \frac{\partial L}{\partial \dot{r}} = \dot{r}$

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Equation of motion, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\frac{\partial L}{\partial \theta}=0 \Rightarrow P_\theta = r^2\dot{\theta} = \text{constant}$

Since, L does not depends on time explicitly. So, total energy of the system is conserved.

Correct option is (c)

1.6. Lagrangian transform,

$$x' = \frac{x-vt}{\sqrt{1-v^2/c^2}} = \gamma(x-vt)\cdot y' = y,\ z' = z,\ t' = \gamma\left(t-\frac{vx}{c^2}\right),\ \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$\therefore \quad \frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial x'}\frac{\partial x'}{\partial x}+\frac{\partial\phi}{\partial t'}\frac{\partial t'}{\partial x} = \gamma\frac{\partial\phi}{\partial x'}-\frac{v\gamma}{c^2}\frac{\partial\phi}{\partial t'}$$

$$\Rightarrow \quad \frac{\partial^2\phi}{\partial x^2} = \frac{\partial}{\partial x'}\left(\frac{\partial\phi}{\partial x}\right)\frac{\partial x'}{\partial x}+\frac{\partial}{\partial t'}\left(\frac{\partial\phi}{\partial x}\right)\frac{\partial t'}{\partial x}$$

$$= \gamma^2\frac{\partial^2\phi}{\partial x'^2}-\frac{v\gamma^2}{c^2}\frac{\partial^2\phi}{\partial x'\partial t'}-\frac{v\gamma^2}{c^2}\frac{\partial^2\phi}{\partial t'\partial x'}+\frac{v^2\gamma^2}{c^4}\frac{\partial^2\phi}{\partial t'^2}$$

And, $\frac{\partial\phi}{\partial t} = \frac{\partial\phi}{\partial x'}+\frac{\partial\phi}{\partial t'}\frac{\partial t'}{\partial t} = \frac{\partial\phi}{\partial x'}(-v\gamma)+\frac{\partial\phi}{\partial t'}\gamma = -v\gamma\frac{\partial\phi}{\partial x'}+\gamma\frac{\partial\phi}{\partial t'}$

$$\therefore \quad \frac{\partial^2\phi}{\partial t^2} = \frac{\partial}{\partial x'}\left(\frac{\partial\phi}{\partial t}\right)\cdot\frac{\partial x'}{\partial t}+\frac{\partial}{\partial t'}\left(\frac{\partial\phi}{\partial t}\right)\frac{\partial t'}{\partial t}$$

$$= v^2\gamma^2\frac{\partial^2\phi}{\partial x'^2}-v\gamma^2\frac{\partial^2\phi}{\partial x\partial t'}-v\gamma^2\frac{\partial^2\phi}{\partial t'\partial x'}+\gamma^2\frac{\partial^2\phi}{\partial t'^2}$$

$$\therefore \quad \frac{\partial^2\phi}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x'^2}+\frac{v^2\gamma^2}{c^4}\frac{\partial^2\phi}{\partial t'^2}-\frac{\gamma^2}{c^2}\frac{\partial^2\phi}{\partial t'^2}$$

$$= \frac{\partial^2\phi}{\partial x'^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t'^2}$$

$$\Rightarrow \quad \frac{\partial^2\phi}{\partial x^2}-\beta\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x'^2}-\beta\frac{\partial^2\phi}{\partial t'^2}$$

Correct option is (b)

1.7. We have Lagrangian,

$$L = \frac{1}{2}\left(\dot{\eta}_1^2+\dot{\eta}_2^2+\dot{\eta}_3^2\right)-\frac{a^2}{2}\left(\eta_1^2+\eta_2^2+\eta_3^2-\eta_1\eta_3\right)$$

Kinetic energy $= \frac{1}{2}\left(\dot{\eta}_1^2+\dot{\eta}_2^2+\dot{\eta}_3^2\right)$

Potential energy $= \frac{a^2}{2}\left(\eta_1^2+\eta_2^2+\eta_3^2-\eta_1\eta_3\right)$

Corresponding matrices,

$$\hat{T} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \hat{V} = a^2\begin{pmatrix}1&0&-1\\0&1&0\\-1&0&1\end{pmatrix}$$

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For frequency (ω) of normal modes

$$\det\left|\hat{V}-\omega^2\hat{T}\right|=0$$

$$\Rightarrow \quad \begin{vmatrix} a^2-\omega^2 & 0 & -a^2 \\ 0 & a^2-\omega^2 & 0 \\ -a^2 & 0 & a^2-\omega^2 \end{vmatrix}=0$$

$$\Rightarrow \quad \left(a^2-\omega^2\right)\left(a^2-\omega^2\right)^2-a^2\cdot a^2\left(a^2-\omega^2\right)=0 \Rightarrow \left(a^2-\omega^2\right)\left[\left(a^2-\omega^2\right)^2-a^4\right]=0$$

$$\Rightarrow \quad \left(a^2-\omega^2\right)=0 \text{ and } \left(a^2-\omega^2\right)^2=a^4$$

$$\Rightarrow \quad \omega^2=a^2 \qquad a^2-\omega^2=\pm a^2$$

$$\Rightarrow \quad \omega^2=a^2 \qquad \omega^2=0 \text{ and } \omega^2=2a^2$$

1.8. According to Coulomb law, the force on Q_2 due to Q_1 is

$$\vec{F}_{21}=\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{21}}\hat{r}_{21}$$

And the force on Q_1 due to Q_2 is

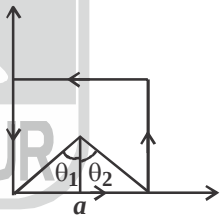
$$F_{12}=\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}\hat{r}_{12}$$

$\therefore \qquad |F_{12}|=|F_{21}|$

Correct answer is (b)

1.9. Magnetic field at distance $\frac{a}{2}$ from finite length wire is given by

$$\vec{B}_1=\frac{\mu_0 I}{4\pi\left(a/2\right)}\left(\sin\theta_1+\sin\theta_2\right) \text{ (outward direction to the page)}$$



Since, $\theta_1=\theta_2=45^\circ$

$$\therefore \qquad \vec{B}_1=\frac{\mu_0 I}{\pi a}\sin 45^\circ\,\hat{z}=\frac{\mu_0 I}{\pi a}\frac{1}{\sqrt{2}}\hat{z}$$

Therefore, the total magnetic field at the centre of square is given by

$$\vec{B}=\vec{B}_1+\vec{B}_2+\vec{B}_3+\vec{B}_4=4\vec{B}_1=\frac{4\mu_0 I}{\pi a}\frac{1}{\sqrt{2}}\hat{z}=\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{z}$$

Correct option is (b)

1.10. The flux through the circular disc is

$$\phi=\pi r^2 B(t)=\pi r^2 B_0 e^{-\alpha t}$$

Therefore, the induce emf $=-\frac{d\phi}{dt}=\pi r^2\alpha B_0 e^{-\alpha t}$

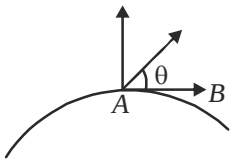
Correct option is (a)

1.11. We know that when we introduce some charge on the conducting the charges is uniformly distributed on the surface and produce a equipotential surface. So, accoding to Gauss law we can say the electric field inside the conductor will be zero.

Now, if we say the electric field out side the conductor is maing a angle θ with surface at the point A.

So, the tangent component of the electric field is $E\cos\theta$. So, the potential difference betweeen A and B points is $E\cos\theta AB$.

But the conducting surface is equipotential surface that says that



$$E\cos\theta \cdot AB = 0$$

$$\Rightarrow \quad \cos\theta = 0 \qquad (\because E \text{ and } AB \text{ can't be zero})$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

So, the electric field out side the conducting surface perpendicular to the conducting surface.

Correct option is (a)

1.12. Kinetic energy of the particle is a function momentum and potential energy of the particle is a function of position. According to Heisenberg's uncertainty principle, position and momentum of the particle cannot be measured simultaneously accurately. Therefore, kinetic energy and potential energy of the particle cannot simultaneously have sharp values.

For a spinless particle of mass *m* moving in central potential $V(r)$, the Hamiltonian of the system can be written as

$$\hat{H} = -\frac{\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{L^2}{2mr^2} + V(r)$$

Therefore, $\left[\hat{H}, \hat{L}^2\right] = 0$ i.e. Energy of the particle and square of the orbital angular momentum can simultaneously have sharp values.

Correct option is (a)

1.13. For a particle moving in the range $(-\infty, \infty)$, the acceptable wave function should be finite, single valued and continuous everywhere in space and the wave function should be square integrable in nature.

$\varphi(x) = A\tan x$ is not finite at $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$ Therefore, the function is not acceptable.

$\varphi(x) = B\cos x$ is not square integrable as

$$\int_{-\infty}^{\infty} |\varphi(x)|^2\, dx = \int_{-\infty}^{\infty} B^2\cos^2 x\, dx = \frac{B^2}{2}\int_{-\infty}^{\infty}(1+\cos 2x)\, dx \neq \text{Finite Quantity}$$

$\varphi(x) = C\exp\left(-\frac{D}{x^2}\right)$ is not finite at $x = 0$ (since $D < 0$)

$\varphi(x) = E\, x\exp\left(-Fx^2\right)$ is finite, single valued and continuous everywhere in space. The function is also square integrable in nature as

$$\int_{-\infty}^{\infty} |\varphi(x)|^2\, dx = \int_{-\infty}^{\infty} E^2x^2\exp\left(-2Fx^2\right) dx = \text{Finite Quantity}$$

Correct option is (d)

1.14. First order energy shift due to perturbation =

$$E_n^{(1)} = \langle n | H_p | n \rangle = \left\langle n \left| \lambda \left(\hat{a}^\dagger \hat{a} \right)^2 \right| n \right\rangle = \lambda \left\langle n \left| \left(\hat{a}^\dagger \hat{a} \right)^2 \right| n \right\rangle = \lambda n^2 \langle n | n \rangle = \lambda n^2$$

Correct option is (c)

1.15. If the average distance between two particles is large compared to their de Broglie wavelengths, then the wave function of those particles will not overlap with each other i.e. two particles will be distinguishable.

Correct option is (a)

1.16. The density of state in 3-D is

$$g(p)\,dp = \frac{V \cdot 4\pi\, p^2 dp}{h^3}$$

For photon gas, $E = pc \Rightarrow dp = \dfrac{dE}{c}$

Therefore, $g(E)\,dE = \dfrac{V \cdot 2 \times 4\pi E^2}{h^3 c^2} \dfrac{dE}{c}$

Photons have two polarization states (One for each dimension transverse to the direction of propagation). So, g(E) should be multiplied by a factor of 2.

$\Rightarrow g(E) \propto E^2$

Correct option is (d)

1.17. According to Moseley's law :

$$\bar{\nu} \propto (Z - \sigma)^2 \qquad \text{... (i)}$$

Here, $\bar{\nu}$ = characteristic frequency emitted by an element

Z = atomic number of element

Here, for strong K_α line

$$\frac{1}{0.1785\ \text{nm}} \propto (27-1)^2 \qquad \text{... (ii)}$$

Here, we have used $\sigma = 1$ as screening factor

and for weak K_α line

$$\frac{1}{0.1930\ \text{nm}} \propto (Z-1)^2 \qquad \text{... (iii)}$$

Now, from equations (ii) and (iii), we have

$$\frac{\left(\dfrac{1}{0.1785\ \text{nm}}\right)}{\left(\dfrac{1}{0.1930\ \text{nm}}\right)} = \frac{(26)^2}{(Z-1)^2}$$

$$\Rightarrow \quad (Z-1)^2 = \frac{0.1785 \times 26^2}{0.1930} = 625 \quad \Rightarrow \quad z - 1 = 25 \Rightarrow 26$$

Hence, correct option is (b).

CAREER ENDEAVOUR

1.18. Hall voltage is defined as :

$$V_H = \frac{BI}{t\,n\,e}$$

Where, B is magnetic field , I is current, t is the thickness, n is charge density and e is electron charge.

$$= \frac{0.2 \times 1 \times 10^{-3}}{200 \times 10^{-6} \times 10^{23} \times 1.6 \times 10^{-19}}$$

$$= \frac{0.2 \times 10^{-1}}{200 \times 1.6} = 0.625 \times 10^{-4} = 62.5\ \mu\text{V}.$$

Here, in this question, the silicon is doped with phosphorus (P) which is pentavalent. Therefore, the Hall voltage will be produced by electrons and will be of negative type.

Hence, correct option is (d).

1.19. The probability in Fermi Dirac statistics is

$$p(E) = \frac{1}{e^{\frac{E - E_F}{k_B T}} + 1}, \text{ where } E - E_F = 0.2 \text{ eV and } T = 700 \text{ K}.$$

$$\text{So, } p(E) = \frac{1}{\exp\left[\frac{0.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 700}\right] + 1} = \frac{1}{\exp\left[\frac{2 \times 1600}{138 \times 7}\right] + 1} = 0.035 = 3.5\,\%.$$

Correct option is (c).

1.20. Here, interatomic distance, $d = 0.3$ nm

Wavelength of X-ray is, $\lambda = 0.03$ nm

From Bragg's law : $n\lambda = 2d \sin\theta$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{2d} \qquad (\because \text{ smallest scattering is considered hence we have taken } n = 1)$$

$$\Rightarrow \quad \sin\theta = \frac{0.03\text{ nm}}{2 \times 0.3\text{ nm}} = \frac{0.03}{0.6} = \frac{1}{20}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{20}\right) = \sin^{-1}(0.05)$$

$$\Rightarrow \quad \theta = 2.86^\circ \simeq 2.9^\circ$$

Hence, correct option is (a).

1.21. Since, we know that the infra-red absorption can be observed only on those molecules which have permanent dipole moment. In the above options, only HCl has permanent dipole moment.

Hence, correct option is (c).

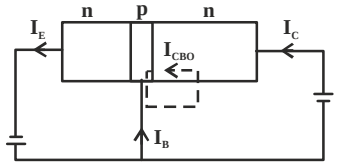
1.22. Charge conjugation because both of these nuclear reactions are outcome of strong interaction. Also, K meson is present as a product of these reactions.

Correct option is (c)

CAREER ENDEAVOUR

1.23. RAM (Random Access memory)
ROM (Read Only Memory)
Both are the computer memories.
Correct option is (b)

1.24. For dc-biasing we apply input loop in forwards biased and output in reverse biased. When input loop is open still these have current in output loop. Which is represent by I_{CBO} it is called leakage current also it is known as reverse saturation current of a reverse-biased diode.



So, current will be flow from collector to base due to motion of electron from based to collectors.
Correct option is (d)

1.25. As the temperature increase the number of free electron at the conduction band increase. So, conductivity of the semiconductor increase. So, in the circuit current will increase as the temperature of the semiconductor is raised.
Correct option is (a)

2.1. $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$

$$\nabla^2\vec{A} = \frac{\partial^2}{\partial x^2}\left(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z\right) + \frac{\partial^2}{\partial y^2}\left(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z\right) + \frac{\partial^2}{\partial z^2}\left(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z\right) = 0$$

Correct option is (c)

2.2. Laurent series expansion of $f(z)$ about $z = 0$, will be

$$f(z) = \frac{\sin z}{z^6} = \frac{1}{z^6}\left[z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + ...\right]$$

$$= \frac{1}{z^5} - \frac{1}{3!}\frac{1}{z^3} + \frac{1}{5!}\frac{1}{z} - .....$$

Residue of $f(z)$ = coefficient of $\left(\frac{1}{z}\right) = \frac{1}{5!}$

Correct option is (b)

2.3. $$F\left[f(x)\right] = g(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-ikx}dx$$

$$F^2\left[f(x)\right] = F\left[g(k)\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(k)\, e^{-ikx}dk$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(k)\, e^{ik(-x)}dk = f(-x) \qquad \left[\because\ F^{-1}\left[g(k)\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(k)e^{ikx}dx = f(x)\right]$$

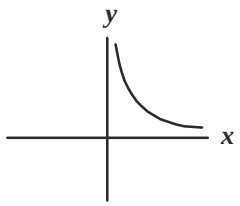
CAREER ENDEAVOUR

2.4. $xy = C$

$\therefore \quad \dot{x}y + \dot{y}x = 0 \Rightarrow \dot{y} = -\dfrac{\dot{x}y}{x}$

The kinetic energy of the particle is

$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) = \frac{1}{2}m\left(\dot{x}^2 + \frac{\dot{x}^2 y^2}{x^2}\right) = \frac{1}{2}m\dot{x}^2\left(1 + \frac{C^2}{x^4}\right)$$



Potential energy, $V = mgy = \dfrac{mgC}{x}$

Therefore, the Lagrangian of the particle is given by

$$L = T - V = \frac{1}{2}m\dot{x}^2\left(1 + \frac{C^2}{x^4}\right) - \frac{mgC}{x}$$

Correct option is (b)

2.5. $\left[q, f(p)\right]_{qp} = \alpha f(p)$

$\Rightarrow \quad \dfrac{\partial f}{\partial p} - 0 = \alpha f(p)$

$\Rightarrow \quad \dfrac{\partial f(p)}{\partial p} = \alpha f(p) \Rightarrow \ell n f(p) = +\alpha p$

$\Rightarrow \quad f(p) = e^{+\alpha p}$

Correct option is (a)

2.6. $\left[X, Y\right] = \left[x\cos\varphi + p\sin\varphi, -x\sin\varphi + p\cos\varphi\right]$

$= -\cos\varphi\sin\varphi\left[x, x\right] + \cos^2\varphi\left[x, p\right] - \sin^2\varphi\left[p, x\right] + \cos\varphi\sin\varphi\left[p, p\right]$

$= 0 + \cos^2\varphi(i) - \sin^2\varphi(-i) + 0 = i\left(\cos^2\varphi + \sin^2\varphi\right) = i$

Correct option is (c)

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2.7. For a quantum particle of mass *m* confined to a square region in x-y plane, the wave function of the particle should be zero at the boundary of the square for the sake of continuous nature of the wave function. Thus, the admissible wave function of the particle should satisfy the following condition:

$$\psi(x = 0, y) = \psi(x = L, y) = \psi(x, y = 0) = \psi(x, y = L) = 0$$

Only $\psi(x, y) = \dfrac{2}{L}\sin\left(\dfrac{n\pi x}{L}\right)\sin\left(\dfrac{n\pi y}{L}\right)$ satisfy the above mentioned condition.

Correct option is (c)

2.8. $L_+L_- = \left(L_x + iL_y\right)\left(L_x - iL_y\right) = L_x^2 + L_y^2 + i\left(L_yL_x - L_xL_y\right) = L_x^2 + L_y^2 + i\left[L_y, L_x\right]$

$= L^2 - L_z^2 + i\left(-i\hbar L_z\right) = L^2 - L_z^2 + \hbar L_z$

Therefore, $\left\langle L_+L_- \right\rangle = \left\langle \psi \,|\left(L^2 - L_z^2 + \hbar L_z\right)|\,\psi \right\rangle = l(l+1)\hbar^2 - l^2\hbar^2 + \hbar(l\hbar) = 2l\hbar^2$

Correct option is (b)

2.9. For the given state of the particle to be normalized, the following condition should be satisfied:

$$\langle \psi(x,t) | \psi(x,t) \rangle = 1$$

$$\Rightarrow \quad \left[C_0 \langle \phi_0 | e^{iE_0 t/\hbar} + C_1 \langle \phi_1 | e^{iE_1 t/\hbar} + ... + C_n \langle \phi_n | e^{iE_n t/\hbar} + ... \right]$$
$$\left[C_0 | \phi_0 \rangle e^{-iE_0 t/\hbar} + C_1 | \phi_1 \rangle e^{-iE_1 t/\hbar} + ... + C_n | \phi_n \rangle e^{-iE_n t/\hbar} + ... \right] = 1$$

$$\Rightarrow \quad |C_0|^2 \langle \phi_0 | \phi_0 \rangle + |C_1|^2 \langle \phi_1 | \phi_1 \rangle + ... + |C_n|^2 \langle \phi_n | \phi_n \rangle + ... = 1$$

$$\Rightarrow \quad |C_0|^2 + |C_1|^2 + ... + |C_n|^2 + ... = 1 \Rightarrow \sum_{n=0}^{\infty} |C_n|^2 = 1$$

Average energy of the particle in the given state will be

$$\langle E \rangle = \langle \psi | \hat{H} | \psi \rangle = |C_0|^2 E_0 + |C_1|^2 E_1 + ... + |C_n|^2 E_n + ... = \sum_{n=0}^{\infty} |C_n|^2 E_n$$

$\psi(x,t)$ will be an Eigen function of the Hamiltonian of the particle only if $\phi_0(x), \phi_1(x), ... \phi_n(x)$ .....all have same energy eigenvalues, i.e., the states are degenerate.

$\psi(x,t)$ is not an Eigen function of the momentum operator as

$$\hat{p}_x \psi(x,t) = (-i\hbar) \frac{\partial}{\partial x} \left[\psi(x,t) \right] \neq \lambda \psi(x,t)$$

Correct option is (a)

2.10. Capacitance of coaxial cable is per unit length

$$C = \frac{2\pi\varepsilon_r \varepsilon_0}{\ell n(b/a)} = \frac{2\pi 3.5 \times 8.85 \times 10^{-12}}{\ell n\left(\frac{0.02}{0.01}\right)} = 280.63 \times 10^{-12} F/m = 280.63\ nF\ /\ km$$

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Correct option is (a)

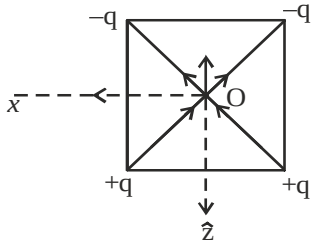
2.11. We know that magnetic field due to sheet carrying uniform current is given by

$$\bar{B} = \frac{\mu_0}{2} \bar{K} \times \hat{n}$$

So, $|B| = \frac{\mu_0 K}{2} = \frac{4\pi \times 10^{-7} \times 50}{2} = 10\pi \times 10^{-6}\ Wb$

Correct option is (d)

2.12. From figure we can say that there will be a net electric field along $-\hat{z}$ direction and field is not uniform.



So, some force will act on the dipole.
If dipole is placed along z direction then torque
on it about the point O. $\tau = \vec{P} \times \vec{E} = PE \sin 180^\circ = 0$.
If the dipole is placed along x direction then potential energy.

$u = -\vec{P}\cdot\vec{E} = PE\cos 90^\circ = 0$, which is not minimum. Because minimum potential energy of dipole is $-pE$.

Correct option is (b)

2.13. Let $E(r,t) = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

We have, $$\nabla^2 E(r,t) = \frac{1}{c^2}\left[\frac{\partial^2 E(r,t)}{\partial t^2} + \omega'^2 E(r,t)\right]$$

$$-k^2 E = \frac{1}{c^2}\left[-\omega^2 + \omega'^2\right]E \qquad (\text{since, } E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \neq 0\ ]$$

$\Rightarrow$ $$k^2c^2 = \omega^2 - \omega'^2$$

$\Rightarrow$ $$\omega^2 = c^2k^2 + \omega'^2$$

Correct option is (b)

2.14. Free electron density of the copper wire is

$$n = \frac{8.94\times10^3}{1.05\times10^{-25}} 1/m^3 = 8.51\times10^{28}\ /\ m^3$$

We know that current density

$$J = \frac{I}{A} = \frac{1}{1\times10^{-6}} = 10^6\ A/m^2$$

Also, $J = nev$, so, $nev = 10^6$

$$v = \frac{10^6}{8.51\times10^{28}\times1.6\times10^{-19}} \simeq 7.4\times10^{-5}\ m/\sec$$

Correct option is (d)

2.15. For $^2p_{1/2}$ term ; $J = \frac{1}{2},\ I = \frac{1}{2}$

The allowed values of hyperfine structure quantum number are

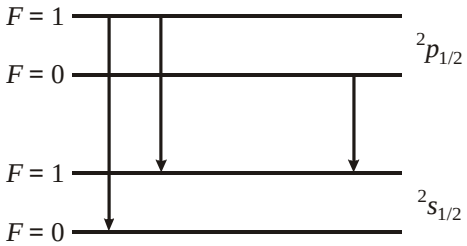
$$F = |J+I|, |J+I-1|, ..., |J-I|$$

$$= \left|\frac{1}{2}+\frac{1}{2}\right|, \left|\frac{1}{2}+\frac{1}{2}-1\right|, ..., \left|\frac{1}{2}-\frac{1}{2}\right| \quad = 1, 0$$

Similarly, for $^2s_{1/2}$ term, $J = \frac{1}{2}, I = \frac{1}{2}$

Thus, $F = 1, 0$

Now, selection rule $\Delta F = 0, \pm1\ (0 \not\leftrightarrow 0)$ will gives transitions as shown in figure.



Hence, total transitions are 3.

Hence, correct option is (a).

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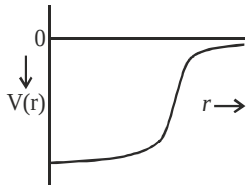
2.16. Intereaction potential can be given as

$$V(r) = -\frac{a}{r} + br$$



where, $-\frac{a}{r} \rightarrow$ because nuclear forces are short range and decreases with decreasing distnace

$br \rightarrow$ because Nuclear forces will be of repulsion type at long range.

Correct option is (b)

2.17. For (a), $\Delta L = (-1 + 1) - (1 - 1) = (0) - (0) = 0$

For (b), $\Delta L = (1) - (1) = 0$

For (c), $\Delta L = (-1) - (1) = -2 \neq 0 \rightarrow$ violates lepton number conservation

For (d), $\Delta L = (1) - (1) = 0$

Correct option is (c)

2.18. For $^3p_1$ term, $l = 1,\ j = 1,\ 2s + 1 = 3 \Rightarrow s = 1$

Therefore, Lande g-factor

$$g = 1 + \left[\frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}\right] = 1 + \left[\frac{1(1+1) - 1(1+1) + 1(1+1)}{2 \times 1 \times (1+1)}\right] = 1 + \left[\frac{2 - 2 + 2}{4}\right] = 1 + \frac{1}{2} = \frac{3}{2}.$$

Hence, correct option is (b).

2.19. Selection rules of rotational Raman spectrum,

$\Delta J = 0, \pm 2$

$\Delta J = 0$, corresponds to the Rayleigh line. However, $\Delta J = \pm 2$, gives anti-Stokes line (+) and stokes line(–) respectively.

The wave number of the rotation Raman lines

$$\nu_{\text{Raman}} = \nu \pm \left[F(J+2) - F(J)\right] = \nu \pm \left[B(J+2)(J+3) - BJ(J+1)\right] \quad \left[\because\ F(J) = BJ(J+1)\right]$$

$$= \nu \pm 4B\left(J + \frac{3}{2}\right)$$

The rotation Raman shift

$$\Delta\nu_{Raman} = 4B\left(J + \frac{3}{2}\right)$$

For J = 0, 1, 2, 3, ..., we get rotational Raman spectrum at distance 6B, 10B, 14B, 18B, ... with equidistant separation 4B.

Correct option is (b)

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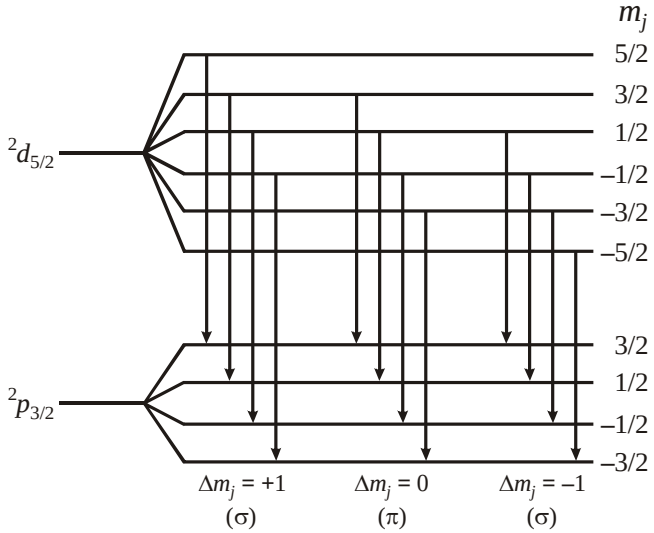
2.20. The electronic transitions of $^2d_{5/2} \rightarrow {}^2p_{3/2}$ is shown in figure (below):



The transitions with selection rules $\Delta m_j = 0, \pm 1$ gives 12 Zeeman components in a weak field.

Hence, correct option is (c).

2.21. Total impedance of the circuit

$$\frac{1}{z} = \frac{1}{R} + \frac{1}{jX_L}$$

Given : $R = 600\,\Omega,\ X_L = 600\,(\Omega)$

$$\therefore \qquad z = \frac{600 \times 600\, j}{600 + j\,600} \qquad \Rightarrow |z| = \frac{600}{\sqrt{2}} = 424.26\,\Omega$$

Correct option is (c)

2.22. The pinch of voltage is given by

$$V_p = \frac{-q \cdot N_D \cdot a^2}{2\varepsilon}$$

$q = 1.6 \times 10^{-19}$ Columb ; N_D = Donor ions concentration $= 10^{21}$

$a = 2 \times 10^{-6}\, m$ (Depletion width)

$\varepsilon = \varepsilon_0 \varepsilon_r \qquad \Rightarrow \quad \varepsilon_{r_{si}} = 11.7$ to 12

$\varepsilon_0 = 8.854 \times 10^{-12}$

$$V_p = \frac{-1.6 \times 10^{-19} \times 10^{21} \times \left(2 \times 10^{-6}\right)^2}{2 \times 8.854 \times 10^{-12} \times 12}$$

$V_p = -3.01$ Volt

2.23. $\dfrac{dy}{dx} = y - z,\ \dfrac{dz}{dx} = -4y + z$

Equation can be expressed as

$$(D-1)\,y + z = 0 \qquad \ldots (1) \qquad [\text{where, } D \equiv \frac{d}{dx}]$$

$$(D-1)z+4y=0 \qquad \ldots (2)$$

(1) × (D–1) $\Rightarrow (D-1)^2 y+(D-1)z=0$

(2) × 1 $\Rightarrow 4y+(D-1)z=0$

Subtracting we get,

$$(D-1)^2 y-4y=0$$

$$\Rightarrow \quad D^2y-2Dy-3y=0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}-\frac{2dy}{dx}-3y=0$$

Let the trial solution $y=e^{mx}$

$$m^2-2m-3=0$$

$$\Rightarrow \quad m^2-3m+m-3=0$$

$$\Rightarrow \quad (m-3)\times(m+1)=0$$

$$\Rightarrow \quad m=3,-1$$

$$\therefore \quad y=Ae^{3x}+Be^{-x}$$

So, $z=y-\frac{dy}{dx}=z=Ae^{3x}+Be^{-x}-3Ae^{3x}+Be^{-x}$

$$=-2Ae^{3x}+2Be^{-x}$$

Correct option is (a)

2.24. $u(x,y,z)=f(x+i\beta y+vt)+g(-x-i\beta y-vt)$

Let $S=(x+i\beta y-vt)$ and $V=(x-i\beta y-vt)$

$$\frac{\partial u}{\partial x}=\frac{\partial f}{\partial S}\cdot\frac{\partial S}{\partial x}+\frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial x}=\frac{\partial f}{\partial S}-\frac{\partial g}{\partial V}$$

$$\therefore \quad \frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial S}\left(\frac{\partial f}{\partial S}-\frac{\partial g}{\partial V}\right)\cdot\frac{\partial S}{\partial x}+\frac{\partial}{\partial V}\left(\frac{\partial f}{\partial s}-\frac{\partial g}{\partial V}\right)\cdot\frac{\partial V}{\partial x}$$

$$=\frac{\partial^2 f}{\partial S^2}-\frac{\partial^2 g}{\partial S\partial V}-\frac{\partial^2 f}{\partial V\partial S}+\frac{\partial^2 g}{\partial V^2}$$

$$\frac{\partial u}{\partial y}=\frac{\partial f}{\partial S}\cdot\frac{\partial S}{\partial y}+\frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial y}=i\beta\frac{\partial f}{\partial S}-i\beta\frac{\partial g}{\partial V}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial S}\left(i\beta\frac{\partial f}{\partial S}-i\beta\frac{\partial g}{\partial V}\right)\cdot\frac{\partial S}{\partial y}+\frac{\partial}{\partial V}\left(i\beta\frac{\partial f}{\partial S}-i\beta\frac{\partial g}{\partial V}\right)\cdot\frac{\partial V}{\partial y}$$

$$=-\beta^2\frac{\partial^2 f}{\partial S^2}+\beta^2\frac{\partial^2 g}{\partial S\partial V}+\beta^2\frac{\partial^2 f}{\partial V\partial S}-\beta^2\frac{\partial^2 g}{\partial V^2}$$

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$$\frac{\partial u}{\partial t} = \frac{\partial f}{\partial S}\frac{\partial S}{\partial t} + \frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial t} = +v\frac{\partial f}{\partial S} - v\frac{\partial g}{\partial V}$$

$$\therefore \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial S}\left(+v\frac{\partial f}{\partial S} - v\frac{\partial g}{\partial V}\right)\frac{\partial S}{\partial t} + \frac{\partial}{\partial V}\left(+v\frac{\partial f}{\partial S} - v\frac{\partial g}{\partial V}\right)\cdot\frac{\partial g}{\partial t}$$

$$= v^2\frac{\partial^2 f}{\partial S^2} - v^2\frac{\partial^2 g}{\partial S\partial V} - v^2\frac{\partial^2 f}{\partial V\partial S} + v^2\frac{\partial^2 g}{\partial V^2}$$

$$\therefore \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

$$\Rightarrow \quad \frac{\partial^2 f}{\partial S^2} - \frac{\partial^2 g}{\partial S\partial V} - \frac{\partial^2 f}{\partial V\partial S} + \frac{\partial^2 g}{\partial V^2} - \beta^2\frac{\partial^2 f}{\partial S^2} + \beta^2\frac{\partial^2 g}{\partial S\partial V} + \beta^2\frac{\partial^2 f}{\partial V\partial S} - \beta^2\frac{\partial^2 y}{\partial V^2}$$

$$-\frac{v^2}{c^2}\frac{\partial^2 f}{\partial S^2} + \frac{v^2}{c^2}\frac{\partial^2 g}{\partial S\partial V} + \frac{v^2}{c^2}\frac{\partial^2 f}{\partial V\partial S} - \frac{v^2}{c^2}\frac{\partial^2 g}{\partial V^2} = 0$$

Coefficient $\frac{\partial^2 f}{\partial V^2}$ should be zero

$$1 - \frac{v^2}{c^2} - \beta^2 = 0 \Rightarrow \beta^2 = \left(1 - \frac{v^2}{c^2}\right) \Rightarrow \beta = \left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

Correct option is (c)

2.25. The rotational energy of a diatomic molecule of moment of inertia I is

$E = \frac{\hbar^2}{2I}\ell(\ell+1)$; $\ell = 0, 1, 2, \ldots.$, where ℓ is the quantum number associated to the rotation.

The partition function is $Z = \sum_i g_i e^{-\frac{E_i}{k_B T}} = \sum_{\ell=0}^{\infty}(2\ell+1)e^{-\frac{\hbar^2}{2I k_B T}\ell(\ell+1)}$; $\ell = 0, 1, 2, \ldots.$

For large value of I and T, the energy separation between the levels is small and we can replace summation by integration. So the approximate rotational partition function is $Z = \int_0^{\infty} 2\ell e^{-\frac{\hbar^2\ell^2}{2I k_B T}} d\ell$.

Letting $\frac{\hbar^2}{2I k_B T} = a$. Then, $Z = 2\int_0^{\infty} \ell e^{-a\ell^2} d\ell$

Now, taking $\ell^2 = t$. Then $2\ell\, d\ell = dt$

Therefore, $Z = \int_0^{\infty} e^{-at} dt = \frac{e^{-at}}{-a}\Big|_0^{\infty} = \frac{1}{a} = \frac{2I k_B T}{\hbar^2}$.

Correct option is (b).

DESCRIPTION

3. $\iint_S \left(\vec{\nabla}\times\vec{A}\right)\cdot\hat{n}\,dS$

Using Divergence theorem

$$=\iiint_V \left(\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right)\right)dV = 0 \qquad \left[\because\ \vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right)=0\right]$$

Correct answer is (0)

4. $$X=\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=a\hat{e}_1+b\hat{e}_2+c\hat{e}_3$$

$$TX = aT\left(\hat{e}_1\right)+bT\left(\hat{e}_2\right)+cT\left(\hat{e}_3\right)$$

$$\Rightarrow T\begin{pmatrix}a\\b\\c\end{pmatrix}=a\begin{pmatrix}1\\0\\1\end{pmatrix}+b\begin{pmatrix}1\\1\\0\end{pmatrix}+c\begin{pmatrix}0\\1\\1\end{pmatrix} \qquad \left[\because T\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}a+b\\b+c\\c+a\end{pmatrix}\right]$$

$$\Rightarrow T\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1&1&0\\0&1&1\\1&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}$$

$$\Rightarrow \qquad T=\begin{pmatrix}1&1&0\\0&1&1\\1&0&1\end{pmatrix}$$

So, $$T^{-1}=\frac{Adj\left(T\right)}{\det\left(T\right)}=\frac{1}{2}\begin{pmatrix}1&1&-1\\-1&1&1\\1&-1&1\end{pmatrix}^{T}=\frac{1}{2}\begin{pmatrix}1&-1&1\\1&1&-1\\-1&1&1\end{pmatrix}$$

5. $$4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$$

Since, $x=0$ is a regular singular point, we assume the solution,

$$y=\sum_{k=0}^{\infty}a_k x^{m+k} \Rightarrow \frac{dy}{dx}=\sum_{k=0}^{\infty}\left(m+k\right)a_k\ x^{m+k-1}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2}=\sum_{k=0}^{\infty}a_k\left(m+k\right)\left(m+k-1\right)x^{m+k-2}$$

Putting these value in the above equation, we get,

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$$4\sum a_k (m+k)(m+k-1)x^{m+k-1} + 2\sum a_k (m+k)x^{m+k-1} + \sum a_k x^{m+k} = 0$$

$$\Rightarrow \quad 2\sum a_k (m+k)(2m+2k-1)x^{m+k-1} + \sum a_k x^{m+k} = 0$$

Now, eqating the coefficient of the lowest power of x i.e. $x^{m-1}$to be equal to zero

$$2a_0 m(2m-1) = 0 \quad \Rightarrow \qquad m = 0 \text{ or } m = \frac{1}{2} \text{ as } a_0 \neq 0$$

Now, equating the coefficient of the x^{m+k} to be equal to zero

$$2a_{k+1}(m+k+1)(2m+2k+1) + a_k = 0 \quad \Rightarrow \quad a_{k+1} = \frac{-a_k}{2(m+k+1)(2m+2k+1)}$$

The first solution of y for $m = 0$, $a_{k+1} = \dfrac{-a_k}{2(k+1)(2k+1)}$

For $k = 0$, $a_1 = -\dfrac{a_0}{2}$

For $k = 1$, $a_2 = -\dfrac{a_1}{2\times 2\times 3} = +\dfrac{a_0}{24}$

For $k = 2$, $a_3 = -\dfrac{a_2}{2\times 3\times 5} = -\dfrac{a_0}{24\times 30}$

$$\therefore \qquad y_1(x) = \sum_{k=0}^{\infty} a_k x^{m+k} = \sum_{k=0}^{\infty} a_k x^k$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 .....$$

$$= a_0 - \frac{x}{2}a_0 + \frac{a_0}{24}x^2 - \frac{a_0}{24\times 30}x^3 .....$$

$$= a_0\left(1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} .....\right)$$

The second solution for $m = \dfrac{1}{2}$

$$a_{k+1} = \frac{-a_k}{2\left(k+\frac{3}{2}\right)(2k+2)} = -\frac{a_k}{2(2k+3)(k+1)}$$

For k =0, $a_1 = -\dfrac{a_0}{6}$, For k = 1, $a_2 = -\dfrac{a_1}{2\times 5\times 2} = +\dfrac{a_0}{120}$, For $k = 2$, $a_3 = -\dfrac{a_2}{2\times 7\times 3} = -\dfrac{a}{120\times 42}$

$$\therefore \qquad y_2(x) = \sum a_k x^{\frac{1}{2}+k} = a_0 x^{1/2} + a_1 x^{3/2} + a_2 x^{5/2} .....$$

$$= a_0 x^{1/2} - \frac{a_0}{6}x^{3/2} + \frac{a_0}{120}x^{5/2} .....$$

$$= a_0\left(x^{1/2} - \frac{x^{3/2}}{6} + \frac{x^{5/2}}{120} - .....\right)$$

$$\therefore \qquad y(x) = Ay_1(x) + By_2(x) = A\left(1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} .....\right) + B\left(x^{1/2} - \frac{x^{3/2}}{6} + \frac{x^{5/2}}{120} - .....\right)$$

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6. (a) $L = \dfrac{q\ln\left(\dot{q}^2\right)}{2} - \lambda q + qf\left(q\right)$

The Euler-Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$L = \frac{q\ln\left(\dot{q}^2\right)}{2} - \lambda q + qf\left(q\right)$$

$$\frac{dL}{d\dot{q}} = \frac{q}{2\dot{q}^2}2\dot{q} = \frac{q\dot{q}}{\dot{q}^2} = \frac{q}{\dot{q}}$$

$$\frac{dL}{dq} = \frac{\ln\left(\dot{q}^2\right)}{2} - \lambda + f\left(q\right) + q\frac{df}{dq}$$

$\therefore$ $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$\Rightarrow$ $$\frac{d}{dt}\left(\frac{q}{\dot{q}}\right) - \frac{\ln\left(\dot{q}^2\right)}{2} + \lambda - f\left(a\right) - q\frac{\partial f}{\partial q} = 0$$

$\Rightarrow$ $$\frac{\dot{q}\dot{q} - q\ddot{q}}{\dot{q}^2} - \frac{\ln\left(\dot{q}^2\right)}{2} + \lambda - f\left(q\right) - q\frac{\partial f}{\partial q} = 0$$

$\Rightarrow$ $$1 - \frac{q\ddot{q}}{\dot{q}^2} - \frac{\ln\left(\dot{q}^2\right)}{2} + \lambda - f\left(q\right) - q\frac{\partial f}{\partial q} = 0$$

(b) Since, L is function of $f\left(\alpha\right)$, arbitrary function. So, lagrangian does not covariant under the transform $q - \alpha q$

(c) Hamiltonian, $H = p\dot{q} - L = \dfrac{q}{\dot{q}} \times \dot{q} - \dfrac{q\ln\left(\dot{q}^2\right)}{2} + \lambda q - qf\left(a\right) = -\dfrac{q\ln\left(\dot{q}^2\right)}{2} + q\left(\lambda + 1\right) - qf\left(q\right)$

(d) Since, L does not depend on time explicitly. So, total energy of the motion is conserved.

7. Since, the lamina is very thin

$\therefore$ $x_3 = 0$

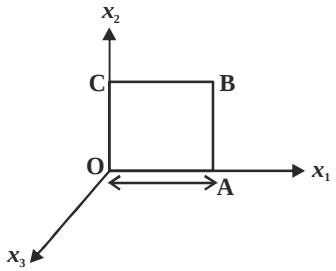
$$I_{11} = \int\left(x_2^2 + x_3^2\right)dm = \sigma\int_0^L x_2^2 dx_2 \int_0^L dx_1 = \frac{1}{3}m\ell^2$$

Similarly, $I_{22} = \dfrac{1}{3}m\ell^2$

$\therefore$ $I_{33} = I_{11} + I_{22} = \dfrac{2}{3}m\ell^2$

CAREER ENDEAVOUR

And $I_{21} = I_{12} = -\iint x_1 x_2 \sigma dx_1 dx_2 = -\sigma \int_0^L x_1 dx_1 \int_0^L x_2 dx_2 = -\frac{1}{4} m\ell^2$



And, $I_{31} = I_{13} = I_{32} = I_{23} = 0$

Therefore, the moment of inertial tensor

$$I = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}m^2 & -\frac{1}{4}m\ell^2 & 0 \\ -\frac{1}{4}m\ell^2 & \frac{1}{3}m\ell^2 & 0 \\ 0 & 0 & \frac{2}{3}m\ell^2 \end{pmatrix}$$

Let I_1, I_2 and I_3 are three principal moment of inertia.
Therefore, the characteristic determinant,

$$\begin{vmatrix} \frac{1}{3}m\ell^2 - I & -\frac{1}{4}m\ell^2 & 0 \\ -\frac{1}{4}m\ell^2 & \frac{1}{3}m\ell^2 - I & 0 \\ 0 & 0 & \frac{2}{3}m\ell^2 - I \end{vmatrix} = 0$$

$$\left(\frac{1}{12}m\ell^2 - I\right)\left(\frac{7}{12}m\ell^2 - I\right)\left(\frac{2}{3}m\ell^2 - I\right) = 0$$

$\therefore \quad I_1 = \frac{1}{12}m\ell^2,\ I_2 = \frac{7}{12}m\ell^2,\ I_3 = \frac{2}{3}m\ell^2$

Corresponding to $I_1 = \frac{1}{12}m\ell^2$

$$\begin{pmatrix} I_{11} - I_1 & I_{12} & I_{13} \\ I_{21} & I_{22} - I_1 & I_{23} \\ I_{31} & I_{32} & I_{33} - I_1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = 0$$

$\therefore \quad \left(\frac{1}{3}m\ell^2 - \frac{1}{12}m\ell^2\right)\omega_1 - \frac{1}{4}m\ell^2\omega_2 + 0 = 0$

$$-\frac{1}{4}m\ell^2\omega_1 + \left(\frac{1}{3}m\ell^2 - \frac{1}{2}m\ell^2\right)\omega_2 + 0 = 0 \Rightarrow 0 + 0 + \left(\frac{2}{3}m\ell^2 - \frac{1}{12}m\ell^2\right)\omega_3 = 0$$

$\therefore \quad \omega_1 = \omega_2$ and $\omega_3 = 0$

$\therefore \quad \bar{\omega} = \hat{x}_1\omega_1 + \hat{x}_2\omega_2$

The principal axis is along $(\hat{x}_1 + \hat{x}_2)$

Similarly, the principal axis corresponding to I_2 is along $\hat{x}_2 - \hat{x}_1$ or $\hat{x}_1 - \hat{x}_2$

And the principal axis corresponding to I_3 is along $\hat{x}_3$

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8. (a) $V(x) = k|x|, \quad k > 0$

Therefore, Hamiltonian, $H = \frac{p_x^2}{2m} + V(x)$

$$\left[H, V(x)\right] = \left[\frac{p_x^2}{2m} + V(x), V(x)\right] = \frac{k}{2m}\left[p_x^2, |x|\right] = \frac{2k}{2m} p_x \left[p_x, |x|\right]$$

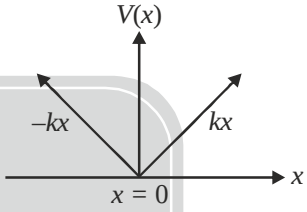
$$= \frac{kp_x}{m}\left[p_x, x\right] = -\frac{i\hbar k}{m} p_x \qquad \text{for } x > 0$$

$$= \frac{kp_x}{m}\left[p_x, -x\right] = \frac{i\hbar k}{m} p_x \qquad \text{for } x < 0$$

(b) $\Delta x \approx 2x$

$$\Delta p \approx \frac{\hbar}{2\Delta x} \approx \frac{\hbar}{4x}$$

$$E = \frac{(\Delta p)^2}{2m} + k|\Delta x| = \frac{\hbar^2}{32x^2 m} + k(2x)$$



For minimum energy,

$$\frac{dE}{dx} = \frac{\hbar^2}{32m}\left(-\frac{2}{x^3}\right) + 2k = 0$$

$$\Rightarrow \qquad x^3 = \frac{\hbar^2}{64km} \Rightarrow x = \frac{1}{4}\left(\frac{\hbar^2}{2km}\right)^{1/3}$$

So, $E_{G.S.} = \dfrac{\hbar^2}{32 \cdot \frac{1}{16}\left(\frac{\hbar^2}{km}\right)^{2/3}} + \frac{2k}{4} \cdot \left(\frac{\hbar^2}{km}\right)^{1/3} = \left(\frac{\hbar^2 k^2}{m}\right)^{1/3}$

9. $V(x) = 0 \qquad 0 \le x \le L$

$= \infty \qquad$ otherwise

$\phi(x) = cx(L - x)$

According to normalized condition,

$$\int_0^L |\phi(x)|^2 \, dx = 1$$

$$\Rightarrow \qquad |c|^2 \int_0^L x^2 (L - x)^2 \, dx = 1 \Rightarrow |c|^2 \int_0^L x^2 \left(L^2 - 2Lx + x^2\right) dx = 1$$

$$\Rightarrow \qquad |c|^2 \int_0^L \left(L^2 x^2 - 2Lx^3 + x^4\right) dx = 1 \Rightarrow |c|^2 \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^L = 1$$

CAREER ENDEAVOUR

$$\Rightarrow \qquad \left|c^2\right|^2\left[\frac{L^5}{3}-\frac{L^5}{2}+\frac{L^5}{5}\right]=1$$

$$\Rightarrow \qquad |c|^2\left[\frac{10-15+6}{30}\right]L^5=1 \Rightarrow \left|c^2\right|\frac{L^5}{30}=1 \Rightarrow |c|=\sqrt{\frac{30}{L^5}}$$

$$\langle E\rangle=\int_0^L \phi^*(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\phi(x)\,dx=\frac{30}{L^5}\int_0^L x(L-x)\left(-\frac{\hbar^2}{2m}\right)(-2)\,dx$$

$$=\frac{\hbar^2}{m}\cdot\frac{30}{L^5}\int_0^L x(L-x)\,dx=\frac{30\hbar^2}{mL^5}\left[L\frac{x^2}{2}-\frac{x^3}{3}\right]_0^L=\frac{30\hbar^2}{mL^5}\left[\frac{L^3}{2}-\frac{L^3}{3}\right]=\frac{30\hbar^2}{mL^5}\times\frac{L^3}{6}=\frac{5\hbar^2}{mL^2}$$

(b) The ground state of the particle in a box

$$\psi_0=\sqrt{\frac{2}{L}}\sin\frac{\pi}{L}x$$

Therefore, the probability of finding the particle in the ground state is

$$\left|\int_0^L {\psi_0}^*\phi\,dx\right|^2=\left|\int_0^L\sqrt{\frac{2}{L}}\sin\frac{\pi}{L}x\cdot\sqrt{\frac{30}{L^5}}x(L-x)\,dx\right|^2$$

$$=\left(\sqrt{\frac{2}{L}\times\frac{30}{L^5}}\right)^2\left|\int_0^L\left(xL\sin\frac{\pi}{L}x-x^2\sin\frac{\pi}{L}x\right)dx\right|$$

$$=\frac{60}{L^6}\times\left|\left(L\times\frac{L^2}{\pi}-\frac{L^3}{\pi}+\frac{4L^3}{\pi^3}\right)\right|^2=\frac{60}{L}\times\left|\frac{4L^3}{\pi^3}\right|^2=\frac{60\times16}{\pi^6}=\frac{960}{961.32}\simeq0.9985$$

10. We have, $H'=\lambda(xp+px)$

The first order energy correction,

$$E^{(I)}=\langle n|H'|n\rangle=\lambda\langle n|xp+px|n\rangle=\langle n|xp-px+2px|n\rangle=\langle n|[x,p]|n\rangle+2\lambda\langle n|px|n\rangle$$

$$=\lambda\langle n|i\hbar|n\rangle+2\lambda\langle n|px|n\rangle=i\hbar\lambda+2\lambda\left\langle n\left|i\sqrt{\frac{\hbar^2}{4}}\left(a^\dagger-a\right)\left(a+a^\dagger\right)\right|n\right\rangle$$

$$=i\hbar\lambda+\frac{i\hbar}{2}\times2\lambda\left\langle n\left|a^\dagger a-a^2+a^{\dagger2}-aa^\dagger\right|n\right\rangle$$

$$=i\hbar\lambda+i\hbar\lambda\left\langle n\left|a^\dagger a-a^2+a^{\dagger2}-aa^\dagger\right|n\right\rangle$$

$$=i\hbar\lambda+i\hbar\lambda\left[\langle n|a^\dagger a|n\rangle-\langle n|a^2|n\rangle+\langle n|a^{\dagger2}|n\rangle-\langle n|aa^\dagger|n\rangle\right]$$

$$=i\hbar\lambda+i\hbar\lambda\left[n-0+0-(n+1)\right]=i\hbar\lambda-i\hbar\lambda=0$$

$$=i\hbar\lambda+i\hbar\lambda\left[n-(n+1)\right]=i\hbar\lambda-i\hbar\lambda=0$$

$$\left[x=\sqrt{\frac{\hbar}{2m\omega}}\left(a+a^\dagger\right),\quad p=i\sqrt{\frac{\hbar m\omega}{2}}\left(a^\dagger-a\right)\right]$$

The first order correction in wave function,

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$$\psi_n^{(1)} = \sum_{m\neq n} \frac{\left|\langle m|H'|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle$$

$$= \sum_{m\neq n} \frac{\left|\langle m|H'|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle = \sum_{m\neq n} \frac{\lambda\left|\langle m|xp+px|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle = \sum_{m\neq n} \frac{\lambda\left|\langle m|xp-px+2px|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle$$

$$= \sum_{m\neq n} \frac{\lambda\left|\langle m|[x_i p]|n\rangle + 2\lambda\langle m|px|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle$$

$$= \sum_{m\neq n} \frac{\lambda\left|\langle m|i\hbar|n\rangle + 2\lambda\left\langle m\left|\frac{i\hbar}{2}\left(a^\dagger - a\right)\left(a+a^\dagger\right)\right|n\right\rangle\right|}{E_n^0 - E_m^0}|m\rangle$$

$$= \sum_{m\neq n} 0 + \frac{\left|i\hbar\lambda\left\langle m\left|a^\dagger a - aa^\dagger - a^2 + a^{\dagger 2}\right|n\right\rangle\right|}{E_n^0 - E_m^0}|m\rangle$$

$$= \sum_{m\neq n} \frac{\left|i\hbar\lambda\left\langle m\left|a^\dagger a - aa^\dagger - a^2 + a^{\dagger 2}\right|n\right\rangle\right|}{E_n^0 - E_m^0}|m\rangle$$

$$= \hbar\lambda\left[\sum_{m\neq n} \frac{\left|\langle m|a^\dagger a|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle - \sum_{m\neq n} \frac{\left|\langle m|a^\dagger a|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle - \sum_{m\neq n} \frac{\left|\langle m|a^2|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle + \sum_{m\neq n} \frac{\left|\langle m|a^{\dagger 2}|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle\right]$$

$$= \hbar\lambda\left[0 + 0 + \sum_{m\neq n} \frac{\left|\langle m|a^{\dagger 2}|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle - \sum_{m\neq n} \frac{\left|\langle m|a^2|n\rangle\right|}{E_n^0 - E_m^0}|m\rangle\right]$$

$$= \hbar\lambda\left[\frac{\sqrt{(n+1)(n+2)}}{\left(n+\frac{1}{2}\right)\hbar\omega - \left(n+\frac{5}{2}\right)\hbar\omega}|n+2\rangle - \frac{\sqrt{n(n-1)}}{\left(n+\frac{1}{2}\right)\hbar\omega - \left(n-\frac{3}{2}\right)\hbar\omega}|n-2\rangle\right]$$

$$= -\frac{\lambda}{2\omega}\left[\sqrt{(n+1)(n+2)}|n+2\rangle + \sqrt{n(n-1)}|n-2\rangle\right]$$

11. Density of states, $d^2k = 2\pi k dk$ and $E = \frac{\hbar^2 k^2}{2m} \Rightarrow dE = \frac{\hbar^2 k}{m}dk$

The probability $f\left(\vec{k}\right)$ that the quantum state of wave-vector $\vec{k}$ is occupied by an electron is given by the Fermi dirac-distribution function

$$f\left(\vec{k}\right) = \frac{1}{1+\exp\left[\frac{E(k)-E_F}{kT}\right]}$$

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Number of electrons, $N = 2\times A\int \frac{d^2\vec{k}}{(2\pi)^2} f(\vec{k})$

Factor of 2 comes from two spin of electron

$$2\times A\int \frac{d^2\vec{k}}{(2\pi)^2} \Rightarrow A\int_0^\infty \frac{kdk}{\pi} \Rightarrow A\int_0^\infty \frac{m}{\pi\hbar^2}dE$$

And we know,

$$N = 2\times A\int \frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{1+\exp\left[\frac{E-E_F}{kT}\right]} = \int_0^\infty dE g_{2D}(E)\frac{1}{1+\exp\left[\frac{E-E_F}{kT}\right]} \Rightarrow \boxed{g_{2D}(E) = \frac{mA}{\pi\hbar^2}}$$

At T = 0K,

$$N = \int_0^\infty dE g_{2D}(E)\, f(E-E_F) = \int_0^{E_F} dE g_{2D}(E) = \frac{Am}{\pi\hbar^2}E_F$$

$$\boxed{n = \frac{N}{A} = \frac{m}{\pi\hbar^2}E_F} \Rightarrow N = \frac{Am}{\pi\hbar^2}E_F$$

At $T \neq 0K$, $n = \frac{1}{A}\int_0^\infty dE g_{2D}(E)\frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{m}{\pi\hbar^2}kT\log\left[1+e^{E_{F/kT}}\right]$ and $n = \frac{m}{\pi\hbar^2}E_F$

So, $\boxed{E_F(T) = kT\log\left[e^{\frac{E_F}{kT}}-1\right]}$ Exact relations

12. The total energy is

$E = \frac{p^2}{2m} + V(x,y,z) = \frac{p^2}{2m} + ax^2 + b(y^2+z^2)^{1/2}$, where $p^2 = p_x^2 + p_y^2 + p_z^2$

The partition function is $Z = \frac{1}{h^3}\int e^{-E/k_BT}d^3p\,d^3x$

$$= \frac{1}{h^3}\left[\int_{-\infty}^{\infty} e^{-p_x^2/2mk_BT}dp_x \int_{-\infty}^{\infty} e^{-p_y^2/2mk_BT}dp_y \int_{-\infty}^{\infty} e^{-p_z^2/2mk_BT}dp_z\right]$$

$$\times \int_{-\infty}^{\infty} e^{-ax^2/k_BT}dx \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-b(y^2+z^2)^{1/2}/k_BT}dy\,dz$$

Now using, $\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2\beta^2}} = \sqrt{2\pi}\beta$, we have

$$Z = \frac{1}{h^3}\left[\sqrt{2\pi m k_BT}\cdot\sqrt{2\pi mk_BT}\cdot\sqrt{2\pi mk_BT}\cdot\sqrt{\frac{\pi k_BT}{a}}\right]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-b(y^2+z^2)^{1/2}/k_BT}dy\,dz$$

$$= \frac{1}{h^3}(2\pi mk_BT)^{3/2}\left(\frac{\pi k_BT}{a}\right)^{1/2}\int_0^\infty\int_0^{2\pi} r\, e^{-\frac{br}{k_BT}}dr\,d\theta$$

where we have subsitute $y = r\cos\theta$ and $z = r\sin\theta$

$$= \frac{2\pi}{h^3}\left(2\pi m k_B T\right)^{3/2}\left(\frac{\pi k_B T}{a}\right)^{1/2}\left(\frac{k_B T}{b}\right)^2$$

The average energy of the particle is

$$\langle E \rangle = k_B T^2 \frac{\partial}{\partial T} \ln Z = 4 k_B T$$

13. (a) The interplaner distance of atomic plane (1 0 0)

$$d_{100} = \frac{a}{\sqrt{h^2 + k^2 + \ell^2}} = a = 6.2911 \text{Å}$$

The interplanar distance of atomic plane (1 1 0)

$$d_{110} = \frac{a}{\sqrt{2}} = 4.448 \text{Å}$$

The interplanar distance of atomic plane (1 1 1)

$$d_{111} = \frac{a}{\sqrt{3}} = 3.6321 \text{Å}$$

(b) KCL has fcc structure, so the number of atom in the unit cell is $n = 4$
The lattice constant of the KCL crystal,

$$a = \left(\frac{nM}{\mathrm{N_A}\ \rho}\right)^{1/3} = \left(\frac{4 \times 74.6}{6.02 \times 10^{26} \times 1990}\right)^{1/3} = \left(0.02490 \times 10^{-26}\right)^{1/3} = \left(249 \times 10^{-30}\right)^{1/3} = 6.2911 \text{Å}$$

14. We know the the threshold energy, $E_{th} = (-Q)\left(1 + \frac{m_x}{m_X}\right)$

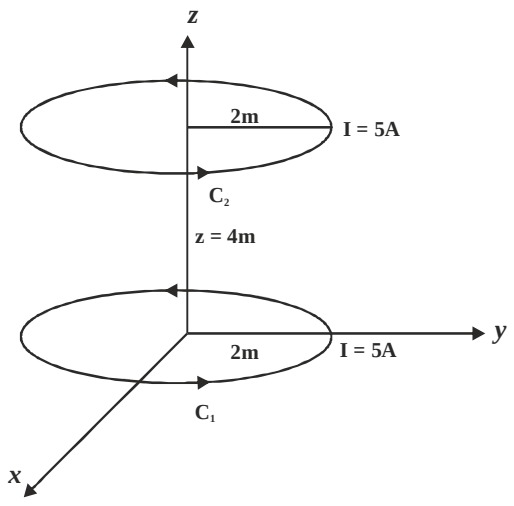
Given : Q = – 0.764 MeV

$m_x = (2p) = 2 \times 1.00814 = 2.01628$ *a.m.u.*

$M_X = M\left({}^3H\right) = 3.0169982$ *a.m.u.*

Therefore, the threshold energy $= 0.764 \times \left(1 + \frac{2.01628}{3.0169982}\right) = 1.02745$ *MeV*

15. The magnetic field at $z = 0$ due current flow in the loop C_2 is



$$\vec{B}_2 = \frac{\mu_0 I}{2}\left[\frac{a^2}{\left(z^2+a^2\right)^{3/2}}\right](\hat{z}) = \frac{\mu_0 I}{2}\left[\frac{4}{(20)^{3/2}}\right](\hat{z}) = \frac{2\mu_0 I}{(20)^{3/2}}(\hat{z})$$

The magnetic field at zero due to current flow in the loop C_1 is

$$\vec{B}_1 = \frac{\mu_0 I}{2a} = \frac{\mu_0 I}{2\times 2} = \frac{\mu_0 I}{4}\hat{z}$$

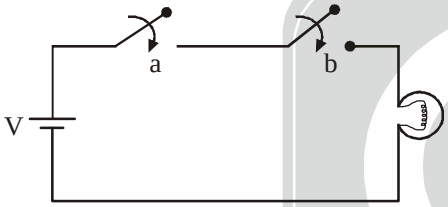
So, total magnetic field at $z = 0$

$$\vec{B} = \left(\frac{\mu_0 I}{4} + \frac{2\mu_0 I}{(20)^{3/2}}\right)\hat{z}$$

$$\vec{B} = \mu_0 I\left[\frac{1}{4} + \frac{2}{(20)^{3/2}}\right]\hat{z} = \mu_0 I(0.272)\hat{z}$$

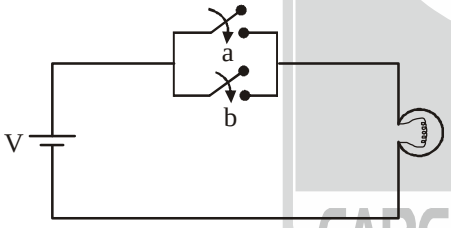
$$= 4\pi\times 10^{-7}\times 5\times 0.272\hat{z} = 15.00\times 10^{-7}\ Tesla = 1.743\ \mu\ Tesla$$

16. (a) AND



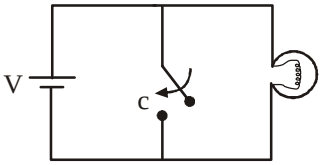
F = ab, Bulb will glow when a and b both switch is closed.

(b) OR



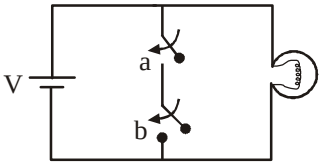
f = a + b
Bulb will glow when any input switch is closed

(c) NOT



$f = \overline{c}$
Bulb will glow when any input switch is closed

(d) NAND



$f = \overline{a \cdot b}$
Bulb will glow when any switch is open

(e) NOR

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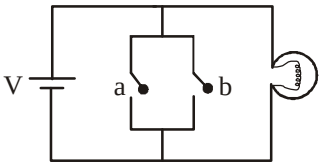


$f = \overline{a + b}$

Bulb will glow when both switch is open

17. Given pinchoff voltage, $V_P = 5V$

Drain to source saturation current, $I_{DSS} = -40$ mA

Drain current, $I_D = -15$ mA

We know that, $I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$

$$-15 = -40\left[1 - \frac{V_{GS}}{5}\right]^2 \Rightarrow \frac{3}{8} = \left[1 - \frac{V_{GS}}{5}\right]^2$$

$$0.612 = 1 - \frac{V_{GS}}{5}$$

$$\frac{V_{GS}}{5} = 0.387$$

$$V_{GS} \simeq 1.94 \text{ Volt}$$

$$\boxed{V_{GS} \equiv 1.94 \text{ Volt}}$$

18. $2d\sin\theta = n\lambda \qquad [\text{given} : 2\theta = 5.8^o, \theta = 2.9^o]$

$$2d\sin\left(2.9^0\right) = \lambda$$

$$\lambda = 2d\sin(2.9) = 2\times 0.3\times 10^{-9}\times 0.05 = 0.03\, nm \text{ or } 0.3\mathring{A}$$

We know, $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{6.62\times 10^{-34}}{0.03\times 10^{-9}} = 2.2\times 10^{-23}\ kgms^{-1}$

Also, $\lambda = \sqrt{\frac{150.4}{V}}\ \mathring{A} \Rightarrow V = \frac{150.4}{\lambda^2} = \frac{150.4}{0.3\times 0.3} = 1671V = 1.67\ kV$

19. Let us take $z < 0$ as region1 and $z > 0$ as region 2
According to boundary condition,

$D_{2n} - D_{1n} = \sigma_{free}$ and $E_{2\tau} = E_{1\tau}$

$$\Rightarrow \quad \frac{D_{2\tau}}{\varepsilon_2} = \frac{D_{1\tau}}{\varepsilon_1}$$

$$\Rightarrow \quad \varepsilon_1 D_{2\tau} = \varepsilon_2 D_{1\tau}$$

$$D_{2n} = 6\hat{e}_z \text{ and } D_{2\tau} = 3\hat{e}_x + 4\hat{e}_y$$

$$\varepsilon_0\left(3\hat{e}_x + 4\hat{e}_y\right) = 2\times\varepsilon_0\left(D_{1\tau}\right)$$

And $D_{2n} - D_{1n} = \sigma_{free} = 5$

$$\Rightarrow \quad D_{1n} = D_{2n} - 5 = 1$$

CAREER ENDEAVOUR

$$\therefore \qquad \vec{D}_1 = \frac{1}{2}\left(3\hat{e}_x + 4\hat{e}_y\right) + \hat{e}_z$$

$$\Rightarrow \qquad \vec{D}_1 = \left(1.5\hat{e}_x + 2\hat{e}_y + \hat{e}_z\right)$$

20. Balmer series for Hydrogen is $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$

If series limit $\left(n_2 = \infty\right)$ is $\lambda_\infty = 360\ nm$

$$\frac{1}{\lambda_\infty} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4} \qquad \Rightarrow R = \frac{4}{\lambda_\infty} = \frac{4}{360\ nm}$$

By Moseley's law:

$$\frac{1}{\lambda} = R\left(z-1\right)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

For minimum λ of this series occur when $n = \infty$,

$$\frac{1}{\lambda} = R\left(Z-1\right)^2 \Rightarrow \left(Z-1\right)^2 = \frac{360\ nm}{4 \times 0.1\ nm} = 900$$

$$\Rightarrow Z - 1 = 30$$

$$\Rightarrow Z = 31$$

21. Cu(Z = 29) = [Ar] $3d^{10}4s^1$ = Ground state electronic configuration

First excited state of Cu = $[Ar]3d^{10}\ 4s^0\ 4p^1$

Second excited state of Cu = $[Ar]\ 3d^{10}\ 4s^0\ 4p^0\ 5s^0\ 5p^1$

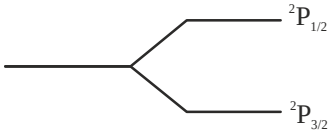
First and second states are written keeping in mind the selection rule $\boxed{\Delta\ell = \pm 1}$

Ground state energy = E_1

First excited state = E_2

Second excited state = E_3

$E_3 \Rightarrow 5p^1 \ \Rightarrow s = \frac{1}{2}, L = \frac{1}{2}$

$E_2 \Rightarrow 4p^1 \Rightarrow s = \frac{1}{2}, L = 1$ 

$E_1 \Rightarrow \ 4s^1 \ \Rightarrow \ s = \frac{1}{2}, L = 0$ ——— $^2S_{1/2}$

State E_2 and E_3 are doubly degenerate (because of splitting) E_3 corresponds to principal quantum number, n = 5, while E_2 corresponds to n = 4. So, E_3 has higher n value. That's why energy of E3 will be more than energy of $E_2\left(E_2 < E_3\right)$

CAREER ENDEAVOUR

22. The given rotational line

$$\bar{v} = 28798 + 3.85m + 0.068m^2$$

We know that the lines of P and R branches of a band are represented by a single formula

$$\bar{v} = v_0 + \left(B'_v + B''_v\right)m + \left(B'_v - B''_v\right)m^2$$

Comparing it with the given equation, we have

$\therefore \quad v_0 = 28798\ cm^{-1}$

$$B'_v + B''_v = 3.85\ cm^{-1}$$

$$B'_v - B''_v = 0.068\ cm^{-1}$$

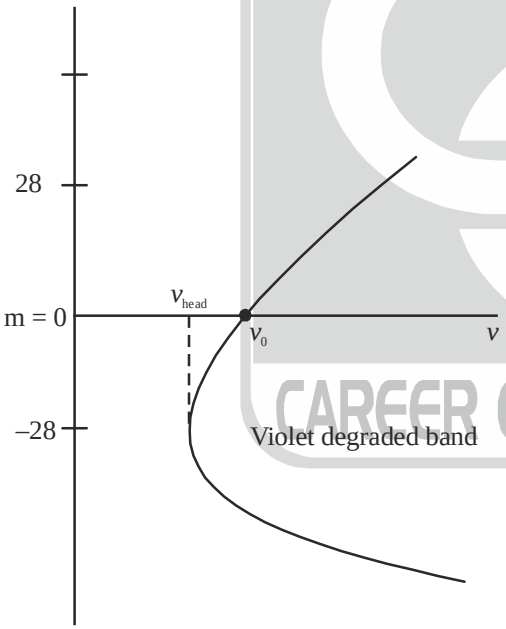
$\therefore \quad B'_v = 1.959\ cm^{-1}$ and $B''_v = 1.891\ cm^{-1}$

Now, the m value corresponding to the vertex of the Fortral Parabola (band-head) is given by

$$m_{vertex} = -\frac{B'_v + B''_v}{2\left(B'_v - B''_v\right)} = -\frac{3.850}{2\times 0.068} = -28.30 \simeq -28$$

Therefore, the band head $v_{head} = v_0 - \frac{\left(B'_v + B''_v\right)^2}{4\left(B'_v - B''_v\right)} = 25798 - \frac{(3.850)^2}{4\times(0.068)} = 25744\ cm^{-1}$

The band head (25744 cm^{-1}) lies toward the lower frequency side of the band origin (25798 cm^{-1}). This says that the band is degrarded toward the low-frequency side, that is toward the violet-degraded band



CAREER ENDEAVOUR

OBJECTIVE QUESTION

1.1. A and B matrices can be diagonalized simultaneously i.e. both A and B can be diagonalized using same matrix.

$$D_A = P^{-1}AP,\ D_B = P^{-1}BP$$

The diagonal matrices D_A and D_B will commute with each other.

$$D_A D_B = D_B D_A$$

$$\Rightarrow \quad P^{-1}APP^{-1}BP = P^{-1}BPP^{-1}AP$$

$$\Rightarrow \quad P^{-1}ABP = P^{-1}BAP \Rightarrow AB = BA$$

Correct option is (c)

1.2. For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $Adj \cdot A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Correct option is (c)

1.3. A complex function $f(z) = u(x, y) + iv(x, y)$ will be analytic in nature if its real part and imaginary part satisfies Cauchy-Reamann equation i.e.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Correct option is (b)

1.4. Homogeneity of time means time is not explicitly present in Lagrangian. Therefore energy is conserved.
Hence correct answer is (c)

1.5. Hamilton's equations (canonical equations) are

$$\frac{dq_i}{dt} = +\frac{\partial H}{\partial p_i},\ \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$$

Hence correct answer is (d)

CAREER ENDEAVOUR

1.6. Rydberg constant for an atom with infinitely large nucleus is given by

$$R_\infty = \frac{me^4}{8\varepsilon_0^2 ch^3} \qquad \text{... (i)}$$

where m is mass of electron.
and Rydberg constant for an atom with nucleus of finite mass M is given by

$$R_M = \frac{\mu e^4}{8\varepsilon_0^2 ch^3} \qquad \text{... (ii)}$$

where, $\mu = \frac{mM}{(m+M)}$, is reduced mass

$$\therefore \quad R_M = \left(\frac{mM}{m+M}\right)\cdot\frac{e^4}{(8\varepsilon_0^2 ch^3)} = \left(\frac{M}{m+M}\right)\cdot\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right)$$

$$\Rightarrow \qquad R_M = \frac{1}{\left(1+\frac{m}{M}\right)} R_\infty \qquad \text{... (iii)}$$

According to given question $R_\infty = R_1$ and $R_M = R_2$ and $\frac{m}{M} = \frac{1}{(7500)}$

Thus, equation (iii), give

$$\Rightarrow \qquad R_2 = \frac{1}{\left(1+\frac{1}{7500}\right)} R_1 \Rightarrow \frac{R_2}{R_1} = \frac{7500}{7501} \quad \text{i.e.,} \quad \frac{R_2}{R_1} < 1$$

Hence, correct option is (a).

1.7. We can write from Ampere law,

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta} \qquad r > R$$

$$= \frac{\mu_0 I r}{2\pi R^2}\hat{\theta} \qquad r < R$$

$\therefore$ $\vec{\nabla} \times \vec{B} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\mu_0 I}{2\pi}\right) = 0 \qquad r > R$ Here, $\vec{\nabla}$ is in cylindrical coordinates

$$\vec{\nabla} \times \vec{B} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\mu_0 I r^2}{2\pi R}\right)\hat{z}$$

$$= \frac{\mu_0 I}{\pi R^2}\hat{z} \neq 0 \qquad r < R$$

Correct option is (c)

1.8. (i) The electric field at p depends on both charge
(ii) The electric flux through the closed surface *S*.

$$\oint E \cdot dS = \frac{q_1}{\varepsilon_0}$$

Therefore, the electric flux crossing the closed surface *S* is independent of q_2.
Correct option is (b)

1.9. Probability density corresponding to the given wave function of the particle is

$$P(x) = |\psi(x)|^2 = A^2 \sin^2\left(\frac{2\pi x}{L}\right)$$

Therefore, Probability density will have a maximum value when $\sin^2\left(\frac{2\pi x}{L}\right) = 1$ i.e. at $x = \frac{L}{4}$ and $\frac{3L}{4}$

Correct option is (d)

1.10. In a Stern-Gerlach experiment, a particle comes out in $|+z;\uparrow\rangle$ state i.e. spin up sate and it can mathematically expressed as

$$|+z;\uparrow\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

We know that, $\left|\frac{1}{2},\frac{1}{2}\right\rangle$ is a simultaneous Eigen state of S^2 and S_z corresponding to eigenvalues $s(s+1)\hbar^2$ and $m_s\hbar$ respectively. Thus, the particle will have definite values of S^2 and S_z.

Correct option is (b)

1.11. According to Partial wave analysis, for a spherically symmetric potential the scattering amplitude and total cross section can be written as

$$f(\theta)=\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l\ P_l(\cos\theta)$$

$$\sigma=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$$

where δ_l is the phase shift of the individual partial waves due to scattering. Now for $\theta=0$, the scattering amplitude will be

$$f(\theta=0)=\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l\ =\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\left[\cos\delta_l\sin\delta_l+i\sin^2\delta_l\right]$$

Therefore, $\sigma=\frac{4\pi}{k}\times$ Imaginary part of $\left|f(\theta=0)\right|$

Correct option is (c)

1.12. For one particle, $V=dx\,dp_x\cdot dy\,dp_y\cdot dz\,dp_z=h^3$

For N particles, $V=(h^3)^N=h^{3N}$

Correct option is (a).

1.13. For Bose-Einstein condensation to take place, the temperature of the Boson gas should be less than T_B,

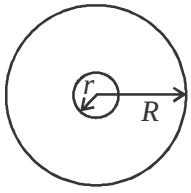
where T_B is given by $T_B=\frac{h^2}{2\pi mk_B}\left\{\frac{N}{V\xi(3/2)}\right\}^{3/2}$

Correct option is (a).

1.14. We have, $I=I_0\sin(\omega t)$

The magnetic field at the centre of large circular surface is

$$B=\frac{\mu_0 IN}{2R}=\frac{\mu_0 NI_0}{2R}\sin\omega t$$



Therefore, the magnetic flux passing through the smaller circular surface.

$$\varphi=\frac{\mu_0 NI_0}{2R}\pi r^2\sin\omega t$$

$$\therefore\qquad emf=-\frac{d\varphi}{dt}=-\frac{\mu_0 NI_0}{2R}\pi r^2\omega\cos\omega t$$

$$emf=-\frac{\mu_0\pi NIr^2}{2R}\cos\omega t$$

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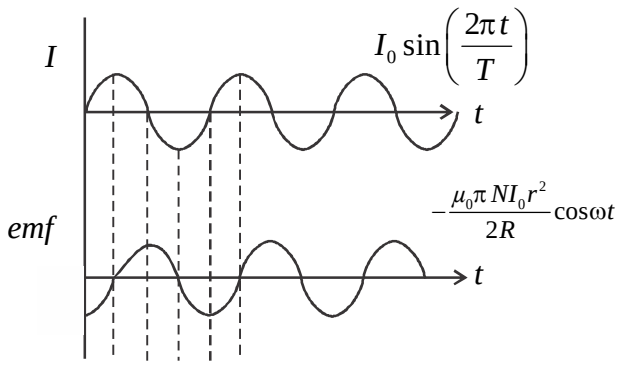


Therefore, induce emf in the small coil lags behind the current in the large coil by $\frac{\pi}{2}$

Correct option is (d)

1.15. The electric dipole moment of the given configuratino with respect to origin,

$$\vec{P} = \sum_i q_i \vec{r}_i = q \times 0 + qa\hat{i} + qa\hat{j} - 2aq\left(\hat{i} + \hat{j}\right) = -aq\hat{i} - aq\hat{j}$$

Correct option is (c)

1.16. *Micro-canonical ensemble* : It is an ensemble in which all element have same macrostate represented by same number of particles, same volume and same energy.

Canonical ensemble : It is an ensemble in which all element have same macrostate represented by same number of particles, same volume and same temperature.

Correct option is (a).

1.17. For one dimensional Kronig Penney model, total number of posible states or wave function is equal to the number of unit cells.

Correct option is (b)

1.18. Divalent solids are Alkaline earth metals. They behave like a conductor

Correct option is (b)

1.19. In $\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R}$, the phonon $\vec{q}_3$ destroys momentum $\hbar\vec{R}$. So conservation of momentum does not hold. Therefore, a finite value of thermal resistance comes out.

Hence, correct option is (a).

1.20. Since, proton belong to Baryon family so its Baryon number is one.

Since, proton do not belong to lepton family. Therefore, Lepton number of proton is zero.

Since, electron is a lepton, its lepton number is equal to one and its Baryon number is zero

Therefore, correct results are one, zero, zero and one.

Correct option is (c)

1.21. 200 MeV and 10 MeV

Correct option is (d)

1.22. Nuclear force have the following properties

(1) These forces are short range forces
(2) These forces are change independent
(3) These forces are strongest known forces
(4) these forces are spin dependent
(5) These forces are velocity dependent

Correct option is (b)

CAREER ENDEAVOUR

1.23. Since, ${}_6C^{14} \rightarrow Z = 6, N = 8, \therefore$ Nuclear spin = 0

(${}_6C^{14}$ have even number of protons and neutrons)

And ${}_{12}Mg^{25} \rightarrow Z = 12$ (even number), N = 13 (odd number)

$13N \rightarrow (^1s_{1/2})^2 (^1p_{3/2})^4 (^1p_{1/2})^2 (^1d_{5/2})^5 \Rightarrow J = \frac{5}{2}$

Therefore, spin = half integer

Correct option is (a)

1.24. The $E_{asymmetry}$ term originates from the asymmetry between the number of protons and the number of neutrons in the nucleus. Nuclear data for stable nuclei indicate that for lighter nuclei, the number of protons is almost equal to that of neutrons, i.e. N = Z. As mass number 'A' increases, the symmetry of proton and neutron number is lost and as the number of neutrons exceeds that of protons to maintain the nuclear stability. This excess of neutrons over protons i.e. (N – Z) is the measure of the asymmetry and it decreases the stability or the binding energy of medium or heavy nuclei.

$E_{asym.} \propto (N - Z)$

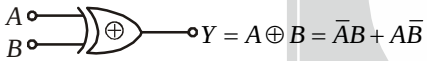
$E_{asym.} \propto (N - Z)/A$

$$\therefore E_{asym.} = a_{asym.} \frac{(N-Z)^2}{A} = a_{asym.} \frac{(A-2Z)^2}{A}$$

arises due to unequal number of protons and neutrons

Correct option is (c)

1.25. XOR ≡ Exclusive OR gate



Truth Table

A	B	$\bar{A}$	$\bar{A}B$	$\bar{B}$	$A\bar{B}$	Y
0	0	1	0	1	0	0
0	1	1	1	0	0	1
1	0	0	0	1	1	1
1	1	0	0	0	0	0

⊕ ≡ XOR operator .

2.1. $\left(a\hat{i} + b\hat{j}\right).\left(-b\hat{i} + a\hat{j}\right) = -ab + ab = 0$

[$\because$ two vectors are said to be orthogonal if their dot product is zero]

Correct option is (a)

2.2. For the existance of fourier transform of a function $f(x)$, $f(x)$ should be either finite or zero at $x = \pm\infty$. But $f(x) = e^{x^2}$ is infinite at $x = \pm\infty$, therefore, Fourier transform of e^{x^2} will not exist.

Correct option is (c)

2.3. Unit vector normal to the surface $\phi : 3x^2 + 4y - z = 0$ at the ponit (1, 1, 7) is

$$\hat{n} = \frac{\vec{\nabla}\phi}{\left|\vec{\nabla}\phi\right|} = \frac{\left(6x\,\hat{i} + 4\hat{j} - \hat{k}\right)}{\sqrt{36x^2 + 16 + 1}}\Bigg|_{(1,1,7)} = \frac{\left(6\,\hat{i} + 4\hat{j} - \hat{k}\right)}{\sqrt{53}}$$

Correct option is (c)

2.4. $y = Ax + Be^{-x} \Rightarrow \dfrac{dy}{dx} = A - Be^{-x} \quad \Rightarrow \dfrac{d^2y}{dx^2} = Be^{-x}$

$$(1+x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y$$

$$= (1+x)\left(Be^{-x}\right) + x\left(A - Be^{-x}\right) - \left(Ax + Be^{-x}\right)$$

$$= Be^{-x} + xBe^{-x} + Ax - xBe^{-x} - Ax - Be^{-x} = 0$$

Correct option is (b)

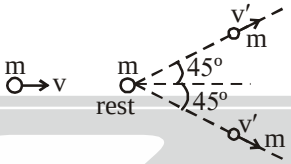
2.5. $\theta_c = 90^o$

angle of scattering in lab frame

$$\tan\theta_1 = \frac{\sin\theta_c}{1+\cos\theta_c} = \frac{\sin 90^0}{1+\cos 90^o} = 1$$

$$\theta_1 = 45^o$$

$$\tan\theta_2 = \frac{\sin\theta_c}{1-\cos\theta_c} = 1$$



$$\theta_2 = 45^o$$

Using conservation of momentum we can conclude that the two masses move with equal speed. Using conservation of kinetic energy

$$v^2 = v'^2 + v'^2 \quad \Rightarrow \quad v' = \frac{v}{\sqrt{2}}$$

Hence correct answer is (a)

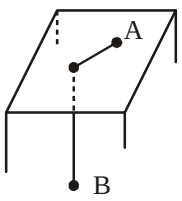
2.6. $\vec{F} = \hat{i}(y+z)$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ y+z & 0 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-1) + \hat{k}(0-1) = -\hat{j} - \hat{k} \neq 0$$

Force is not conservative so we cannot define potential.

Hence correct answer is (d)

2.7.



Particle A moves on a surface therefore it has 2 degree of freedom. Particle B moves in space so it has three degree of freedm. However between the particles there is a constraint that length of string is constant. Therefore total degree of freedom for A + B is 2 + 3 –1 = 4.

Hence correct answer is (b)

CAREER ENDEAVOUR

2.8. Kinetic energy = qV

$$mc^2 - m_0c^2 = qV$$

$$\frac{m-m_0}{m_0} = \frac{qV}{m_0c^2} = \frac{10.2\times10^6 eV}{0.51\times10^6 eV}$$

$$\% \text{ increase in mass} = \frac{m-m_0}{m_0}\times100$$

$$= \frac{10.2}{0.51}\times100 = 2000\%$$

Hence correct answer is (b)

2.9. According Maxwell equation, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$\Rightarrow \iint\left(\vec{\nabla}\times\vec{E}\right)\cdot d\vec{s} = -\frac{\partial}{\partial t}\iint\vec{B}\cdot d\vec{s}$$

$$\Rightarrow \oint\vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\mu_0 I_0 \sin\omega t\, n\pi r^2 \text{ for } r < a$$

$$\Rightarrow \vec{E} = -\frac{n\mu_0 I_0\,\omega\cos\omega t\,\pi r^2}{2\pi r}\hat{\theta}$$

$$= -\frac{n\mu_0 I_0\,\omega\cos\omega t}{2} r\,\hat{\theta}$$

And for $r > a$

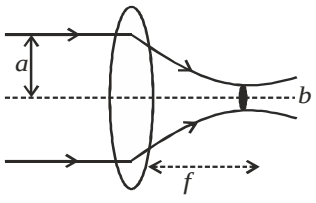
$$\vec{E} = -\frac{n\mu_0 I_0\omega\cos\omega t\,\pi a^2}{2\pi r}\hat{\theta}$$

$$= -\frac{n\mu_0 I_0\omega a^2\cos\omega t}{2r}\hat{\theta}$$

Correct option is (c)

2.10. The radius of the focal spot

$$b = \frac{\lambda f}{a}$$



We have $\lambda = 600\times10^{-9} m$

$f = 10\,cm = 10^{-1} m$

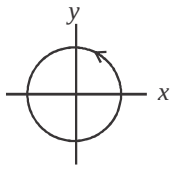
$a = 10\times10^{-3} m$

$$\therefore\ b = \frac{600\times10^{-9}\times10^{-1}}{10\times10^{-3}} = 6\,\mu m$$

Correct option is (b)

CAREER ENDEAVOUR

2.11. Two components of left circular light as



$$E_x = E_0 \cos(\omega t - kz)$$

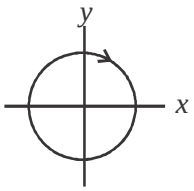
$$E_y = E_0 \sin(\omega t - kz)$$

Let after travelling 'd' distance the left circular polarized light become right circular polarized.

$\therefore$ $E_x = E_0 \cos(\omega t - kz)$

$$E_y = -E_0 \sin(\omega t - kz)$$

Therefore, the phase introduce between two component



$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\, d = \pi$$

$$\Rightarrow \quad d = \frac{\lambda}{2(n_e - n_0)} = \frac{600}{2 \times 0.01} = 30\ \mu m$$

Correct option is (c)

2.12. $$\frac{I_{\max}}{I_{\min}} = \frac{25I_0}{9I_0} = \frac{25}{9}$$

$$\left(\frac{a+b}{a-b}\right)^2 = \frac{25}{9} \Rightarrow \frac{a+b}{a-b} = \pm\frac{5}{3}$$

Taking positive value

$$3a + 3b = 5a - 5b \Rightarrow 2a = 8b \Rightarrow \frac{a}{b} = \frac{4}{1}$$

$\therefore$ $$\frac{I_1}{I_2} = \left(\frac{a}{b}\right)^2 = \frac{16I_0}{I_0}$$

Correct option is (a)

2.13. Uncertainty in the y component of position of the electron is $\Delta y = 1\, nm$. According to Heisenberg's uncertainty principle, the uncertainty in the y-component of velocity of the electron after passing through the slit is

$$\Delta v_y \approx \frac{\hbar}{m\,\Delta y} = \frac{\left(1.06 \times 10^{-34}\right)}{\left(9.1 \times 10^{-31}\right) \times \left(10^{-9}\right)} m/\sec = 1.164 \times 10^5\, m/\sec$$

Correct option is (b).

2.14. $$\left[\left[\hat{A}, \hat{B}\right], \left[\hat{B}, \hat{A}\right]\right] = -\left[\left[\hat{A}, \hat{B}\right], \left[\hat{A}, \hat{B}\right]\right] = 0 \quad (\text{since, } \left[\hat{B}, \hat{A}\right] = -\left[\hat{A}, \hat{B}\right])$$

Correct option is (c)

2.15. Given state of the electron:

$$\phi_s = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} = \frac{\sqrt{3}}{2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\sqrt{3}}{2}\left|+\hat{z}, \uparrow\right\rangle + \frac{1}{2}\left|-\hat{z}, \downarrow\right\rangle = C_\uparrow \left|+\hat{z}, \uparrow\right\rangle + C_\downarrow \left|-\hat{z}, \downarrow\right\rangle$$

Probability of finding the z-component of spin along direction $-\hat{z} = \left|C_\downarrow\right|^2 = \frac{1}{4} = 0.25$.

Correct option is (d)

CAREER ENDEAVOUR

2.16. For Balmer series, the first line is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5R}{36}$$

$$\Rightarrow \quad R = \frac{36}{5\lambda} \qquad \text{... (i)}$$

Now, the first line of the Lyman series is given by

$$\frac{1}{\lambda_{\text{Lyman}}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4} = \frac{3}{4}\left[\frac{36}{5\lambda}\right] = \frac{27}{5\lambda}$$

$$\Rightarrow \quad \frac{1}{\lambda_{\text{Lyman}}} = \frac{5\lambda}{27}.$$

Hence, correct option is (b).

2.17. $\vec{A}$ will solenoidal if $\vec{\nabla} \cdot \vec{A} = 0 \Rightarrow 5 + 3 - a = 0 \Rightarrow a = 8$

Correct option is (c)

2.18. The mean free path is $\lambda = \dfrac{k_B T}{\sqrt{2}\pi d^2 P}$, where d is the diameter of the molecule.

Given : when $P = p_0,\ T = T_0,\ \lambda = \lambda_0$

$$\therefore \quad \lambda_0 = \frac{k_B T_0}{\sqrt{2}\pi d^2 p_0} \qquad \text{... (i)}$$

Now, if $P = 1.5\, p_0$ and $T = 0.75 T_0$, we have

$$\lambda_{\text{new}} = \frac{k_B (0.75) T_0}{\sqrt{2}\pi d^2 (1.5\, p_0)} \qquad \text{... (ii)}$$

Dividing (ii) by (i), $\dfrac{\lambda_{\text{new}}}{\lambda_0} = \dfrac{0.75}{1.50} = \dfrac{1}{2} \Rightarrow \lambda_{\text{new}} = \dfrac{\lambda_0}{2}$

Correct option is (b).

2.19. The quantum statistics applies where the interparticle distance between the particles is comparable to the de-Broglie wavelength of them.

If n is the number density of the particle then,

$$n = \frac{N}{V} = \frac{N}{\frac{4}{3}\pi r^3}$$, where N is the total number of particles

$\Rightarrow \quad r \cong n^{-1/3}$, where r is the interparticle distance.

Also, the de-Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mk_B T}}$$

So for quantum statistics, $r \simeq \lambda \Rightarrow n^{1/3}\lambda \simeq 1$

CAREER ENDEAVOUR

$$\Rightarrow \quad \frac{hn^{1/3}}{(2mk_BT)^{1/2}} \simeq 1 \Rightarrow \frac{n^{1/3}}{T^{1/2}} \simeq \frac{(2mk_B)^{1/2}}{h} \Rightarrow \frac{n}{T^{3/2}} \simeq \frac{(2mk_B)^{3/2}}{h^3} \simeq 10^{22} \Rightarrow \frac{n}{T^{3/2}} \gg 1$$

Correct option is (c).

2.20. For a perfect free electron gas in a metal, the angular frequency and wavenumber are related by the following relation: $\omega = \frac{\hbar k^2}{2m}$

So, Phase velocity $v_p = \frac{\omega}{k} = \frac{\hbar k}{2m}$ and Group velocity $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m}$.

Therefore, relation between phase velocity and group velocity is $v_p = \frac{1}{2} v_g$.

Correct option is (b)

2.21. The wavelength of plasma oscillation.

$$\lambda_p = \frac{2\pi \times c}{\omega_p} = \frac{2\pi \times c}{18\pi\sqrt{n_0}} = \frac{10^8}{3\sqrt{n_0}} \qquad \left(n_0 = 10^{28}/m^3\right)$$

$$= 0.33\mu m$$

Correct option is (b)

2.22. For cubic system, the relation between interatomic distance (d) and cell-edge (a) is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \text{where, } (hkl) = \text{miller indices} \qquad \text{... (i)}$$

From Bragg's law : $2d \sin\theta = n\lambda$ ... (ii)

$$\therefore \quad \sin\theta = \frac{n\lambda}{2d}$$

For minimum θ, $n = 1$, d must be maximum.

For maximum d, $\sqrt{(h^2 + k^2 + l^2)}$ will be minimum. According to given options only "(a) (100)" shows minimum $\sqrt{(h^2 + k^2 + l^2)}$, which gives smallest θ.

Hence, correct option is (a).

2.23. Unit-cell-edge $(a) = 0.542$ nm

The interatomic separation $(d) = ?$

Since diamond has fcc structure : $4r = a\sqrt{2}$

$$2r = \frac{a\sqrt{2}}{2} = \frac{a \times 1.414}{2} = 0.707 \times 0.542$$

$$\Rightarrow \quad d = 2r = 0.383 \text{ nm}$$

Hence, correct option is (c).

CAREER ENDEAVOUR

2.24. ${}_4Be^9$ has Z = 4, N = 9 – 4 = 5

$5N \rightarrow ({}^1s_{1/2})^2 ({}^1p_{3/2})^3$

Therefore, $J = \frac{3}{2}$, l = 1 (for p) and parity = $(-1)^l = (-1)^{+1} = -1$ (odd)

Therefore, spin and parity = $\frac{3}{2}$ and odd.

Correct option is (a)

2.25. Closed loop gain for negative feed-back $A_{CL} = \frac{A_{OL}}{1 + A_{OL}\beta}$

where, A_{OL} (open loop gain) , A_{CL} (closed loop gain), $\beta \rightarrow$ feedback ratio

$$100 = \frac{250}{1 + 250 \cdot \beta} \quad \Rightarrow \quad 1 + 250 \times \beta = 2.5$$

$$250 \times \beta = 1.5 \quad \Rightarrow \quad \beta = 6 \times 10^{-3} = 0.006$$

Correct option is (d)

DESCRIPTION

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$

Let, $y = C \cdot e^{mx}$ be the trial solution

$$\Rightarrow \quad m^2 + 2m + 5 = 0 \Rightarrow m = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm 2i$$

So, $y = A\,e^{(-1+2i)x} + B\,e^{(-1-2i)x}$

Given : $x = 0, y = 0 \quad \Rightarrow \quad A + B = 0$

$$x = 0, \frac{dy}{dx} = 1 \quad \Rightarrow \quad A(-1+2i) - (1+2i)B = 1 \Rightarrow 2i(A - B) = 1 \Rightarrow A - B = -\frac{i}{2}$$

So, $A = -\frac{i}{4}$, $B = +\frac{i}{4}$

Thus, $y(x) = \frac{i}{4}\left[-e^{(-1+2i)x} + e^{(-1-2i)x}\right] = -\frac{i}{4}\left[e^{-x}\left(e^{2ix} - e^{-2ix}\right)\right] = -\frac{i}{4}\left[e^{-x}\left(2i\sin 2x\right)\right] = \frac{1}{2}e^{-x} \cdot \sin 2x$

4. $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

The eigen value equation,

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0, \ \lambda = \pm 1$$

Let, the eigenvector corresponding, $\lambda = 1$ is X_1

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$\therefore \quad AX_1 = +X_1$

$\Rightarrow \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\Rightarrow \quad x_2 = +x_1$

Let $x_1 = k \Rightarrow x_2 = +k$

$\therefore \quad X_1 = \begin{pmatrix} k \\ k \end{pmatrix}$

Applying normalization condition i.e. $X_1^{\dagger}X_1 = 1 \Rightarrow k = \dfrac{1}{\sqrt{2}}$

So, $\quad X_1 = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Similarly, the eigenvector corresponding to the eigen value $\lambda = -1$ is $X_2 = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Diagonalizing matrix,

$\therefore \quad P = \begin{pmatrix} X_1 & X_2 \end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

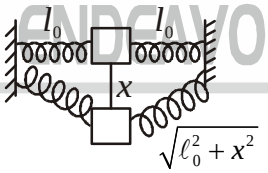
5. $I = \displaystyle\int_0^{\infty} \frac{dx}{(1+x)^4}$

Lets $t = (1+x)$

$\therefore \quad dt = dx$

$\therefore \quad I = \displaystyle\int_1^{\infty} \frac{dt}{t^4} = \left[\frac{t^{-4+1}}{-4+1}\right]_1^{\infty} = -\frac{1}{3}[0-1] = \frac{1}{3} = 0.33$

6. Kinetic energy $T = \dfrac{1}{2}m\dot{x}^2$



potential energy $V = \dfrac{1}{2}k\left(\sqrt{\ell_0^2 + x^2} - \ell_0\right)^2$

Gravitational potential energy term is not introduced because motion is taking place in horizontal plane.

Lagrangian $L = T - V = \dfrac{1}{2}m\dot{x}^2 - \dfrac{1}{2}k\left(\sqrt{\ell_0^2 + x^2} - \ell_0\right)^2$

Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$m\ddot{x} + 2k\left(\sqrt{\ell_0^2 + x^2} - \ell_0\right)\frac{x}{\sqrt{\ell_0^2 + x^2}} = 0$$

$$m\ddot{x} + 2kx\left(1 - \frac{\ell_0}{\sqrt{\ell_0^2 + x^2}}\right) = 0$$

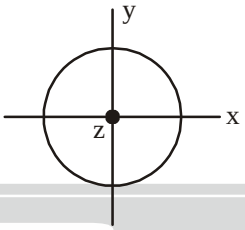
for $x << \ell_0$ $\quad m\ddot{x} + 2kx\left(1 - \left(1 + \frac{x^2}{\ell_0^2}\right)^{-1/2}\right) = 0$

$$m\ddot{x} + 2kx\left(1 - 1 + \frac{x^2}{2\ell_0^2}\right) = 0 \quad \Rightarrow \quad m\ddot{x} + \frac{kx^3}{\ell_0^2} = 0 \quad \therefore \ \ddot{x} \propto -x^3$$

Motion is not simple harmonic.

7. x, y, z axes are symmetric therefore these are Principal axes

Thus $I_{xy} = I_{yx} = I_{yz} = I_{zy} = I_{xz} = I_{zx} = 0$



$$I_{xx} = I_{diameter} = \frac{MR^2}{4} = I_{yy}$$

$$I_{zz} = I_{xx} + I_{yy} = \frac{MR^2}{2}$$

$$\text{inertia tensor II} = \frac{MR^2}{4}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Here I_{xx}, I_{yy}, I_{zz} are principal moment of inertia.

8. $\Delta t = 10^{-7}$ sec $\quad \Delta x = 50$ *meter*

In observer's frame $\Delta t' = 0$

$$\Delta t' = \left(\Delta t - \frac{\Delta x v}{c^2}\right)\gamma; \quad 0 = \left(10^{-7} - \frac{50.v}{c^2}\right)\gamma; \quad v = \frac{c^2 \times 10^{-8}}{5} = \frac{3}{5}c$$

Spatial separation between these events in observer's frame

$$\Delta x' = (\Delta x - v\Delta t)\gamma = \frac{(\Delta x - v\Delta t)}{\sqrt{1 - v^2/c^2}} = \frac{50 - \frac{3}{5} \times 3 \times 10^8 \times 10^{-7}}{\sqrt{1 - \left(\frac{3}{5}\right)^2}}$$

$$= \frac{50 - 18}{4/5} = \frac{32 \times 5}{4} = 40 \text{ meters}$$

9. We know that displacement current,

$$I_d = \left(\frac{A\varepsilon_0}{d}\right)\frac{\partial V}{\partial t}$$

$$= \left(\frac{A\varepsilon_0}{d}\right)\frac{\partial}{\partial t}\left[360\sin\left(2\pi \times 10^6 t\right)\right] = \left(\frac{A\varepsilon_0}{d}\right) \times 2\pi \times 10^6\left[360\cos\left(2\pi \times 10^6 t\right)\right]$$

$$= \frac{10 \times 10^{-4}}{2 \times 10^{-3}} \times 8.85 \times 10^{-12} \times 2\pi \times 360 \times 10^6 \cos\left(2\pi \times 10^6 t\right) = 0.01\cos\left(2\pi \times 10^6 t\right)$$

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10. We have, $V(r) = \frac{a}{3}\left(4 - \frac{r^2}{R^3}\right) \quad r \le R$

$$= \frac{aR}{r} \quad r > R$$

$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r} = \frac{a}{3}\frac{2r}{R^3}\hat{r} \quad r \le R$$

$$= \frac{aR}{r^2}\hat{r} \quad r > R$$

$$\therefore \quad \vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{2a}{3R^3}r\right) = \frac{2a}{R^3} \quad r \le R$$

$$= \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{aR}{r^2}\right) = 0 \quad r > R$$

According to Poisson's equation,

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0\frac{2a}{R^3}$$

Therefore, total charge, $Q = \int_0^R \rho 4\pi r^2 dr = \frac{2a\varepsilon_0}{R^3}\times\frac{4\pi R^3}{3} = \frac{8\pi a\varepsilon_0}{3}$.

11. We have, $\vec{E} = E_0\left(\frac{\sqrt{3}}{2}\hat{j} - \frac{1}{2}\hat{i}\right)e^{i\left(\omega t - \frac{\sqrt{3}}{2}\alpha x - \frac{1}{2}\alpha y\right)}$

$$\vec{k} = \left(\frac{\sqrt{3}}{2}\alpha\hat{i} + \frac{1}{2}\alpha\hat{j}\right)$$

For plane progressive wave

$$\vec{K}\times\vec{E} = \omega\vec{B}$$

$$\Rightarrow \quad \vec{B} = \frac{1}{\omega}\left(\vec{k}\times\vec{E}\right) = \frac{1}{\omega}\left(\frac{\sqrt{3}}{2}\alpha\hat{i} + \frac{1}{2}\alpha\hat{j}\right)\times\left(\frac{\sqrt{3}}{2}\hat{j} - \frac{1}{2}\hat{i}\right)E_0 e^{i\left(\omega t - \frac{\sqrt{3}}{2}\alpha x - \frac{1}{2}\alpha y\right)}$$

$$= \frac{E_0}{\omega}\left(\frac{3}{4}\alpha\hat{z} + \frac{1}{4}\alpha\hat{z}\right)e^{i\left(\omega t - \frac{\sqrt{3}}{2}\alpha x - \frac{1}{2}\alpha y\right)}$$

$$= \frac{E_0}{\omega}\alpha\hat{z}\, e^{i\left(\omega t - \frac{\sqrt{3}}{2}\alpha x - \frac{1}{2}\alpha y\right)}$$

12. The ground state wave function of infinitely deep potential will is given

$$\psi = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$$

We have, $H' = V_0\cos^2\left(\frac{\pi x}{a}\right)$

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Therefore, the first order energy correction,

$$\therefore \qquad E^{(I)} = \langle \psi | H' | \psi \rangle = \frac{2V_0}{a}\int_0^a \cos^2\left(\frac{\pi x}{a}\right)\sin^2\left(\frac{\pi x}{a}\right)dx = \frac{V_0}{2a}\int_0^a \sin^2\frac{2\pi x}{a}\,dx$$

$$= \frac{V_0}{4a}\int_0^a\left(1-\cos\frac{4\pi x}{a}\right)dx = \frac{V_0}{4a}\left[x - \frac{\sin\frac{4\pi x}{a}}{\frac{4\pi}{a}}\right]_0^a = \frac{V_0}{4a}[a-0] = \frac{V_0}{4}$$

13. $-i\hbar\dfrac{\partial\psi}{\partial x} = p_x\psi$

$$\Rightarrow \qquad \frac{d\psi}{dx} = \frac{ip_x}{\hbar}\psi \;\Rightarrow \psi = e^{\frac{ip_x}{\hbar}x}$$

The wave function is for a free particle and it can carry momentum $p_x = +\hbar k$ or $p_x = -\hbar k$

$$\therefore \qquad |\psi\rangle = A\cdot e^{ikx} + B\cdot e^{-ikx}$$

Applying normalization condition,

$$|A|^2 + |B|^2 = 1 \quad \text{Given: } |A|^2 = 0.9 \Rightarrow A = \frac{3}{\sqrt{10}}$$

$$\Rightarrow \qquad 0.9 + |B|^2 = 1 \Rightarrow B = \frac{1}{\sqrt{10}}$$

So, $$|\psi\rangle = \frac{3}{\sqrt{10}}e^{ikx} + \frac{1}{\sqrt{10}}e^{-ikx}$$

14. We have, $\psi(r) = Ce^{-\left(x^2+y^2+z^2\right)} = \psi_x(x)\psi_y(y)\psi_z(z)$

$$\psi_x(x) = e^{-x^2}$$

$$\phi(p_x) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{-x^2}e^{-ik_x x}dx = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{-x^2+(-ik_x)x}dx = \frac{1}{\sqrt{2\pi\hbar}}e^{\frac{(-ik_x)^2}{4}}\sqrt{\frac{\pi}{1}} = \frac{1}{\sqrt{2\hbar}}e^{-\frac{k_x^2}{4}}$$

Therefore, $\phi(p) = \phi(p_x)\cdot\phi(p_y)\phi(p_z)$

$$\phi(p) = \frac{C}{2\sqrt{2\hbar^3}}e^{-\frac{1}{4}\left(k_x^2+k_y^2+k_z^2\right)} = \frac{C}{2\sqrt{2\hbar^3}}e^{-\frac{1}{4}\left[\frac{p_x^2}{\hbar^2}+\frac{p_y^2}{\hbar^2}+\frac{p_z^2}{\hbar^2}\right]}$$

Therefore, the probability density in the momentum space

$$P(p) = |\psi(p)|^2 = \frac{C^2}{8\hbar^3}e^{-\frac{1}{2\hbar^2}\left(p_x^2+p_y^2+p_z^2\right)}$$

Therefore, according normalization condition,

$$\int_{-\infty}^{\infty}P(p)\,dp = 1 \Rightarrow \frac{|C|^2}{8\hbar^3}\int_{-\infty}^{\infty}e^{-p_x^2/2\hbar^2}dp_x\int_{-\infty}^{\infty}e^{-p_y^2/2\hbar^2}dp_y\int_{-\infty}^{\infty}e^{-p_z^2/2\hbar^2} = 1$$

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$$\Rightarrow \quad \frac{|C|^2}{8\hbar^3}\times\sqrt{\pi 2\hbar^2}\times\sqrt{2\hbar^2\pi}\times\sqrt{2\hbar^2\pi}=1$$

$$\Rightarrow \quad \frac{|C|^2}{8}\times(2\pi)^{3/2}=1$$

$$\Rightarrow \quad |C|=\left[\frac{8}{(2\pi)^{3/2}}\right]^{1/2}$$

15. The quantum-mechanical energy of a rotating diatomic molecule is given by

$$E_J=\frac{h^2}{8\pi^2 I}J(J+1)$$

where, I is the moment of inertia of the molecule about the axis of rotation.
The lowest rotational energy level corresponds to J = 1, and for this level in CO

$$E_{J=1}=\frac{h^2\times 2}{8\pi^2 I}=\frac{h^2}{4\pi^2 I}$$

Substituting the value of h in M.K.S. units and the given value of I, we have

$$E_{J=1}=\frac{\left(6.62\times10^{-34}\,J-\sec\right)^2}{4\times(3.14)^2\times\left(1.46\times10^{-46}\ kg.m^2\right)}=7.61\times10^{-23}\,J$$

The reduced mass of CO:

$$\mu_{CO}=\frac{M_C M_O}{M_C+M_O}$$

$$=\frac{(12\times16)/\left(6.023\times10^{23}\right)^2}{(12+16)/\left(6.023\times10^{23}\right)}$$

$$=\frac{12\times16}{28\times\left(6.023\times10^{23}\right)}=1.14\times10^{-23}\,g$$

$\mu_{co}=1.14\times10^{-23}\,\text{g}=1.14\times10^{-26}\,\text{kg}$

Moment of inertia

$I=\mu r^2$; where r is bond length

$=\left(1.14\times10^{-26}\,\text{kg}\right)\left(0.1132\times10^{-9}\,\text{m}\right)^2$

$=1.46\times10^{-46}\,\text{kg}-\text{m}^2$

The angular frequency of the co molecular

$$\omega=\sqrt{\frac{2E}{I}}=10.2\times10^{11}\,\text{rad/sec}$$

Frequency

$$2\pi\nu=\omega \ \Rightarrow\ \nu=\frac{\omega}{2\pi}=\frac{10.2\times10^{11}}{2\pi}=1.623\times10^{11}\ \text{Hz}$$

CAREER ENDEAVOUR

16. Given : 1 kg water (353 K) + 1 kg water (293 K)

$m_1 = 1$ kg, $m_2 = 1$ kg

Let the final temperature after mixing waters be T_f. Then,

By the principle of calorimetry, $m_1 s_1 (T_f - T_1) = m_2 s_2 (T_2 - T_f)$, where $s_1 = s_2 =$ specific heat of water.

$$\Rightarrow \quad T_f = \frac{T_1 + T_2}{2} = \frac{353 + 293}{2} \qquad [\because m_1 = m_2]$$

$$\Rightarrow \quad T_f = 323 \text{ K}$$

Now change in entropy is

$$\Delta S = (\Delta S)_1 + (\Delta S)_2 = \int \frac{dQ_1}{T} + \int \frac{dQ_2}{T} = m_1 s_1 \int_{T_1}^{T_f} \frac{dT}{T} + m_2 s_2 \int_{T_2}^{T_f} \frac{dT}{T} \qquad [\because dQ = msdT]$$

$$= m_1 s_1 \ell n\left(\frac{T_f}{T_1}\right) + m_2 s_2 \ell n\left(\frac{T_f}{T_1}\right) = 238\left[\ell n\left(\frac{323}{353}\right) + \ell n\left(\frac{323}{293}\right)\right] = 2.06 \text{ J K}^{-1}.$$

17. Electrons are fermions. So they follow Fermi-Dirac statistics. Their distribution is

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \Rightarrow f(E) = \begin{cases} 1 & \text{for} \quad E < E_F \\ 0 & \text{for} \quad E > E_F \end{cases} \text{ when } T \to 0 \text{ K}$$

The density of state is

$$g(p)dp = \frac{4\pi V}{h^3} p^2 dp,$$

where $p = \sqrt{2mE} \Rightarrow dp = \sqrt{\frac{m}{2E}}\, dE$

$$\therefore \quad g(E)dE = \frac{4\pi V}{h^3} \cdot 2mE\sqrt{\frac{m}{2E}}\, dE$$

Since, electron spin degeneracy is 2 so we should multiply the density of state by the factor of two

$$\therefore \qquad g(E) = \frac{8\pi V}{h^3} 2mE\sqrt{\frac{m}{2E}}$$

The total number of particles are $N = \int_0^{E_F} f(E)\, g(E)\, dE = \frac{8\pi V}{h^3}\sqrt{2}m^{3/2} \int_0^{E_F} E^{1/2} dE$

$$\Rightarrow \qquad N = \frac{8\pi V}{h^3}\sqrt{2}m^{3/2}\left(\frac{2E_F^{3/2}}{3}\right) \qquad \text{... (i)}$$

and electrons density, $n = \frac{N}{V} = \frac{8\pi}{h^3}\sqrt{2}m^{3/2}\left(\frac{2}{3}E_F^{3/2}\right)$ ... (ii)

Also, the total energy is

$$\langle E \rangle = \int_0^{\infty} E\, f(E)\, g(E)\, dE$$

CAREER ENDEAVOUR

$$= \int_0^{E_F} E \cdot \frac{8\pi V}{h^3}\sqrt{2}m^{3/2}E^{1/2}dE$$

$$= \frac{8\pi V}{h^3}\sqrt{2}m^{3/2}\left(\frac{2E_F^{5/2}}{5}\right) \quad \text{... (iii)}$$

The average energy per electron

$$= \frac{\langle E\rangle}{N} = \frac{\frac{2}{5}E_F^{5/2}}{\frac{2}{3}E_F^{3/2}} = \frac{3}{5}E_F \quad \text{... (iv)}$$

Now, using equation (ii), $n = \frac{8\pi}{h^3}\sqrt{2}m^{3/2}\frac{2}{3}E_F^{3/2}$

$$\Rightarrow \quad E_F = \left(\frac{3nh^3}{16\sqrt{2}\pi m^{3/2}}\right)^{2/3} = \left(\frac{3nh^3}{16\sqrt{2}\pi}\right)^{2/3}\frac{1}{m}$$

Therefore, average energy per electron $= \frac{\langle E\rangle}{N} = \frac{3}{5}\left(\frac{3nh^3}{16\sqrt{2}\pi}\right)^{2/3}\frac{1}{m}$

18. The Einstein relationship between mobility diffusion constant.

$$\frac{D}{\mu} = \frac{k_BT}{e}$$

$$\Rightarrow \quad D = \frac{k_BT}{e}\times\mu = \frac{k_BT}{e}\times\frac{d}{t}\times\frac{1}{E} = \frac{1.38\times10^{-19}\times300}{1.6\times10^{-19}}\times\frac{1.5\times10^{-2}\times10^{-2}}{300\times10^{-6}\times30} = 4.315\times10^{-4}\ m^2/\text{sec}$$

19. According to momentum conservation,

$$\hbar\vec{k}' = \hbar\vec{q} + \hbar\vec{k}$$

where, $\vec{q}$ is the wave vector of phonon , $\vec{k}$ & $\vec{k}'$ are the wave vector of electron before and after collision, respectively.

$$\Rightarrow \quad \vec{q} = \vec{k}' - \vec{k}$$

$$\Rightarrow \quad q^2 = k'^2 + k^2 - 2kk'\cos\alpha$$

Since, energy of the electron much much larger than the phonon. So, we can take $k' \simeq k$

$$\therefore \quad q^2 = 4k^2\sin^2\frac{\alpha}{2}$$

$$\Rightarrow \quad q = 2k\sin\frac{\alpha}{2} \Rightarrow \alpha = 2\sin^{-1}\left(\frac{q}{2k}\right)$$

20. $$\psi(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\sin\theta e^{-i\phi}$$

Operate parity operator on ψ :

$$\hat{p}\left[\psi(r,\theta,\phi)\right] = \psi(r,\pi-\theta,\pi+\phi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\sin(\pi-\theta)e^{-i(\pi+\phi)}$$

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$$= -\frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\sin\theta\, e^{-i\phi} = -\psi(r,\theta,\phi)$$

So, the parity of the function is odd.

21. The Q value of the reaction is given by

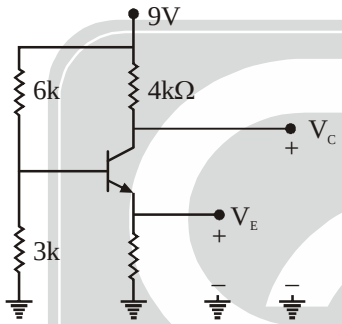
$$Q = M_0 + M_n - (M_{He} + M_C) + E_{\text{excitation}}$$
$$= (-2.215 + 1.79)\,MeV = -0.4251\,MeV$$

$$\therefore \quad E_{th} = -Q\left(1+\frac{M_n}{M_0}\right) = 0.4251\times\left(1+\frac{1.008986}{16}\right) = 0.4519\ MeV$$

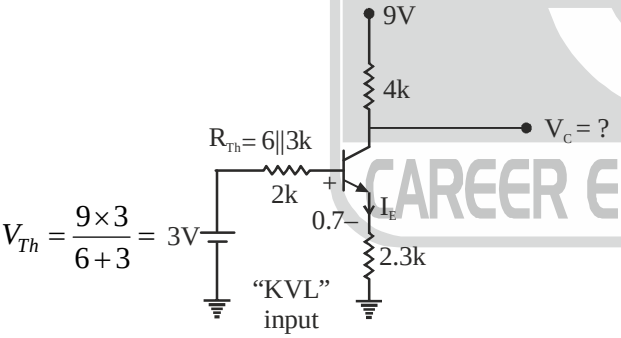
22. To calculate $V_c = ?$

$\beta = 200$; $V_{BE} = 0.7$ Volt

Applying DC - Analysis capacitors are open circuited because f = 0 for DC source so, $X_c = \frac{1}{2\pi fc} = \infty$



Reducing circuit "Thevenin equivalent"



Input KVL, $3 = 2\,I_B + 0.7 + 2.3\times I_E$

$I_E = 201\times I_B$; $I_E = 1\text{mA}$

Output KVL, $9 = 4\times I_C + V_C$

$\because \quad \beta = 200$

$I_E = I_C = 1\,\text{mA}$ $\quad \because \beta$ is very large

So, $\boxed{V_C = 5 \text{ Volt}}$

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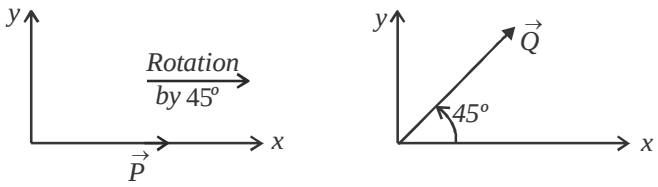
OBJECTIVE QUESTION

1. $\vec{P} = \hat{i} = \cos 0^o \hat{i} + \cos 90^o \hat{j}$

$\vec{Q} = \left(\hat{i} + \hat{j}\right)/\sqrt{2} = \cos 45^o \hat{i} + \sin 45^o \hat{j}$

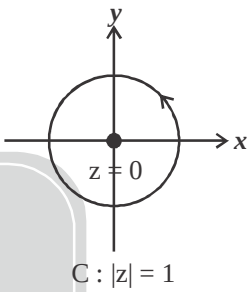


Correct option is (a)

2. Determinant of matrix = product of the eigenvalues = 0

Correct option is (d)

3. $f(z) = \frac{1}{z^2}$ has a pole of order 2 at $z = 0$



Therefore, $\int_C \frac{dz}{z^2} = 2\pi i$ [Residue at $z = 0$]

$$= 2\pi i\left[\frac{1}{(2-1)!}\frac{d}{dz}\left\{z^2 \cdot \frac{1}{z^2}\right\}_{z=0}\right] = 0$$

Correct option is (a)

4. Total force acting on particle

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

$\therefore \quad \vec{F} = qE_0 \hat{j} + qv_0 B_0 \hat{k}$

So, particle will not be confined to any plane.

5. Static friction is given as $f_s \le \mu N \quad \therefore \quad f_s \le \mu\, mg$

Correct option is (a)

6. $L = \frac{1}{2}m\left(v_x^2 + v_y^2\right) + a\left(xv_y - yv_x\right)$

$\therefore \qquad p_x = \frac{\partial L}{\partial v_x} = mv_x - ay$

And $\qquad p_y = \frac{\partial L}{\partial v_y} = mv_y + ax$

Hence correct answer is (c)

7. The distance between two event is

$\Delta r = 6\times10^5\ km = 6\times10^8\ m$

The time difference between two event is

$\Delta t = 1\text{sec}$

$c\,\Delta t = 3\times10^8 \times 1 = 3\times10^8\ m$

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$\because \quad \Delta r > c\Delta t$

Therefore, interval between the events is space like

Hence correct answer is (c)

8. As soon as we put same charge on the sphere it will distribute uniformly over the whole surface. So, field inside the sphere will be zero. As a result potential will be constant.

Correct option is (b)

9. Let σ is the surface charge density on the cross-section area appear due to polarization. So, the depolarization field

$$E_p = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{L}{\sqrt{L^2 + R^2}}\right) \quad L >> R$$

Thus, $E_p = 0$

Correct option is (d)

10. We know that, $\bar{\nabla} \cdot \bar{B} = 0$

$$\Rightarrow \iiint (\vec{\nabla} \cdot \vec{B})\, dV = 0$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{S} = 0$$

So, magnetic flux through a close surface is zero.

This say that monopole magnetic does exist.

Correct option is (b)

11. We know in free space for progressive wave.

$$\omega \bar{B} = \bar{k} \times \bar{E}$$

$$\Rightarrow \vec{B} = +\frac{k}{\omega}(\hat{z} \times \hat{i})\cos(\omega t - kz)$$

$$\Rightarrow \vec{B} = \frac{1}{c}\cos(\omega t - kz)\,\hat{j}$$

Correct option is (a)

12. Phase velocity $v_p = \frac{\omega}{k} = c + \frac{m}{k}$ and Group velocity $v_g = \frac{d\omega}{dk} = c \Rightarrow v_g < v_p \ (\because \ m > 0 \text{ and } k > 0)$

Correct option is (b)

13. Normalized wave function for a particle in a one dimensional infinite potential well of width L centered at $x = 0$, is

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}}\cos\left(\frac{n\pi x}{L}\right) & \text{for } n = 1, 3, 5, ... \\ \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) & \text{for } n = 2, 4, 6, ... \end{cases}$$

Correct option is (c)

14. $\left[x, P^2\right] = 2P\left[x, P\right] = 2i\hbar P$ (Since, $\left[\hat{A}, \hat{B}^n\right] = n\hat{B}^{n-1}\left[\hat{A}, \hat{B}\right]$)

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15. State of the spin half particle is $|\psi\rangle = \left|S_z = \frac{\hbar}{2}; \uparrow\right\rangle = \begin{bmatrix}1\\0\end{bmatrix}$

Therefore, $\langle S_x\rangle = \langle\psi \mid S_x \mid \psi\rangle = [1 \quad 0]\frac{\hbar}{2}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = 0$

$$\langle S_y\rangle = \langle\psi \mid S_y \mid \psi\rangle = [1 \quad 0]\frac{\hbar}{2}\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = 0$$

$$\langle S_x^2\rangle = \langle\psi \mid S_x^2 \mid \psi\rangle = [1 \quad 0]\frac{\hbar^2}{4}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{\hbar^2}{4}$$

$$\langle S_y^2\rangle = \langle\psi \mid S_y^2 \mid \psi\rangle = [1 \quad 0]\frac{\hbar^2}{4}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{\hbar^2}{4}$$

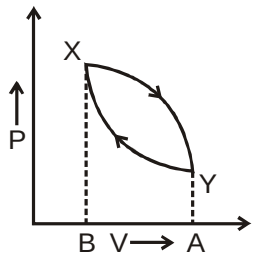
Correct option is (c)

16. For $s = \frac{3}{2}$, means either 3 electron or 3 unpaired subshell. As the shell is 70% filled, therefore it can not have only three electrons. Thus we have 7 d electrons. Thus $J = L + s = 3 + \frac{3}{2} = \frac{9}{2}$

According to the Hund's rule for more than half filled shell, we take highest value of J for ground state.
Hence, correct option is (b).

17. The hyperfine splitting of the spectral lines of an atom is due to the coupling between the electron spin and the nuclear spin.
Hence, correct option is (c).

18. Work done = Area between P-V curve and volume axis.
So, W_{XY} = Area XYABX and W_{YX} = Area YXBAY
Total work done, $W = W_{XY} + W_{YX}$
Since, W_{XY} is positive as volume is increased and W_{YX} is negative as volume is decreased.
So, W = Area XYABX – Area YXBAY
Since Area XYABX > Area YXBAY , W is positive.



So work done by the gas is positive if the direction of the thermal cycle is clockwise.
Correct option is (a).

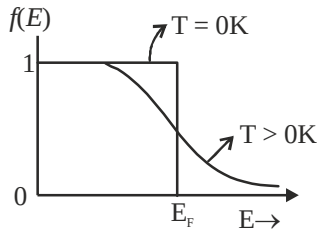
19. The second derivative of Gibbs free energy with respect to temperature and pressure are discontinuous in second order phase transition.

i.e. $\left[\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)_T\right]_P = \left(\frac{\partial V}{\partial T}\right)_P = \alpha V$ and $\left[\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right)_P\right]_T = -\left(\frac{\partial S}{\partial T}\right)_T = -\frac{C_v}{T}$

So, isothermal compressibility (α) and specific heat at constant volume (C_v) are discontinuous in second order phase transition.

Correct option is (b).

20. $$f(E)=\frac{1}{\exp\left[\frac{E-E_F}{kT}\right]+1}$$



So, At T = 300 K and E = E_F

$$f(E_F)=\frac{1}{\exp\left[\frac{E_F-E_F}{kT}\right]+1}=\frac{1}{2}$$

Correct option is (b)

21. Ionic solid MnS will have NaCl type crystal structure.
Hence, correct option is (c).

22. **For optical brands :** $\frac{A}{B}=-\frac{m_1}{m_2}$. This indicates that the two atoms move in opposite directions and their amplitudes are inversely proportional to their masses so that the center of mass of the unit cell remains unchanged.

For acoustical branch : $\frac{A}{B}=+1$. This means that the two atoms of different masses move in the same direction with the same amplitude and there is a movement of their centers of masses as well.
Hence, correct option is (b).

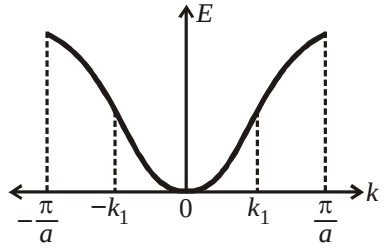
23. The phonon contribution to the electrical resistivity at temperature *T* varies as:

$\rho \propto T$ (at temperatures higher than Debye temperature i.e. $T > \Theta_D$).

and $\rho \propto T^5$ (at temperatures lower than Debye temperature i.e. $T < \Theta_D$).
Hence, correct option is (d).

24. The effective mass (*m**) can be represented as

$$\in(k)=\frac{\hbar^2k^2}{2m} \Rightarrow \left(\frac{d^2\in}{dk^2}\right)=\frac{\hbar^2}{m} \quad m^*=\hbar^2\left[\frac{d^2E}{dk^2}\right]^{-1}$$



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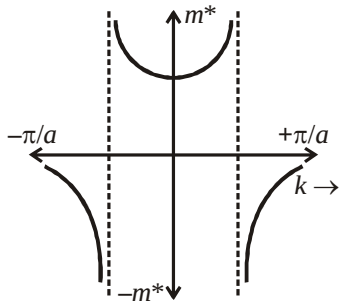

From this figure, it is clear that *m** is negative near the top of the band.
Hence, correct option is (a).

25. Water is a polar molecule or dielectric material. Its molecule has a permanent dipole moment that's why its dielectric constant and refractive index do not validate the expression $n = \varepsilon^{1/2}$.
Hence, correct option is (a).

26. ${}_4Be^9 \rightarrow$ number of protons (Z) = 4 and number of neutrons (N) = 9 – 4 = 5

For one proton, there will be $\rightarrow$ uud (quark structure) and for one neutron, there will be $\rightarrow$ udd (quark structure).
For 4 proton, (4u, 4u, 4d) and for 5 neutrons (5u, 5d, 5d).
Therefore, total up quarks for ${}_4Be^9$ = 13 and total down quarks for ${}_4Be^9$ = 14
Correct option is (b)

27. The basic equations of β^+-disintegrations is
${}^{17}N_7$ has 7 protons and 6 neutrons, ${}^{13}C_6$ has 6 protons and 7 neutrons. So, one proton is changed into neutron.

$$p \rightarrow n + e^+ + \nu_e$$

Out of these options, **correct option is (b)**

28. In the process of neutron decay into proton and electron, the conservation of angular momentum will be violated (according to Fermi theory)
Correct option is (d)

29. At the pn-junction of a semiconductor, when no external bias, due to diffusion there will be no majority charge carries. But negative minority carrier on the p-side and positive minority carrier on the n-side.
Correct option is (c)

30. By putting the value of given option check which is satisfy this relation $R = P\bar{Q}$

(a) $P = 1,\ Q = 1,\ R = 0$
Put the value in the given expression

$$0 = 1.\bar{1}$$
$$1 \equiv 0$$
$$\bar{0} = 1$$

So, satisfying the condition,

(b) $P = 1,\ Q = 1,\ R = 1$

$1 \neq 1.\bar{1}$ **Not satisfying** the condition

(c) $P = 0,\ Q = 0,\ R = 0$

$0 = 0.\bar{0}$ **Satisfying** the condition

(d) $P = 0,\ Q = 1,\ R = 1$

$1 \neq 0.\bar{1}$ **Not satisfying** the condition

So, **(b) and (d) is not satisfying.**

31. $$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix} = \left(\hat{i} + \hat{j} + \hat{k}\right)$$

Correct option is (a)

32. $$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

Let, $x = C.e^{mt}$ be the trial solution.

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So, $\left(m^2+2m+1\right)C.e^{mt}=0 \Rightarrow m=-1,-1$

Therefore, solution will be $x=\left(C_1+C_2t\right)e^{-t}$

At $t=0$, $x=1 \Rightarrow C_1=1 \Rightarrow x=\left(1+C_2t\right)e^{-t}$

Now, $\frac{dx}{dt}=C_2e^{-t}+\left(1+C_2t\right)\left(-e^{-t}\right)$

At $t=0$, $\frac{dx}{dt}=0 \Rightarrow C_2-1=0 \Rightarrow C_2=1$

Therefore, $x=\left(1+t\right)e^{-t}$ and $x\left(t=1\right)=\frac{2}{e}$

Correct option is (b)

33. Number of independent components of a symmetric tensor of rank 2 in 3-D will be $\frac{1}{2}3(3+1)=6$.

Number of independent components of an anti-symmetric tensor of rank 2 in 3-D will be $\frac{1}{2}3(3-1)=3$.

Correct option is (b)

34. $f\left(x\right)=x^4-x^2$

$\Rightarrow \quad f'\left(x\right)=4x^3-2x$

For maximum or minimum of $f\left(x\right)$: $f'\left(x\right)=0$

$\Rightarrow 4x^3-2x=0$

$\Rightarrow x=0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

Now, $f''\left(x\right)=12x^2-2$

$f''\left(x=0\right)=-2<0$. So, at $x=0$ $f\left(x\right)$ has a maximum value

$f''\left(x=\pm\frac{1}{\sqrt{2}}\right)=4>0$. So, at $x=\pm\frac{1}{\sqrt{2}}$ $f\left(x\right)$ has a minimum value

So, $f\left(x\right)$ has three extremum points i.e. statement *R* is correct.

When plot of $f\left(x\right)$ vs *x*, cuts *x*-axis, then $f\left(x\right)=0$

$\Rightarrow \quad x^4-x^2=0$

$\Rightarrow \quad x^2\left(x^2-1\right)=0$

$\Rightarrow \quad x=0, 0, \pm1$ i.e. plot will cut *x*-axis at three points.

For $0<x<1$, $x^4<x^2$ and $f\left(x\right)<0$

So, the corresponding part of the plot lies in the 4th quadrant.

Correct option is (b)

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35. $F\left[\frac{df}{dx}\right] = \int_{-\infty}^{\infty} \frac{df}{dx} e^{ikx} dx$

$= e^{ikx} \cdot f(x)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} ike^{ikx} \cdot f(x) dx$

$= 0 - ik \int_{-\infty}^{\infty} f(x) e^{ikx} dx = -ikF(k)$

Correct option is (c)

36. $V(x) = \frac{1}{2} m\omega^2 \left(3x^2 + 3y^2 + 2z^2 + 2xy\right)$

$$\hat{V} = m\omega^2 \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{T} = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If ω' be frequency of normal modes

$\det\left|\hat{V} - \omega'^2 \hat{T}\right| = 0$

$$\det \begin{vmatrix} 3m\omega^2 - m\omega'^2 & m\omega^2 & 0 \\ m\omega^2 & 3m\omega^2 - m\omega'^2 & 0 \\ 0 & 0 & 2m\omega^2 - m\omega'^2 \end{vmatrix} = 0$$

$\left(2m\omega^2 - m\omega'^2\right)\left[\left(3m\omega^2 - m\omega'^2\right)^2 - \left(m\omega^2\right)^2\right] = 0$

$2\omega^2 - \omega'^2 \Rightarrow \omega' = \sqrt{2}\,\omega$

$\left(3\omega^2 - \omega'^2\right) - \left(\omega^2\right)^2 = 0$

$3\omega^2 - \omega'^2 = \pm\omega^2 \Rightarrow \omega' = \sqrt{2}\,\omega, 2\omega$

$\therefore$ frequencies of normal modes are

$\sqrt{2}\,\omega, \sqrt{2}\,\omega, 2\omega$.

Hence correct answer is (c)

37. Kinetic energy $= m_0 c^2$

$mc^2 - m_0 c^2 = m_0 c^2 \Rightarrow m = 2m_0$

$\frac{m_0}{\sqrt{1 - v^2/c^2}} = 2m_0 \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \quad \therefore v = \frac{\sqrt{3}}{2} c$

38. The cut-off frequency of elecromagnetic wave inside a waveguide is given by

$$\omega = c\sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}$$

For square cross-section, $a = b = W$

Therefore, the minimum ω is $= c\sqrt{\left(\frac{\pi}{W}\right)^2} = \left(\frac{\pi c}{W}\right)$ (take $m = 1,\ n = 0$)

Correct option is (c)

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39. Since, the displacement of the charge particle is always perpendicular to the magnetic force. So, the work done by the magnetic field is zero.

Correct option is (c)

40. $\bar{B} = \frac{\mu_0}{2}\bar{K} \times \hat{n}$

from $z > 0$, $\hat{n} = \hat{z}$ and $\vec{K} = k\hat{y}$

$$B = \frac{\mu_0 k}{2}(\hat{y} \times \hat{z}) = \frac{\mu_0 n}{2}\hat{x}$$

Correct option is (c)

41. For nth order maxima

$d \sin\theta = n\lambda$ (for largest maximum θ = 90°)

$d = n\lambda$

$$n = \frac{d}{\lambda} = \frac{5\times10^{-6}}{1.01\times10^{-6}} = \frac{500}{101} \simeq 4 \quad \text{(since, } n \text{ is an integer)}$$

Correct option is (a)

42. Condition for the first order minima is $a \sin\theta = \lambda$, where a is the slit width and θ is the angle of diffraction for first order minima. Width of the central maximum is given by,

$$W \approx \frac{2\lambda}{a}D = \frac{2\hbar}{ap}D$$

where D is the distance between the slit and screen & p is momentum of the electrons.

Thus the width of the central maxima can be increased by one of the following ways: (i) by increasing D, (ii) by decreasing p, (iii) decreasing a.

Correct option is (c).

43. Given wave function of the particle: $\psi(x, t=0) = C_0\psi_0 + C_1\psi_1$

Wave function of the particle at time 't' will be $\psi(x,t) = C_0\psi_0 e^{-i\frac{E_0}{\hbar}t} + C_1\psi_1 e^{-i\frac{E_1}{\hbar}t}$

Wave function of the particle will repeat itself after time

$$t = \frac{2\pi\hbar}{E_0 - E_1} = \frac{h}{E_0 - E_1} = \frac{\left(6.6\times10^{-34}\right)}{10\times1.6\times10^{-19}} = 0.41\times10^{-15}\,\text{sec}$$

Correct option is (b)

44. $\hat{p}_x\psi(x,t) = -i\hbar\frac{\partial}{\partial x}\left[Ae^{i\omega t}\cos kx\right] = -i\hbar Ae^{i\omega t}(-k\sin kx) \neq \lambda\psi(x,t)$

$$\hat{H}\psi(x,t) = i\hbar\frac{\partial}{\partial t}\left[Ae^{i\omega t}\cos kx\right] = -i\hbar Ae^{i\omega t}(i\omega)\cos kx = (\hbar\omega)\psi(x,t) = \lambda\psi(x,t)$$

Correct option is (b)

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45. Wavelength of the particle in **Region-I:** $\lambda = \dfrac{2\pi}{k_I} = \dfrac{2\pi\hbar}{\sqrt{2mE}}$

Wavelength of the particle in **Region-II:** $2\lambda = \dfrac{2\pi}{k_{II}} = \dfrac{2\pi\hbar}{\sqrt{2m(E-V)}}$

Thus, $\sqrt{\dfrac{E-V}{E}} = \dfrac{1}{2} \Rightarrow 1-\dfrac{V}{E} = \dfrac{1}{4} \Rightarrow \dfrac{V}{E} = \dfrac{3}{4}$

Correct option is (c)

46. Since, the frequencies do not coincide in Raman and vibrational spectra thus by mutual exclusion principle molecule should have centre of symmetry. This could be satisfied by linear molecule e.g. CO_2(O = C = O).
Correct option is (a)

47. Energy, $E = \dfrac{hc}{\lambda} = \left(\dfrac{1}{\lambda}\right)hc = \bar{\nu}hc$

where $\bar{\nu} = \dfrac{1}{\lambda}$; wave number

For P : Given wave number $\bar{\nu} = 10\, m^{-1} = 10\times10^2\, cm^{-1}$

$$E = \bar{\nu}hc = 10\times10^2\, m^{-1}\times 6.63\times10^{-34}\, J\cdot S\times 3\times10^8\, m/s = 19.89\times10^{-23}\, J$$

For Q : $E = 10^{-23}\, J$

For R : Given frequency $\nu = 10^4\, MHz = 10^4\times10^6\, Hz$

Energy $E = h\nu = 6.63\times10^{-34}\, J.S\times10^4\times10^6\, s^{-1} = 6.63\times10^{-24}\, J$

Thus, increasing order of energy is

$$19.89\times10^{-23}\, J > 10^{-23}\, J > 6.63\times10^{-24}\, J$$

$\Rightarrow \quad R < Q < P$

Hence correct option is (b)

48. For X-ray, $E = eV = \dfrac{hc}{\lambda}$

$$\Rightarrow \mathrm{V} = \frac{\mathrm{hc}}{\mathrm{e}\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times0.0825\times10^{-9}} = 150\times10^2\,\mathrm{V} = 15\times10^3\,\mathrm{V} = 15\,\mathrm{kV}.$$

Hence, correct option is (c).

49. The spin-orbit interaction energy is given by

$$\Delta E_{S,L} = \frac{a}{2}\left[\, j(j+1) - \ell(\ell+1) - s(s+1)\right]$$

where $\vec{j} = \vec{\ell} + \vec{s}$ (for one electron atom)
where *a* is coupling constant.

Ground state configuration of sodium spectrum (that emit D line) is ^2S. So $\ell = 0, s = \dfrac{1}{2} \Rightarrow j = \ell + s = \dfrac{1}{2}$

Ground state energy level for $^2S_{1/2}\left(S_J^{(2s+1)}\right)$ is

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$$\Delta E_{So}\left({}^2S_{1/2}\right)=\frac{a}{2}\left[\frac{1}{2}\left(\frac{1}{2}+1\right)-0-\frac{1}{2}\left(\frac{1}{2}+1\right)\right]=0$$

Excited state term is ${}^2P$, i.e. $\ell=1,\ s=\frac{1}{2},\ j=1\pm\frac{1}{2}=\frac{3}{2},\frac{1}{2}$

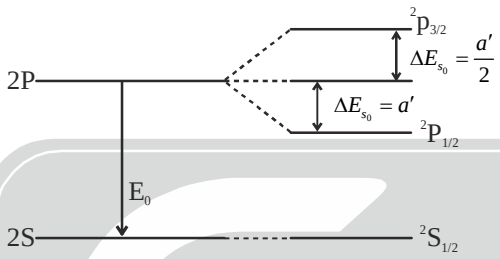
This level will split into two energy level ${}^2P_{1/2}$, ${}^2P_{3/2}$.

For ${}^2P_{1/2}$: Let a' be the split-coupling constant for ${}^2P$ level.

$$\Delta E_{s_0}\left({}^2P_{1/2}\right)=\frac{a'}{2}\left[\frac{1}{2}\left(\frac{1}{2}+1\right)-1(1+1)-\frac{1}{2}\left(\frac{1}{2}+1\right)\right]=-a'$$

$$\Delta E_{s_0}\left({}^2P_{3/2}\right)=\frac{a'}{2}\left[\frac{3}{2}\left(\frac{3}{2}+1\right)-1(1+1)-\frac{1}{2}\left(\frac{1}{2}+1\right)\right]=\frac{a'}{2}$$

Therefore energy level is



For the first lines transition $\left({}^2p_{3/2}\rightarrow{}^2S_{1/2}\right)$

$D_1=16973.4\ cm^{-1}=E_0+\frac{a}{2}$, where E_0 is central energy.

For the second lines transition $\left({}^2p_{1/2}\rightarrow{}^2S_{1/2}\right)$

$D_2=16956.2\ cm^{-1}=E_0-a$

$\Rightarrow\quad \frac{3}{2}a=(16973.4-16956.2)\,cm^{-1}$

$\Rightarrow\quad a=\frac{2}{3}\times17.2=11.4\ cm^{-1}$

Hence correct option is (b)

50. (P) for pure rotational Raman spectrum molecule should have permanent electric dipolemoment but CH_4 does not have permanent dipole. So, it will not show pure rotational Raman spectrum.
(Q) SF_6 also has not permanent dipolemoment. So, it will not show pure rotational spectrum.
(R) IR-absorption occurs from the stretching and bending at the covalent bonds in molecules. Since, N_2 has symmetric bonds and bond streching does not change the dipole moment of the molecule. So, it will not show IR spectrum.
(S) For vibrational Raman and infrared absorption lines observe if the polarisability of the molecule change as molecule vibrates. C_2H_6 follow the above condition.
(T) Since, H_2O_2 has premanent dipole moment so it will shows the pure rotational spectrum
Hence, correct option is (c).

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51. In case of photons, $n(\omega)=\dfrac{1}{e^{\frac{\hbar\omega}{k_BT}}-1}$ and density of states is

$$g(\omega)\,d\omega=\frac{2V}{h^3}4\pi\left(\frac{\hbar\omega}{c}\right)^2\left(\frac{\hbar d\omega}{c}\right)=\frac{V\omega^2 d\omega}{\pi^2c^3}$$

So the average energy in cavity, $\langle E\rangle=\int_0^\infty E\,n(E)\,g(E)\,dE=\int_0^\infty \hbar\omega n(\omega)\,g(\omega)\,d\omega=\int_0^\infty \frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}\cdot\frac{V\omega^2}{\pi^2c^3}d\omega$

Letting $\dfrac{\hbar\omega}{k_BT}=x$, then

$$\langle E\rangle=\frac{(k_BT)^4V}{\pi^2\hbar^3c^3}\int_0^\infty\frac{x^3\,dx}{e^x-1}$$

$\Rightarrow\quad \langle E\rangle\propto T^4$ ... (i)

Also, the number of photons are given by

$$\langle N\rangle=\int_0^\infty n(E)\,g(E)\,dE=\frac{V}{\pi^2c^3}\int_0^\infty\frac{\omega^2 d\omega}{e^{\hbar\omega/k_BT}-1}=\frac{V}{\pi^2c^3}\left(\frac{k_BT}{\hbar}\right)^3\int_0^\infty\frac{x^2}{e^x-1}dx$$

$\Rightarrow\quad \langle N\rangle\propto T^3$ ... (ii)

From equations (i) and (ii), if temperature is doubled, then $\langle E\rangle\to 16\langle E\rangle$ and $\langle N\rangle\to 8\langle N\rangle$

Correct option is (c).

52. For isothermal process, $dT=0$

$\Rightarrow\quad dU=0$ for an ideal gas

From first law of thermodynamics, $\Delta Q=\Delta U+P\Delta V$

$\Rightarrow\quad T\Delta S=P\Delta V=\Delta W$

$\Rightarrow\quad \Delta W=\dfrac{PV}{nR}\Delta S\quad[\because$ for an ideal gas $PV=nRT]$

Correct option is (a).

53. The Maxwell-Boltzmann distribution is given by

$$f(v)=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2e^{-\frac{mv^2}{2k_BT}}$$

For most probable speed, $\dfrac{df(v)}{dv}=0$

$$\Rightarrow\quad 2ve^{-\frac{mv^2}{2k_BT}}-\frac{2mv}{2k_BT}v^2e^{-\frac{mv^2}{2k_BT}}=0$$

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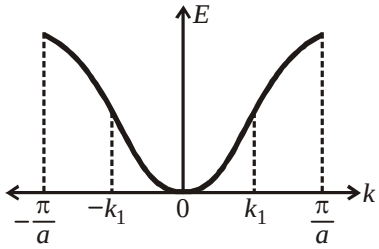
$$\Rightarrow \quad 2v - \frac{mv^3}{k_B T} = 0 \qquad \left[\because e^{-\frac{mv^2}{2k_B T}} \neq 0 \text{ as this means } v \text{ is infinite}\right]$$

$$\Rightarrow \quad v = \sqrt{\frac{2k_B T}{m}} \qquad [\because v \neq 0]$$

Correct option is (c).

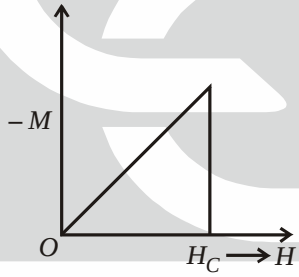
54. *E-k* diagram is represented in the figure. Also it is expressed as $E = \frac{\hbar^2 k^2}{2m}$ ... (i)



It is clear from equation (i) that the plot of *E* versus *k* will be parabolic in the interval $\left(-\frac{\pi}{a}\right) < k < \left(\frac{\pi}{a}\right)$.

Hence, correct option is (c).

55. Below H_C, the sample behaves as a diamagnetic. Its magnetization varies linearly with *H* (slope = –1) and Meissner effect will be followed by the superconductor. The statement "the sample exhibits mixed states of magnetization near H_C" is not true for type-I superconductor. It is true for type-II superconductors.

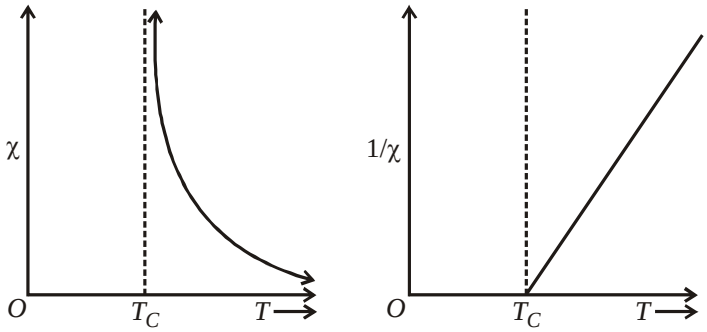


Hence, correct option is (d).

56. The magnetic susceptibility of a ferromagnetic material is given from Curie-Weiss equations

$$\chi = \frac{C}{(T - T_C)} \qquad \text{... (i)}$$

Above T_C, atomic magnets will be randomly oriented and the material will become paramagnetic.



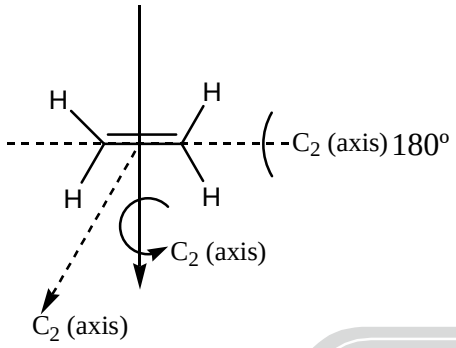
At T = 200 K and T_C = 100 K; $\chi_{200\,K} = \dfrac{C}{(200-100)} = \left(\dfrac{C}{100}\right)$ ... (ii)

At T = 300 K ; $\chi_{300\,K} = \dfrac{C}{(300-100)} = \left(\dfrac{C}{200}\right)$ ... (iii)

From equations (ii) and (iii), we get $\boxed{\chi_{200\,K} = 2\chi_{300\,K}}$.

Hence, correct option is (a).

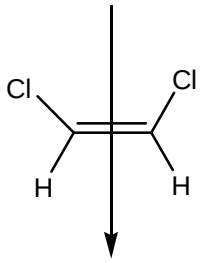
57. (a)



(1) $3C_2$
(2) $2\sigma_v$
(3) $1\sigma_h$

$C_2 + 2C_2 + 2\sigma_v + 1\sigma_h \rightarrow D_{2h}$

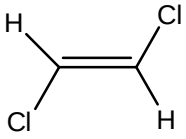
(b)



(1) $1C_2$
(2) $2\sigma_v$

$1C_2 + 2\sigma_v \rightarrow C_{2v}$

(c)



(1) $1C_2$
(2) $1\sigma_h$

$1C_2 + 1\sigma_h \rightarrow C_{2h}$

Correct option is (c)

58. Energy eigenvalues of an electron in a one dimensional potential well of infinitely high walls is

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2} \Rightarrow n = \frac{L}{\pi\hbar}\sqrt{2mE}$$

Energy density of states is $\dfrac{dn}{dE} = \dfrac{L}{\pi\hbar}\sqrt{2m}\dfrac{1}{2\sqrt{E}} = \dfrac{L}{\pi\hbar}\sqrt{\dfrac{m}{2E}}$

Correct option is (a).

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59. **I**n Compton effect experiment, the relation between the wavelength of the scattered photon (λ') and wavelength of the incident photon (λ) are related as

$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\varphi) \quad \text{i.e. } \lambda' > \lambda$$

Energy conservation Law: $h\nu + m_0c^2 = h\nu' + m_0c^2 + E_{k,electron} \Rightarrow h\nu = h\nu' + E_{k,electron}$

Kinetic energy acquired by the electron is $E_{k,electron} = \dfrac{\dfrac{h^2\nu^2}{m_0c^2}(1-\cos\varphi)}{1+\dfrac{h\nu}{m_0c^2}(1-\cos\varphi)}$.

Thus, kinetic energy will be largest if $\varphi = 180^0$ i.e. incident and scattered photons move in the opposite direction.

If the electron acquires a kinetic energy equal to energy of the incident photon, then energy and momentum conservation cannot be satisfied simultaneously.

Correct option is (b)

60. Form of Coulamb potential will be replaced by Yukawa potential

$$V = \frac{e^{-mr}}{4\pi r}(\text{in natural units})$$

Correct option is (b)

61. Since, photons are bosons having integral spins. Therefore, the particle will be a boson.

Correct option is (c)

62. Binding energy is given as

$$\text{BE} = [\text{mass of constituents} - \text{mass of atom}]c^2$$

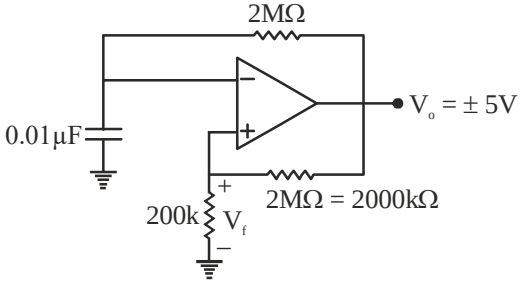
$\therefore$ $B.E.\ of\ {}^{238}_{92}U$ is

$$= \left[92\times1.0078 + (238-92)\times1.0087 - 238.0508\right]c^2$$

$$= 1804.59\ MeV \quad [use\ 1\ amu = 931\ MeV]$$

Correct option is (d)

63. Given circuit is positive feedback "Schmitt trigger"



Output of positive feedback circuit is $V_0 = \pm V_{SAT}$

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$$V_f = \frac{V_0 \times 200}{200 + 2000} \quad \text{Put } V_0 = +5 \text{ (apply voltage division rule)}$$

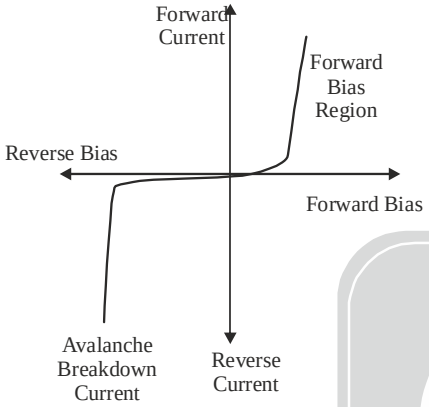
$$V_f = \frac{5}{11}$$

$$V_f = \frac{-5}{11} \qquad \text{Put } V_0 = -5$$

V_f (feed back voltage) $= \pm\frac{5}{11}$ Volt

Correct option is (a)

64.



Correct option is (c)

65. (a) $P\bar{Q} + PQ = P$

$= P\left[\bar{Q} + Q\right] = P$ Correct.

(b) $\overline{PQ} = \bar{P} + \bar{Q}$ Correct by De-morgan's law.

(c) $P\bar{Q} = \left(\overline{\bar{P} + Q}\right)$ By De-morgan's law $\overline{x + y} = \bar{x} \cdot \bar{y}$

$\bar{\bar{P}} \cdot \bar{Q} \equiv P\bar{Q}$ correct $\bar{\bar{x}} = x$ Axioms.

(d) $P\bar{Q} + Q = P$ By Distributive law $x + xy = (x + x)(x + y)$

$(P + Q)(\bar{Q} + Q) = P + Q \neq P$ incorrect

Correct option is (d)

66. $\iint_S \vec{V} \cdot \hat{n} ds = \iiint_v \left(\vec{\nabla} \cdot \vec{V}\right) dv$ (using Divergence theorem)

$$= \iiint_v \vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) dv = 4\pi \iiint_v \delta^3(\vec{r})\, dv$$

$$= 4\pi \iiint_v \delta(x)\delta(y)\delta(z)\, dv = 4\pi$$

Correct option is (d)

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67. $\vec{\nabla}\times\left(\frac{\vec{r}}{r^3}\right)=0$

Therefore, $\vec{V}=\frac{\vec{r}}{r^3}$ is a conservative field.

So, $\oint_C \vec{V}\cdot d\vec{r}=0$ and $\vec{V}=\vec{\nabla}\phi$ and $\int_P^Q \vec{V}\cdot d\vec{r}=$ independent of path

But, magnetic field $\vec{B}$ is not conservative.

Correct option is (d)

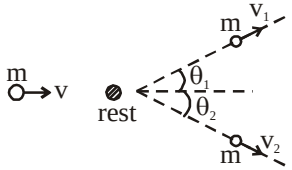
68. Energy line intersects potential at P, Therefore at P speed is zero.

Hence correct answer is (a)

69. At S potential energy is not minimum therefore speed cannot be maximum at this point.

Hence correct answer is (c)

70.



using conservation of momentum we get

$v=v_1\cos v_1+v_2\cos\theta_2$ ... (i)

$0=v_1\sin\theta_1-v_2\sin\theta_2$ ... (ii)

square and add to get

$v^2=v_1^2+v_2^2+2v_1v_2\cos(\theta_1+\theta_2)$ ... (iii)

If collision is elastic

$\frac{1}{2}mv^2=\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2$

$v^2=v_1^2+v_2^2$

from equation (iii) we get

$v_1^2+v_2^2=v_1^2+v_2^2+2v_1\cdot v_2\cos(\theta_1+\theta_2)$

$\Rightarrow \quad \cos(\theta_1+\theta_2)=0$

$\theta_1+\theta_2=90^o$ i.e. two particles move at 90^o to each other.

Hence correct answer is (b)

71. If collision is inelastic

$v^2>v_1^2+v_2^2$

$v^2-v_1^2-v_2^2>0$

from equation (iii) $2v_1v_2\cos(\theta_1+\theta_2)>0$

$\cos(\theta_1+\theta_2)>0$

$\theta_1+\theta_2<90^o$

Hence correct answer is (a)

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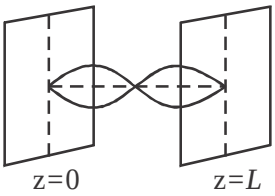
72. We have, $\vec{A}(r,t) = A_0\hat{i}\cos(kz+\alpha)\cos(kct)$, $\varphi(r,t) = 0$

$$\therefore \quad \vec{E} = -\frac{\partial\vec{A}}{\partial t} - \nabla\vec{\varphi} = -\frac{\partial A}{\partial t} + 0 = +A_0 kc\cos(kz+\alpha)\sin(kct)$$

For standing wave the boundary condition is

$$E(0,t) = E(L,t) = 0$$



$$\therefore \quad \cos\alpha = 0; \qquad \alpha = \frac{\pi}{2}$$

And $\cos(kL+\alpha) = 0$

$$(kL+\alpha) = (2m+1)\times\frac{\pi}{2} \qquad (m = 0,1,2,3,....)$$

$$kL + \frac{\pi}{2} = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2} \qquad \Rightarrow k = \left(\frac{2\pi}{L}\right)$$

$$\therefore \quad k = \frac{2\pi}{L} \text{ and } \alpha = \frac{\pi}{2}$$

Correct option is (b)

73. The energy density, $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (Akc)^2 \sin^2 kct\cos(kz+\alpha)$

At $t = 0$ and $z = 0$, u = 0

Correct option is (a)

74. Energy eigenvalues of a particle located in three dimensional cubic well of width L with impenetrable walls, are

$$E_{n_x n_y n_z} = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) \quad \left[\text{where } n_x, n_y, n_z = 1,2,3,.....\right]$$

Sum of third level energy and fourth level energy = 2nd excited state energy + 3rd excited state energy =

$$E_{122} + E_{113} = \frac{9\pi^2\hbar^2}{2mL^2} + \frac{11\pi^2\hbar^2}{2mL^2} = \frac{10\pi^2\hbar^2}{mL^2}$$

Correct option is (a)

75. Degeneracy of the fourth level i.e. third excited state is 3 and corresponding combinations of (n_x, n_y, n_z) are (1,1,3), (1,3,1) and (3,1,1).

Correct option is (c).

76. $$\langle A\rangle = \langle\psi|\hat{A}|\psi\rangle = \left[\frac{3}{5}\langle\psi_1| + \frac{4}{5}\langle\psi_2|\right]\hat{A}\left[\frac{3}{5}|\psi_1\rangle + \frac{4}{5}|\psi_2\rangle\right] = \left[\frac{3}{5}\langle\psi_1| + \frac{4}{5}\langle\psi_2|\right]\left[\frac{3}{5}|\psi_2\rangle + \frac{4}{5}|\psi_1\rangle\right]$$

$$= \frac{12}{25} + \frac{12}{25} = \frac{24}{25} = 0.96$$

Correct option is (d)

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77. $\hat{A}^2|\psi_1\rangle = \hat{A}\hat{A}|\psi_1\rangle = \hat{A}|\psi_2\rangle = |\psi_1\rangle$ i.e. $\hat{A}^2$ is an Eigen function of $|\psi_1\rangle$.

$\hat{A}^2|\psi_2\rangle = \hat{A}\hat{A}|\psi_2\rangle = \hat{A}|\psi_1\rangle = |\psi_2\rangle$ i.e. $\hat{A}^2$ is an Eigen function of $|\psi_2\rangle$.

Correct option is (a).

78. Both transitions are between multiplets. Therefore, both are anomalous Zeeman effect.

Correct option is (b)

79. In anomolous Zeeman effect, each *J* level splits in 2J + 1 levels. Here, J values are 5, 4, $\frac{5}{2}$ and $\frac{3}{2}$. Therefore, 2J + 1 values are equal to 11, 9, 6 and 4.

Correct option is (c)

80. Number of possible micro-states for a collection of N particles having spin quantum numbers

$$\Omega = (2S+1)^N$$

$$= \left(2\times\frac{1}{2}+1\right)^3 = 8 \quad \left[\because s = \frac{1}{2}\right]$$

Correct option is (d).

81. For $S_z = \frac{1}{2}$, only the following combinations of $S_z(1), S_z(2), S_z(3)$ are possible.

$$\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right),\left(\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$$

S_z	$\left[S_z(1), S_z(2), S_z(3)\right]$	P
$\frac{3}{2}$	$\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$	$\frac{1}{8}$
$\frac{1}{2}$	$\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right),\left(\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$	$\frac{3}{8}$
$-\frac{1}{2}$	$\left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right),\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right),\left(-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)$	$\frac{3}{8}$
$-\frac{3}{2}$	$\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)$	$\frac{1}{8}$

$$\langle S_z\rangle = \frac{1}{8}\times\frac{3}{2}+\frac{3}{8}\times\frac{1}{2}+\frac{3}{8}\times\left(-\frac{1}{2}\right)+\frac{1}{8}\times\left(-\frac{3}{2}\right) = 0$$

So the total number of possible microstates are 3.

Correct option is (a).

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82. The following microstates are possible:

$S_z(1)$	$S_z(2)$	$S_z(3)$	S_z
+½	+½	+½	$+\frac{3}{2}$
+½	+½	–½	+½
+½	–½	+½	+½
–½	+½	+½	+½
–½	–½	+½	+½
–½	+½	–½	–½
+½	–½	–½	–½
–½	–½	–½	$-\frac{3}{2}$

Since each state is equally probable, only above configurations are possible. Hence,

ensemble average $= \dfrac{\left(+\frac{3}{2}\right)+3\times\left(+\frac{1}{2}\right)+3\times\left(-\frac{1}{2}\right)+\left(-\frac{3}{2}\right)}{4} = 0.$

Correct option is (b).

83. Given : $\ell n\, Z = N \ell n\left[(V-bN)\,(k_BT)^{3/2}\right]$

The Helmhotz free energy is

$F = -k_BT\ \ell n\ Z$

$= -k_BT\ N\ \ell n\left[(V-bN)\,(k_BT)^{3/2}\right]$

The pressure is given by

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = Nk_BT\,\frac{(k_BT)^{3/2}}{(V-bN)\,(k_BT)^{3/2}}$$

$\Rightarrow \quad P(V-bN) = N\,k_BT$

Correct option is (a).

84. The internal energy is

$$U = -\frac{\partial}{\partial\beta}\ell n\, Z = k_BT^2\,\frac{\partial}{\partial T}\ell n\, Z \qquad \left[\because \beta = \frac{1}{k_BT}\right]$$

$$= k_BT^2\,\frac{\partial}{\partial T}\left[N\,\ell n\left\{(V-bN)\,(k_BT)^{3/2}\right\}\right]$$

$$= N\,k_BT^2\,\frac{3k_B{}^{3/2}T^{1/2}}{2(k_BT)^{3/2}}$$

$$= \frac{3}{2}\,N\,k_BT$$

Correct option is (c).

CAREER ENDEAVOUR

85. Unit cell dimension (a) = 1 nm = 1×10^{-9} m and size of the crystal = 1 cm.

$\therefore$ Number of unit cells in the crystal $= \dfrac{1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}}{10^{-9}\text{ m} \times 10^{-9}\text{ m} \times 10^{-9}\text{m}} = \dfrac{10^{-2} \times 10^{-2} \times 10^{-2}}{10^{-9} \times 10^{-9} \times 10^{-9}} \dfrac{\text{m}^3}{\text{m}^3} = \dfrac{10^{-6}}{10^{-27}} = 10^{21}.$

$\therefore$ Number of atoms in the crystal = (number of atoms/unit cell) × number of unit cells $= 4 \times 10^{21}$ atoms.

Hence, correct option is (b).

86. The amplitude of diffracted X-ray beam for fcc lattice having four atoms at (0, 0, 0), $\left(\frac{1}{2}, 0, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}, \frac{1}{2}\right)$ positions is given by

$$F = f\left[1 + e^{\pi i(h+l)} + e^{\pi i(h+k)} + e^{\pi i(k+l)}\right] \qquad \text{... (i)}$$

It is obvious that the amplitude of diffracted X-ray beam is non-zero only if h, k and l are all even or all odd and has a value equal to $4f$. The amplitude of diffracted X-ray beam vanishes for all other odd-even combinations of h, k and l.

Hence, for (010) :

$$F = f\left[1 + e^{\pi i(0+0)} + e^{\pi i(0+1)} + e^{\pi i(1+)}\right] = f\left[1 + e^{0} + e^{\pi i} + e^{\pi i}\right] = f\left[1 + 1 - 1 - 1\right] = 0$$

And for (200) : $F = f\left[1 + e^{\pi i(2+0)} + e^{\pi i(0+0)} + e^{\pi i(0+2)}\right] = f\left[1 + e^{2\pi i} + e^{0\cdot\pi i} + e^{2\pi i}\right] = f\left[1 + 1 + 1 + 1\right] = 4f.$

Hence, correct option is (b).

87. Since, each nuclei of ^{235}U releases 3 neutrons but only 20% of the ^{235}U atoms in bomb undergoes fission, hence total number of neutrons released are

$$= \frac{6.1 \times 10^3}{235} \times 6.023 \times 10^{23} \times 3 \times \left(\frac{20}{100}\right) = 0.0938 \times 10^{26} = 9.38 \times 10^{24}$$

Correct option is (b)

88. First we will solve (ii),

Let the total mass of ^{235}U is m kg.

Therefore, the number of nuclei in m kg mass are $= \dfrac{m}{235} \times 6.023 \times 10^{23}$

According to question, each ^{235}U gives 200 MeV but 20% of ^{235}U nuclei in the bomb undergoes fission. Hence, the total energy produced after the fission of m mass atomic bomb

$$= \frac{m}{235} \times 6.023 \times 10^{23} \times \left(\frac{20}{100}\right) \times 200 \text{ MeV}$$

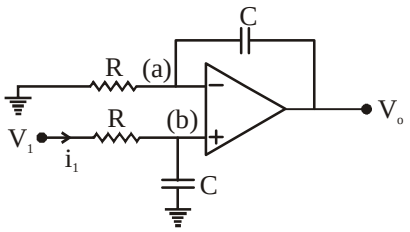
$$10^{14}\text{ joule} = \frac{m}{235} \times 6.023 \times 10^{23} \times 40 \times 1.6 \times 10^{-13}\text{ joule}$$

$$\Rightarrow \qquad m = \frac{235 \times 10^{14}}{6.023 \times 10^{23} \times 40 \times 1.6 \times 10^{-13}} = 0.6096 \times 10^{(14-23+13)} = 0.6096 \times 10^4\, g = 6.096\, kg \approx 6.1\, kg$$

Correct option is (c)

CAREER ENDEAVOUR

89.



An ideal op-amp has infinite input resistance. So, there will be no current pass through op-amp. So, V_a and V_b will be virtual short i.e. $V_a = V_b$.

$$V_1 = i_1 \cdot z$$

$$\frac{V_1}{i_1} = z = R + \frac{1}{CS}$$

$$z(j\omega) = R + \frac{1}{j\omega C} \qquad \because \quad S = j\omega$$

$$z(j\infty) = R$$

Correct option is (b)

90. Applying voltage divider rule at node 'b'

$$V_b = \frac{V_{i(s)} \times \frac{1}{CS}}{R + \frac{1}{CS}} \quad \Rightarrow \quad V_b = \frac{V_{i(s)}}{1 + RCS} = V_a \quad [\text{Virtual ground concept}]$$

By applying KCL at node 'a'

$$\frac{V_a}{R} + \frac{V_a - V_{o(s)}}{\frac{1}{CS}} = 0$$

$$V_{0(s)} \cdot CS = V_a\left[\frac{1}{R} + CS\right] \quad \Rightarrow \quad V_{0(s)} \cdot CS = V_a\left[\frac{1 + RCS}{R}\right]$$

Put value of V_a

$$V_{o(s)} \cdot CS = \frac{V_{i(s)}}{1 + RCS} \cdot \frac{1 + RCS}{R}$$

$$\frac{V_{0(s)}}{V_{i(s)}} = ? \quad \frac{V_{i(s)}}{RCS} = V_{0(s)}$$

$$\frac{V_{0(s)}}{V_{i(s)}} = \frac{1}{RCS} \quad \Rightarrow \quad V_0(t) = \frac{1}{RC}\int V_i(t) \cdot dt \quad \text{By taking inverse Laplace, this is given in option (a)}$$

(b) $V_a = V_b$ (Virtual ground concept) perfectly equal.

(c) V_i and i_1 (through R) are in equal phase

(d) Current will be in opposite direction in both Resistor (180°) out of phase

Correct option is (a)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.30 : Carry ONE mark each.

1. $\phi = x^2 y + xy \Rightarrow \vec{\nabla}\phi = (2xy + y)\hat{i} + (x^2 + x)\hat{j}$

$\vec{\nabla}\phi\big|_{(1,1,1)} = 3\hat{i} + 2\hat{j} \Rightarrow \left|\vec{\nabla}\phi\right|_{(1,1,1)} = \sqrt{13}$

Correct option is (d)

2. $f(x) = \sin x,\ (0 \le x \le \pi)$

Average value of $f(x) = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} \sin x dx = \frac{1}{\pi}[-\cos x]_0^{\pi} = \frac{2}{\pi}$

Correct option is (b)

3. At equilibrium points $\frac{dV}{dr} = 0$

and at unstable equilibrium there is local maxima

Hence correct answer is (c)

4. quantity $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

is invariant under Lorentz transformation as shown below.

According to Lorentz transformation

$x' = (x - vt)\gamma$

$y' = y$ $\qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$z' = z$

$t' = (t - xv/c^2)\gamma$

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial \psi}{\partial t'}\frac{\partial t'}{\partial x} = \gamma\frac{\partial \psi}{\partial x'} - \frac{\gamma v}{c^2}\frac{\partial \psi}{\partial t'}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) = \frac{\partial}{\partial x'}\left(\frac{\partial \psi}{\partial x}\right)\frac{\partial x'}{\partial x} + \frac{\partial}{\partial t'}\left(\frac{\partial \psi}{\partial x}\right)\frac{\partial t'}{\partial x}$$

$$= \gamma^2\frac{\partial^2 \psi}{\partial x'^2} - \frac{\gamma^2 v}{c^2}\frac{\partial^2 \psi}{\partial x' \partial t'} - \frac{\gamma v}{c^2}\frac{\partial^2 \psi}{\partial t' \partial x'} + \frac{\gamma^2 v^2}{c^4}\frac{\partial^2 \psi}{\partial t'^2}$$

Similarly, $\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial x'}\cdot\frac{\partial x'}{\partial t} + \frac{\partial \psi}{\partial t'}\cdot\frac{\partial t'}{\partial t}$

$$\frac{\partial \psi}{\partial t} = -v\gamma\frac{\partial \psi}{\partial x'} + \gamma\frac{\partial \psi}{\partial t'}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial t}\right) = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right)\frac{\partial x'}{\partial t} + \frac{\partial}{\partial t'}\left(\frac{\partial \psi}{\partial t}\right)\frac{\partial t'}{\partial t}$$

$$= v^2\gamma^2\frac{\partial^2 \psi}{\partial x'^2} - v\gamma^2\frac{\partial^2 \psi}{\partial x' \partial t'} - v\gamma^2\frac{\partial \psi}{\partial x' \partial t'} + \gamma^2\frac{\partial^2 \psi}{\partial t'^2}$$

CAREER ENDEAVOUR

since $y' = y, z' = z$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y'^2}, \quad \frac{\partial^2}{\partial z^2} = \frac{\partial}{\partial z'^2}$$

$$\frac{\partial}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \gamma^2 \frac{\partial}{\partial x^2} - \frac{\gamma^2 v}{c^2}\frac{\partial^2}{\partial x' \partial t'} - \frac{\gamma^2 v}{c^2}\frac{\partial^2}{\partial x' \partial t'} + \frac{\gamma^2 v^2}{c^4}\frac{\partial^2}{\partial t'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial}{\partial z'^2} - \frac{1}{c^2} v^2 \gamma^2 \frac{\partial}{\partial x'^2}$$

$$+ \frac{v\gamma^2}{c^2}\frac{\partial^2}{\partial x' \partial t'} + \frac{v\gamma^2}{c^2}\frac{\partial^2}{\partial x' \partial t'} - \frac{\gamma^2}{c^2}\frac{\partial^2}{\partial t'^2}$$

$$= \gamma^2\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2}\frac{\gamma^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2}{\partial t'^2}$$

$$= \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}$$

Correct option is (c)

5. The image charge,

$$q' = -\frac{qa}{d} = -\frac{qR}{2R} = -\frac{q}{2}$$

And distance from the centre

$$d' = \frac{a^2}{d} = \frac{R^2}{2R} = \frac{R}{2}$$

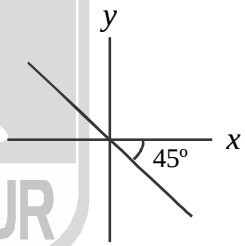
Correct option is (a)

6. We have,

$$\vec{E} = \hat{i}E_0 \cos(kz - \omega t) - \hat{j}E_0 \cos(kz - \omega t)$$

So, $E_x = E_0 \cos(kz - \omega t)$

$E_y = -E_0 \cos(kz - \omega t)$



$$\therefore \quad \frac{E_y}{E_x} = -1$$

$$\therefore \quad E_y = -E_x$$

Therefore, it is a linear plarized light at –45°to x-axis.

Correct option is (b)

7. $\because \quad \Delta E \Delta t \simeq \hbar$ where $\Delta E = \Gamma$

$\Delta t = \tau$

$\Rightarrow$ Life time, $\tau = \frac{\hbar}{\Gamma}$ i.e. inversely proportional to Γ

Correct option is (c)

8. **Correct option is (d)**

9. Coulomb degeneracy of n^{th} state of hydrogen atom is $g_n = n^2$. For $n = 4$ state, $g_4 = 16$

Correct option is (b)

CAREER ENDEAVOUR

10. $H = \frac{p^2}{2m} + V(|\vec{r}|) + \phi(|\vec{r}|)\,\vec{L}\cdot\vec{S} = \frac{p^2}{2m} + V(r) + \phi(r) \quad [L_xS_x + L_yS_y + L_zS_z]$

$= \frac{p^2}{2m} + V(|\vec{r}|) + \frac{1}{2}\phi(|\vec{r}|)\,(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$

$[H, S^2] = 0 \qquad [H, \vec{L} + \vec{S}] \neq 0$

$[H, L^2] = 0$

$[H, \vec{L}_z] = 0$

Correct option is (a)

11. Term with largest multiplicity (2S + 1) will be ground state term. ^5S term has largest multiplicity equal to 5.

Correct option is (a)

12. For ^{3}F term ; $2s+1=3 \Rightarrow s=1$ and $l=3$

Therefore, total angular momentum $J=(l+s)$ to $|l-s| = (3+1)$ to $(3-1) = 4, 3, 2$.

Now, total number of $m_j = \sum_{J=2}^{4}(2J+1) = (2\times4+1)+(2\times3+1)+(2\times2+1) = 9+7+5 = 21.$

Therefore, degeneracy of spectral term ^{3}F is 21.

Hence, correct option is (d).

13. For $^3D_3$, $2s+1=3 \Rightarrow s=1$ and $l=2$ and $J=3$

Therefore, g-factor $(g) = 1 + \left[\frac{J(J+1)+s(s+1)-l(l+1)}{2J(J+1)}\right]$

$= 1 + \left[\frac{3(3+1)+1(1+1)-2(2+1)}{2\times3\times(3+1)}\right] = 1 + \left[\frac{12+2-6}{24}\right] = 1 + \frac{1}{3} = \frac{4}{3}.$

Hence, correct option is (d).

14. Infrared spectra will be observed when there is a change in the electric dipole moment of a molecule.

Hence, correct option is (b).

15. The second law of thermodynamics states that the entropy of the universe always increases in a irreversible process and remains constant in a reversible process, i.e., $dS \geq 0$.

Correct option is (b).

16. The degree of freedom of each rigid diatomic molecules, f = (3N–C) = 3×2–1 = 5

So, the phase space dimension of each diatomic molecules = $2f$ = 10

So, the phase space dimension of 10 diatomic molecules = 100

Correct option is (d)

17. The specific heat of Fermi gas is given by

$C_v = \frac{\pi^2}{2}Nk\left(\frac{T}{T_F}\right)$. So, $C_v \propto T$

Correct option is (a)

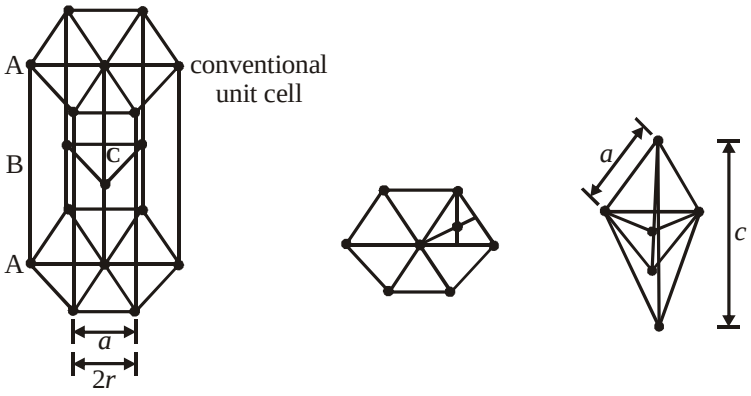
CAREER ENDEAVOUR

18. The first order phase transition involves the concept of latent heat. Moreover, the vaporization needs latent heat of vaporization and first derivative of Gibbs free energy is discontinious

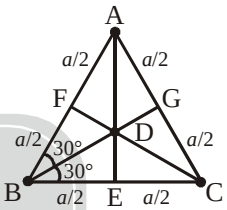
Correct option is (a).

19. Let a is interatomic distance within a layer and c is height of the unit cell

Unit cell of hexagonal closed packed structure can be made as:



From this figure, $\frac{c}{a} = \sqrt{\frac{8}{3}}$.



In figure : $BD = \frac{a/2}{\cos 30°} = \frac{a/2}{\sqrt{3}/2} = \frac{a}{\sqrt{3}}$

In the center of spheres in contact, then

$$a^2 = \left(\frac{c}{2}\right)^2 + \left(\frac{a}{\sqrt{3}}\right)^2 \Rightarrow a^2 - \frac{a^2}{3} = \frac{c^2}{4} \Rightarrow \frac{2a^2}{3} = \frac{c^2}{4} \Rightarrow \frac{c}{a} = \sqrt{\frac{8}{3}} = 1.633.$$

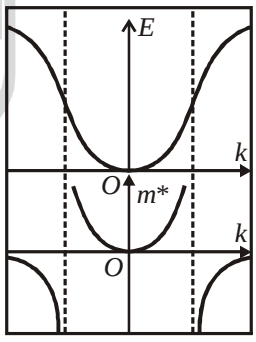
Hence, correct option is (d).

20. Youngs Modulus, Bulk Modulus and Poissions Ratio.

Correct option is (c)

21. An electron moving in a crystal has an effective mass m^*,

$$m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$$

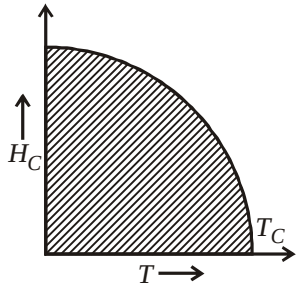


$\Rightarrow$ m^* can be positive in the lower half of the energy band and negative in the upper half.

Hence, correct option is (c).

CAREER ENDEAVOUR

22. $\because \quad H_C(T) = H_C(0)\left[1 - \frac{T^2}{T_C^2}\right]$, where $H_C(0)$ is critical field at 0 K. Now, if T increases, T^2 will also increase.



Thus, $H_C(T)$ will decrease and vice-versa

Hence, correct option is (c).

23. Since, $R \propto A^{1/3} = R_0 A^{1/3}$

$\therefore \quad R^3 \propto A = R_0{}^3 A \qquad \ldots (1)$

Therefore, volume of nucleus (V) = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R_0^3 A) \Rightarrow \boxed{V \propto A}$

Correct option is (a)

24. The decrease of binding energy from ^{56}Fe to ^{235}U nuclear is due to the long range nature of Coulomb forces.

Correct option is (b)

25. The reaction cross-section σ is given by

$$\sigma = \frac{R}{n v N}$$

where, R is the reaction rate density

n is the number density of the particles in the neutron beam.

v is the velocity of neutron

N is the number of density of the nuclei in the tangent.

Correct option is (a)

26. Since, neutron, proton and Δ^+ are member of Baryon family so they will have B = 1

But pion belong to Hadrons family (meson). Therefore, it will have B = 0

Correct option is (a)

27. **Correct option is (c)**

28. *FET* is unipolar because it has only majority carriers.

FET is high impedance device

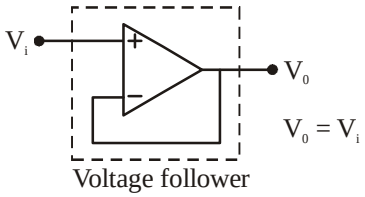
FET is having low gain

FET "VCCS" voltage control current source.

Correct option is (a)

CAREER ENDEAVOUR

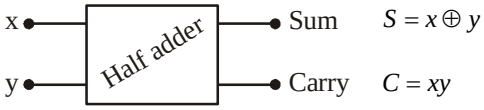
29. Drawing circuit as per statement

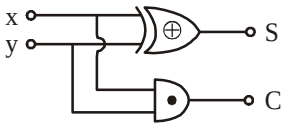


Voltage follower

Note: Used for impedance matching.
Correct option is (c)

30.





Hence, it has 2 input and 2 output (Sum, Carry)
Note : Similarly for full addar it has 3 input and 2 output.
Correct option is (d)

Q.31 – Q.85 : Carry TWO marks each.

31. Sum of the eigenvalues = Trace of the matrix

$\Rightarrow \lambda_1+\lambda_2+\lambda_3+\lambda_4=0 \Rightarrow 1+(-1)+\lambda_3+\lambda_4=0 \Rightarrow \lambda_3+\lambda_4=0$

So, $\lambda_3=+1, \lambda_4=-1$

Correct option is (d)

32. Eigenvalue equation : $\begin{vmatrix}1-\lambda & i\\ -i & 1-\lambda\end{vmatrix}=0 \Rightarrow (\lambda-1)^2+i^2=0 \Rightarrow \lambda^2-2\lambda=0 \Rightarrow \lambda=0, 2$

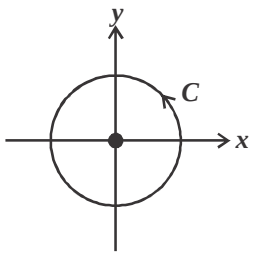
Correct option is (c)

33. $\left(\dfrac{3+4i}{3-4i}\right)^{-1}=\dfrac{3-4i}{3+4i}=\dfrac{(3-4i)(3-4i)}{(3+4i)(3-4i)}=\dfrac{(3-4i)^2}{9-16i^2}=\dfrac{-7-24i}{25}=-\dfrac{7}{25}-i\dfrac{24}{25}$

Correct option is (d)

34. $f(z)=\dfrac{1}{z^2+a^2}$ has simple poles at $z=\pm ia$

For $a = 2$, poles at $z=\pm 2i$ lies outside the circle $|z| = 1$.



C : |z| = 1

So, $\oint_C \dfrac{dz}{z^2+a^2}=0$

For $a=\dfrac{1}{2}$, poles at $z=\pm\dfrac{i}{2}$ lies within the circle

$$\oint_C \frac{dz}{z^2+a^2}=2\pi i\left[\text{Res. } f\left(z=\frac{i}{2}\right)+\text{Res. } f\left(z=-\frac{i}{2}\right)\right]$$

$$=2\pi i\left[\left.\frac{1}{2z}\right|_{z=i/2}+\left.\frac{1}{2z}\right|_{z=-\frac{i}{2}}\right]=0$$

Correct option is (b)

35. **Property :** $L\left[f(x)\right] = f(s)$ then $L\left[x^n f(x)\right] = (-1)^n \dfrac{d^n}{ds^n}\left[f(s)\right]$

$$L\left[t\sin\pi t\right] = (-1)^1 \frac{d}{ds}\left[L(\sin\pi t)\right] = -\frac{d}{ds}\left[\frac{\pi}{s^2+\pi^2}\right] = \frac{2\pi s}{\left(s^2+\pi^2\right)^2}$$

Correct option is (c)

36. $$f(x) = x = \sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin(nx)$$

$$\Rightarrow \qquad x = 2\sin x - \frac{2}{2}\sin 2x + \frac{2}{3}\sin 3x - \frac{2}{4}\sin 4x$$

Using, Perseval's formula, we get

$$\int_{-\pi}^{\pi}\left(x^2\right)dx = \pi\left[4 + \frac{4}{2^2} + \frac{4}{3^2} + \frac{4}{4^2}.....\right]$$

$$\Rightarrow \qquad \frac{2\pi^3}{3} = 4\pi\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}.....\right]$$

$$\Rightarrow \qquad \frac{\pi^2}{6} = \sum_{n=1}^{\infty} n^{-2}$$

Correct option is (c)

37. $$F\left[e^{-x^2}\right] = \int_{-\infty}^{\infty} e^{-x^2}\cdot e^{ikx}dx = \int_{-\infty}^{\infty} e^{-x^2+ikx}dx = \sqrt{\pi}\exp\left[-\frac{k^2}{4}\right] \qquad \left(\text{Using } \int_{-\infty}^{\infty} e^{-ax^2+bx}dx = \sqrt{\frac{\pi}{a}}\, e^{b^2/4a}\right)$$

Correct option is (b)

38. $$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

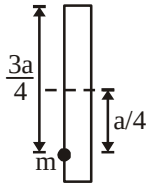
$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$$

Hence correct answer is (a)

39. $$I_0 = I_{rod} + I_{particle}$$

$$= \frac{1}{3}(4m)a^2 + m\left(\frac{3a}{4}\right)^2$$

$$= \frac{4ma^2}{3} + \frac{9ma^2}{16} = \frac{91}{48}ma^2$$



Hence correct answer is (b)

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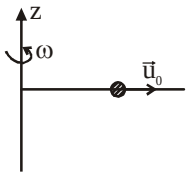
40. $\vec{F}_{coriolis} = 2m\,\vec{v}\times\vec{\omega}$

$= 2m u_0 \hat{r}\times\omega\hat{z}$

$= 2m u_0\omega\left(-\hat{\theta}\right)$



Hence correct answer is (b)

41. $L = a\dot{x}^2 + b\dot{y}^2 - kxy$

$$p_x = \frac{\partial L}{\partial \dot{x}} = 2a\dot{x} \quad \Rightarrow \quad \dot{x} = \frac{p_x}{2a}$$

$$p_y = \frac{\partial L}{\partial \dot{y}} = 2b\dot{y} \quad \Rightarrow \quad \dot{y} = \frac{p_y}{2b}$$

$$H = \sum_i p_i \cdot \dot{q}_i - L = p_x\,\dot{x} + p_y\,\dot{y} - a\dot{x}^2 - b\dot{y}^2 + kxy$$

$$H = \frac{p_x^2}{2a} + \frac{p_y^2}{2b} - \frac{p_x^2}{4a} - \frac{p_y^2}{4b} + kxy$$

$$H = \frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$$

Hence correct answer is (c)

42. $\left\{\vec{a}\cdot\vec{r}, \vec{b}\cdot\vec{p}\right\} = \left\{a_i\, r_i, b_j\, p_j\right\}$

$= a_i b_j \left\{r_i,\, p_j\right\} = a_i b_j\, \delta_{ij}$

$= a_i\, b_i = \vec{a}\cdot\vec{b}$

Hence correct answer is (d)

43. force $= -x\left(k_1 + k_2\right)$, $U = -\int F\,dx = \frac{1}{2}\left(k_1 + k_2\right)x^2$

$$L = T - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}\left(k_1 + k_2\right)x^2$$

at any instant $\vec{r}$ and $\vec{p}$ are along same line. Therefore

$$\vec{L} = \vec{r}\times\vec{p} = 0$$

total energy $E = \frac{1}{2}m\,\dot{x}^2 + \frac{1}{2}\left(k_1 + k_2\right)x^2$

Hence correct answer is (b)

44. $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$; $2m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

$$1 - \frac{v^2}{c^2} = \frac{1}{4} \quad \Rightarrow \quad v = \frac{\sqrt{3}}{2}c$$

Hence correct answer is (a)

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45. Energy $u \propto \dfrac{Q^2}{R}$

$$\Rightarrow \quad \frac{u_1}{u_2} = \frac{Q^2/R}{4Q/(R/2)} \Rightarrow u_2 = 8u_1$$

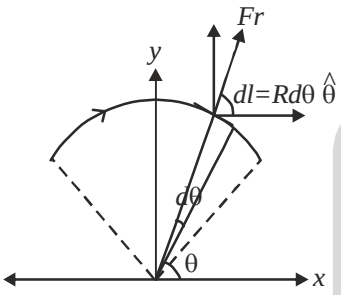
Correct option is (b)

46. $$\sigma = -\varepsilon_0 \left.\frac{\partial V_{out}}{\partial r}\right|_{r=R}$$

$$= \varepsilon_0 E_0 \left(\cos\theta + \frac{2R^3}{r^3}\cos\theta\right)\Bigg|_{r=R} = 3\varepsilon_0 E_0 \cos\theta$$

Correct option is (c)

47.



The force on the elementary length

$$dF = -IRd\theta\hat{\theta} \times -B_0\hat{k} = IRB_0 d\theta\hat{r} = I\,R\,B_0(\cos\theta\,\hat{j} + \sin\theta\,\hat{i}) \times d\theta$$

So, $\vec{F} = \int_{-\pi/6}^{\pi/6} I\,R\,B_0(\cos\theta\,\hat{j} + \sin\theta\,\hat{i}) \times d\theta = IRB_0\hat{j}$

Correct option is (b)

48. For normal incidence, $\theta_1 = \theta_2 = 0$

$$R = R_\perp = R_{||} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

$$T = T_\perp = T_{||} = \frac{4n_1 n_2}{(n_1 + n_2)^2} = \frac{4n}{(n+1)^2}$$

Correct option is (a)

49. We have, $\vec{E} = \vec{E}_0 \exp\left[i(xk\cos\alpha + yk\sin\alpha - \omega t)\right]$

Comparing this with plane wave equation

$$E = E_0 e^{i\left(\vec{k}\vec{r} - \omega t\right)}$$

We can write, $\vec{k} = \hat{i}k\cos\alpha + \hat{j}k\sin\alpha$

Correct option is (c)

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50. $\omega^2 = \omega_0^2 + k^2c^2 \Rightarrow 2\omega d\omega = c^2\left(2kdk\right) \Rightarrow \frac{d\omega}{dk} = \frac{c^2}{\omega}k \Rightarrow v_g = \frac{c^2}{v_p} \Rightarrow v_p v_g = c^2$

Correct option is (c)

51. For the m[th] order bright fring

$2d = m\lambda$

Given: $2\left(d + \Delta d\right) = \left(m + 92\right)\lambda$

$\Rightarrow \quad 2\Delta d = 92\lambda$

$\Rightarrow \quad \lambda = \frac{2 \times 25.3 \times 10^{-6}}{92} = 550\ nm$

52. $\psi = u\left(x\right)e^{ikz}$

$\Rightarrow \psi^* = u\left(x\right)e^{-ikz}$

So, probability current density, $J = \frac{i\hbar}{2m}\left[\psi\nabla\psi^* - \psi^*\nabla\psi\right]$

$$= \frac{i\hbar}{2m}\left[\psi\frac{\partial u}{\partial x}e^{-ikz} - iku^2\left(x\right) - \psi^*\frac{\partial u}{\partial x}e^{ikz} - iku^2\left(x\right)\right]$$

$$= \frac{i\hbar}{2m}\left(-2iku^2\left(x\right)\right) = \frac{\hbar k}{m}u^2\left(x\right) = vu^2\left(x\right)$$

Correct option is (a)

53. According to Schrodinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\left(x\right)\psi\left(x\right) = E\psi\left(x\right)$$

$\Rightarrow \quad -\frac{\hbar^2}{2m}\frac{d}{dx}\left(-A2\alpha^2xe^{-\alpha^2x^2}\right) + V\left(x\right)\psi\left(x\right) = E_0\psi\left(x\right)$

$\Rightarrow \quad -\frac{\hbar^2}{2m}\left(-2A\alpha^2e^{-\alpha^2x^2} + 4A\alpha^4x^2e^{-\alpha^2x^2}\right) + V\left(x\right)\psi\left(x\right) = E_0\psi\left(x\right)$

$\Rightarrow \quad -\frac{\hbar^2}{2m}\left(-2\alpha^2 + 4\alpha^4x^2\right) + V\left(x\right) = E_0$

$\Rightarrow \quad V\left(x\right) = E_0 + \frac{\hbar^2}{2m}\left(-2\alpha^2 + 4\alpha^4x^2\right) \qquad \left(\because\ E_0 = \frac{\hbar^2\alpha^2}{m}\right)$

$$= E_0 - E_0 + 2E_0\alpha^2x^2 = 2E_0\alpha^2x^2$$

Correct option is (c)

54. $V(r) = \vec{S}_1 \cdot \vec{S}_2 V_0(r)$

$$= \frac{1}{2}\left(\hat{S}^2 - \hat{S}_1^2 - \hat{S}_2^2\right)V_0(r)$$

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$$\Rightarrow \qquad E_p = \frac{1}{2}\left[s(s+1) - s_1(s_1+1) - s_2(s_2+1)\right]V_0(r)$$

For singlet state $s = 0, \quad s_1 = \frac{1}{2}, \quad s_2 = \frac{1}{2}$

$$\Rightarrow \qquad E_p = \frac{1}{2}\left[-\frac{1}{2}\times\frac{3}{2} - \frac{1}{2}\times\frac{3}{2}\right]V_0(r) = -\frac{3}{4}V_0(r)$$

For triplet state $s = 1, s_1 = \frac{1}{2}, s_2 = \frac{1}{2}$

$$E_p = \frac{1}{2}\left[2 - 2\times\frac{3}{4}\right] = \frac{1}{2}\left[2 - \frac{3}{2}\right] = +\frac{1}{4}V_0(r)$$

Correct option is (d)

55. $\psi_0 = A\exp\left(\frac{\alpha^2 x^2}{2}\right)$

Applying normalization condition,

$$\int_{-\infty}^{\infty} \psi_0^{*}\psi_0 dx = 1$$

$$\Rightarrow \qquad \int_{-\infty}^{\infty} |A|^2 e^{-\alpha^2 x^2} dx = 1 \qquad \left[\text{using } \int_{-\infty}^{\infty} e^{-ax^2+bx}dx = \sqrt{\frac{\pi}{a}}\, e^{b^2/4a}\right]$$

$$\Rightarrow \qquad |A|^2\sqrt{\frac{\pi}{\alpha^2}} = 1 \Rightarrow \boxed{|A|^2 = \frac{\alpha}{\sqrt{\pi}}}$$

So, $\psi_0 = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\alpha^2 x^2/2}$

So, first order correction to energy,

$$E_0^{(I)} = \left\langle \psi_0 \left| E_0\left(\frac{\alpha x}{10}\right)^4 \right| \psi_0 \right\rangle = \frac{E_0\alpha^4}{10^4}\frac{\alpha}{\sqrt{\pi}}\int_{-\infty}^{\infty} x^4 e^{-\alpha^2 x^2} dx$$

$$= \frac{E_0\alpha^5}{10^4\sqrt{\pi}} 2\int_0^{\infty} x^4 e^{-\alpha^2 x^2} dx$$

[Put $\alpha^2 x^2 = z \Rightarrow x^2 = \frac{z}{\alpha^2} \Rightarrow 2xdx = \frac{1}{\alpha^2}dz \Rightarrow dx = \frac{dz}{2\alpha^2 \cdot x}$]

$$= \frac{E_0\alpha^5}{10^4\sqrt{\pi}}\int_0^{\infty} z^{3/2} e^{-z}\frac{dz}{\alpha^5} = \frac{E_0}{10^4\sqrt{\pi}}\int_0^{\infty} e^{-z} z^{3/2} dz = \frac{E_0}{10^4\sqrt{\pi}}\sqrt{\frac{5}{2}}$$

$$= \frac{E_0}{10^4\sqrt{\pi}}\frac{3}{2}\times\frac{1}{2}\sqrt{\pi} = \left(\frac{3}{4}E_0\right)10^{-4}$$

Correct option is (c)

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56. Atomic number of boron, $z = 5$.

The electronic configuration of boron is given by $1s^2, 2s^2, 2p^1$ i.e., there is one unpaired electron in $2p$ subshell.

Thus, $l = 1,\ s = \frac{1}{2}$

According to L-S coupling, J will have values from $(l + s)$ to $|l - s|$.

Thus, $J = \left(1 + \frac{1}{2}\right)$ to $\left|1 - \frac{1}{2}\right| = \frac{3}{2},\ \frac{1}{2}$.

For less than half filled orbital, lowest J value corresponds to ground state.

Thus, the ground state is characterized by $L = 1,\ S = \frac{1}{2},\ J = \frac{1}{2}$

Hence, correct option is (b).

57. For the spectroscopic term ^3p we have $S = 1$

Multiplicity $= 2s + 1 = 3 \Rightarrow l = 1$

Now, for spin-orbit interaction, total angular momentum can have values from $(l + s)$ to $|l - s|$.

Thus, $J = (1+1)$ to $|1-1| = 2, 1, 0$.

So, the term is $^3p_{0,1,2}$ i.e., $^3p_0$, $^3p_1$ and $^3p_2$

Now, degeneracy of individual terms are given by

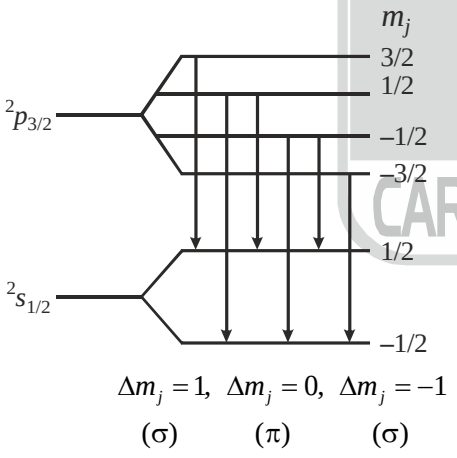
For $J = 0 \Rightarrow (2J + 1) = 1$

For $J = 1 \Rightarrow (2J + 1) = 3$

For $J = 2 \Rightarrow (2J + 1) = 5$

Hence, correct option is (a).

58.



According to selection rule, $\Delta m_j = 0, \pm 1\ (0 \not\leftrightarrow 0)$, total number of permitted transitions from $^2p_{3/2}$ to $^2s_{1/2}$ due to a weak magnetic field is 6.

Hence, correct option is (c).

59. For pure rotational spectrum of a diatomic rigid rotor, the separation between two consecutive lines $\Delta\bar{\nu}$ in the spectrum is given by

$$\Delta\bar{\nu} = 2B = \frac{2h}{8\pi^2 Ic}$$

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where, I is moment of inertia.

Thus, the separation $\Delta\bar{\nu}$ is inversely proportional to the moment of inertia of the rotor.

Hence, correct option is (b).

60. The frequency of the light of wavelength 1.5 μm incident on material is

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{1.5\times10^{-6}} = 2\times10^{14}\ \text{Hz}$$

This will be the exciting line i.e. Rayleigh line and the frequency of Raman line is $\nu_{\text{Raman}} = 20\times10^{12}$ Hz

Therefore, the Raman shift

$$\Delta\nu = \nu - \nu_{\text{Raman}} = 2\times10^{14} - 20\times10^{12}\ \text{Hz}$$
$$= 180\times10^{12}\ \text{Hz}$$

In terms of wavelength, the Raman shift is given by

$$\Delta\lambda = \frac{3\times10^{8}}{180\times10^{12}} = 1.67\times10^{-6}\ \text{m}$$
$$= 1.67\ \mu\text{m}$$

Hence, correct option is (c).

61. According to Wien's displacement law of blackbody radiation, $TV^{1/3} = \text{constant}$.

$$\Rightarrow \quad T_1V_1^{1/3} = T_2V_2^{1/3}$$

where $T_1 = 2000\,\text{K}$, $V_1 = 10\ \text{cm}^3$, $T_2 = ?$, $V_2 = 640\ \text{cm}^3$

Therefore, $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{1/3} = 2000\left(\frac{10}{640}\right)^{1/3} = 500\ \text{K}$

Correct option is (d).

62. Given equation : $\frac{p\nu}{k_BT} \approx 1 + \frac{B(T)}{\nu}$ ... (i)

For a vander Waal's gas, $\left(p + \frac{a}{\nu^2}\right)(\nu - b) = k_BT$

$$\Rightarrow \quad p = \frac{k_BT}{\nu - b} - \frac{a}{\nu^2}$$

$$\Rightarrow \quad \frac{p\nu}{k_BT} = \frac{\nu}{\nu - b} - \frac{a}{\nu k_BT}$$

$$= \left(1 - \frac{b}{\nu}\right)^{-1} - \frac{a}{\nu k_BT}$$

$$= 1 + \frac{b}{\nu} - \frac{a}{\nu k_BT} \quad \left(\text{considering } \frac{b}{v} << 1\right)$$

$$= 1 + \left(b - \frac{a}{k_BT}\right)\frac{1}{\nu}$$

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So, $B(T) = b - \dfrac{a}{k_B T}$

For an ideal Fermi gas, $\dfrac{pv}{k_B T} = N \dfrac{f_{5/2}(z)}{f_{3/2}(z)}$, where $f_\nu(z) = z - \dfrac{z^2}{2^\nu} + \dfrac{z^3}{3^\nu} + \cdots$

For an ideal Bose gas, $\dfrac{pv}{k_B T} = N \dfrac{g_{5/2}(z)}{g_{3/2}(z)}$, where $g_\nu(z) = z + \dfrac{z^2}{2^\nu} + \dfrac{z^3}{3^\nu} + \cdots$

For an ideal gas, $\dfrac{pv}{k_B T} = N$

Clearly, $B(T)$ can be negative for Vander waals' gas only, i.e., $B(T) < 0$

$\Rightarrow \quad b - \dfrac{a}{k_B T} < 0$ or $T < \dfrac{a}{b k_B}$

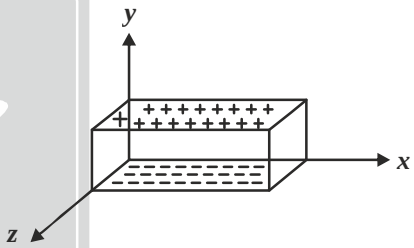
Correct option is (a).

63. The region of co-existance of a liquid and a vapour is a region of first order phase transition and the specific heats C_p and C_V both diverges in it.

Correct option is (a).

64. Hall voltage, $V_H = \dfrac{IB}{enb}$

$$\Rightarrow \qquad n = \frac{IB}{ebV_H} = \frac{1\times10^{-3}\times0.5}{1.6\times10^{-19}\times10^{-2}\times6\times10^{-3}} = 5.2\times10^{19}\,m^{-3}$$



and this is *n*-type

Correct option is (b)

65. $\because$ Slope of the $\ell n\,\sigma$ versus T^{-1} graph is $-\dfrac{E}{2k_B}$, also from the given figure, slope of A < slope of B.

$$\Rightarrow \quad -\frac{(E)_A}{2k_B} < -\frac{(E)_B}{2k_B} \quad \Rightarrow \quad E_A > E_B$$

Hence, correct option is (a).

66. Clausius-Mossotti equation ; $\dfrac{\varepsilon - 1}{\varepsilon + 2} = \dfrac{N\alpha}{3\varepsilon_0}$ ... (i)

where, α is total polarizability.

For $\varepsilon = 5.6$, $\dfrac{N\alpha}{3\varepsilon_0} = \left(\dfrac{5.6-1}{5.6+2}\right) = \dfrac{4.6}{7.6} = 0.605$ ... (ii)

Now, since refractive index, $n = 1.5$

$\Rightarrow \quad \sqrt{\varepsilon_r} = 1.5 \quad \Rightarrow \quad \varepsilon_r = 2.25$

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Again from equation (i), $\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0} \Rightarrow \frac{N\alpha_e}{3\varepsilon_0} = \left(\frac{2.25-1}{2.25+2}\right) = 0.2941.$

$$\therefore \quad \frac{\text{Electric polarizability}}{\text{Total polarizability}} = \frac{\alpha_e}{\alpha} = \frac{\left(\frac{N\alpha_e}{3\varepsilon_0}\right)}{\left(\frac{N\alpha}{3\varepsilon_0}\right)} = \frac{0.294}{0.605} = 0.486 \approx 0.5.$$

Hence, correct option is (a).

67. Parmanent magnets are made from hard ferromagnetic materials.
Correct option is (b)

68. From option (a) entropy always decreases on cooling a superconductor below its critical temperature. This indicates that superconducting state is more ordered than normal state.
From option (b) electronic contribution to heat capacity also has exponential form.
From option (c) A type-I superconductor is also perfect diamagnet
From option (d) Critical temperature of superconductor vary with isotopic mass and this is known as isotopic effect.
Correct option is (d)

69. $F(\vec{q}) = \int \exp\left(i\vec{q}\cdot\frac{\vec{r}}{\hbar}\right)\rho(r)d^3r$

Now, given $\rho(r) = \delta(r)$ $\qquad \left[\because \int f(x)\delta(x)dx = f(0)\right]$

$$= \int \exp\left(iq\cdot\frac{\vec{r}}{\hbar}\right)\delta(r)d^3r = 1$$

Correct option is (a)

70. In ground state deutron remains 96% of time in ^3S state for which (L = 0, S = 1) and 4% of time in 3D state for which (L = 2, S = 1)

Correct option is (c)

71. (a) $p \to n + e^+$; here Lepton number and angular momentum is not consreved. Hence reaction is not allowed.

(b) $p \to e^+ + \nu_e$; Baryon number and angular momentum is not conserved. Hence, reaction is not allowed.

(c) $p \to \pi^+ + \gamma$ **;** this violates the Baryon number conservation. Hence, this is not allowed reaction.

(d) $\bar{p} + n \to \pi^- + \pi^0$

Charge (Q): $-1+0 \to -1+0 \Rightarrow \Delta Q = 0$

Baryon number (B): $-1+1 \to 0+0 \Rightarrow \Delta B = 0$

Strangeness (S): $0+0 \to 0+0 \Rightarrow \Delta S = 0$

This reaction is allowed.

Correct option is (d).

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72. Given circuit is standard Wein Bridge Oscillator. The condition for sustaining oscillation in the Wein bridge oscillator is Gain $A \geq 3$

Therefore, Gain $= 3 = 1 + \dfrac{R_2}{R_1}$

- Frequency of oscillation $f_0 = \dfrac{1}{2\pi RC}$

frequency can be calculated by finding feedback factor "β" and equating it's imaginary part to zero

$$3 = 1 + \frac{R_2}{R_1} \qquad \because \quad R_1 = 12\ k\Omega$$

$$2 = \frac{R_2}{12} \qquad R_2 = 24\ k\Omega$$

$$f = \frac{1}{2\pi\ RC} \Rightarrow 300 = \frac{1}{2\pi \times R \times 0.01\mu F}$$

$$R = 53\ k\Omega$$

Correct option is (d)

73. The gain of the transistor is equal to $-g_m R_C \simeq -\dfrac{R_C}{r_e}$

$$\therefore \qquad Av = -\frac{R_C}{r_E} = -\frac{2\times10^3}{20} = -100$$

$$\therefore \qquad |Av| \approx 100$$

$$\therefore \qquad V_{out}(t) = AvV_{in}(t)$$

Now, the input should be such that the transistor does not go in to saturation region

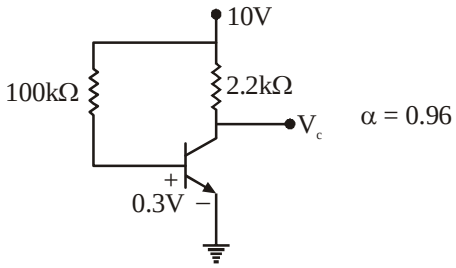
$$\therefore \qquad V_0(\max) = 10\,V$$

$$\therefore \qquad V_{in} = \frac{10}{100} = \frac{10\times1000}{100\times1000} = 100\ mV$$

Thus theoretically on input signal of 100 mV can be applied.

Correct option is (d)

74.



$\beta = \dfrac{\alpha}{1-\alpha}$, put value of α

$\Rightarrow \quad \beta = 24$

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Given $I_{CBO} = 20\,\mu A$

We know $I_C = \beta I_B + (\beta + 1) I_{CBO}$

By putting value $I_C = 24 I_B + (25 \times 20 \mu A)$

$$I_C = 24 I_B + (500\,\mu A) \quad \text{... (1)}$$

Apply KVL at collector-base junction

$$10 = 100 \cdot I_B + 0.3$$

$$I_B = 0.097\ mA$$

Put the value of I_B in equation (1)

$$I_C = (24 \times 0.097 mA) + (0.5\ mA)$$

$$I_C = (2.328 + 0.5)\, mA = 2.828\ mA$$

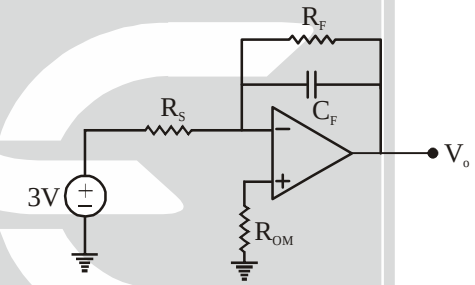
Apply KVL at output

$$10 = I_C \times 2.2 + V_C$$

$$V_C = 3.778 \text{ Volt}$$

Correct option is (a)

75. 3V is applied for $0 \le t \le 4 \text{sec}$.



$$\frac{V_0(s)}{V_i(s)} = \frac{-z(s)}{R_S} \equiv \frac{\dfrac{R_F \times \dfrac{1}{C_F S}}{R_F + \dfrac{1}{C_F S}}}{R_S}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{\dfrac{-R_F}{1 + R_F C_F S}}{R_S}$$

$$\frac{V_0}{V_i}(s) = \frac{-R_F}{R_S} \cdot \frac{1}{1 + R_F C_F S}$$

Put the given value

$$\frac{V_0}{V_i}(s) = \frac{-20}{30} \cdot \frac{1}{1 + 2S} \quad \Rightarrow \quad \frac{V_0}{V_i}(s) = \frac{-2}{3} \cdot \frac{1}{2\left(S + \frac{1}{2}\right)}$$

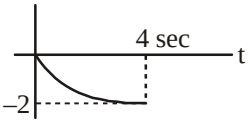
Note: $V_i = 3 \quad \Rightarrow \quad V_i(s) = \dfrac{3}{S}$ By Laplas transform

$$V_0(s) = \frac{-1}{3}\left[\frac{1}{S + 0.5}\right]\frac{3}{S} \quad \Rightarrow \quad V_0(s) = -\frac{1}{S(S + 0.5)}$$

$$V_0(s) = -\left[\frac{2}{S} - \frac{2}{S + 0.5}\right] \quad \Rightarrow \quad V_0(s) = -2\left[\frac{1}{S} - \frac{1}{S + 0.5}\right]$$

By applying Inverse Laplace Transforms

$$V_0(t) = -2\left[1 - e^{-0.5t}\right] \quad \Rightarrow \quad V_0(t) = -1.73 \text{ Volt}$$



It is Ramp function.

Nearest option is (d)

76. $Y = \overline{ABCD} + \overline{AB}CD + \bar{A}B\bar{C}D + \bar{A}BCD$

$= \overline{ABC}D + \overline{AB}CD + \bar{A}B\left(\bar{C} + C\right)D$

$= \overline{ABC}D + \overline{AB}CD + \bar{A}BD$

$= \overline{ABC}D + \bar{A}CD + \bar{B}CD + \bar{A}BD$

$= \bar{A}\left(\bar{B}D + \bar{C}D + D + CD + BD\right) + \bar{B}CD$

$= \bar{A}\left(D + D + \bar{C}D\right) + \bar{B}CD$

$= \bar{A}\left(D\right) + \bar{B}CD$

$= \bar{A}D + \bar{B}CD$

77. Consider the given differential equation,

$$y'' + p(x)\,y' + q(x)\,y(x) = 0$$

We can apply Frobenius power series method

$$y = \sum_{n=0}^{\infty} C_n x^{k+n}$$

Indicial equation will be

$$k^2 + (p-1)k + q = 0$$

By Taylor series expansion, $p = 4$, $q = 2$

$\Rightarrow \quad k^2 + (4-1)k + 2 = 0$

$\Rightarrow \quad k^2 + 3k + 2 = 0$

$\Rightarrow \quad k = -1, -2$

Correct option is (b)

78. $W = \begin{vmatrix} x_1 & x_2 \\ x'_1 & x'_2 \end{vmatrix} \neq 0$

For solution to be linearly independent

$$\begin{vmatrix} x & y \\ 1 & \dfrac{dy}{dx} \end{vmatrix} = W \qquad \left[\because\ W \neq 0 \text{ condition } \ell\mathrm{n}\left(x = 0\right) = 1,\ y = 0,\ \text{at } x = \frac{1}{2}\right]$$

$$x\frac{dy}{dx} - y = W$$

At $x = \dfrac{1}{2}$, $y = 0 \ \Rightarrow\ \dfrac{1}{2}\dfrac{dy}{dx} = W \Rightarrow \dfrac{dy}{dx} = 2W$

$$y = 2Wx + c$$

CAREER ENDEAVOUR

At $x=\frac{1}{2}, y=0$

$\Rightarrow \quad 0=2W\times\frac{1}{2}+c$

$\Rightarrow \quad c=-W \Rightarrow y=(2x-1)W$

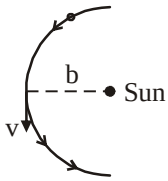
Correct option is (d)

79. Energy of comet in parabolic orbit is zero.

$E=0$

near the sun

$$\frac{1}{2}mv^2-\frac{GMm}{b}=0 \Rightarrow v=\sqrt{\frac{2GM}{b}}$$



angular momentum

$$L=mvb=m\sqrt{\frac{2GM}{b}}b \Rightarrow L=m\sqrt{2GMb}$$

Hence correct answer is (d)

80. Nearest to the Sun, force is maximum, therefore acceleration is maximum.

Hence correct answer is (a)

81. Given : $\vec{E}=\hat{x}E_0\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\vec{k}=\hat{z}(k\cos\varphi+ik\sin\phi)$

$E=\hat{x}\,E_0\,e^{i}\left[\hat{z}(k\cos\varphi+ik\sin\varphi)\cdot\vec{r}-\omega t\right]$

$=\hat{x}E_0\,e^{-k\sin\varphi z}e^{i}(k\cos\varphi z-\omega t)$

This says that the amplitude decays as the wave propagates.

Correct option is (a)

82. We know,

$\vec{k}\times\vec{E}=\omega\vec{B}$

$\Rightarrow \quad \vec{B}=\frac{k}{\omega}(\hat{k}\times\vec{E})$

We have,

$\vec{E}=\hat{x}E_0\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}=\hat{x}E_0\,e^{-kz\sin\varphi}\,e^{i(kz\cos\varphi-\omega t)}$ and $\hat{k}=\hat{z}$

$\vec{B}=\frac{k}{\omega}E_0\,e^{-kz\sin\varphi}\,e^{i(kz\cos\varphi-\omega t)}[\hat{z}\times\hat{x}]$

$=\hat{y}\frac{k}{\omega}E_0\,e^{-kz\sin\varphi}\,e^{i(kz\cos\varphi-\omega t)}$

Correct option is (b)

CAREER ENDEAVOUR

83. The wavefunction of nth state is $|\psi_n\rangle = \sqrt{2/L}\,\sin\left(\frac{n\pi}{L}x\right)$

Therefore, $|\psi_2\rangle = \sqrt{2/L}\,\sin\left(\frac{2\pi}{L}x\right)$

The probability density in the first excited state, is

$$P = \langle\psi_2|\psi_2\rangle = \frac{2}{L}\sin^2\left(\frac{2\pi}{L}x\right)$$

Probability of finding the particle, will be maximum if, $\sin\frac{2\pi}{L}x = \pm 1$

$$\Rightarrow \frac{2\pi}{L}x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{L}{4} \Rightarrow x = \frac{L}{4}, \frac{3L}{4}$$

Correct option is (d)

84. The wave function of the lowest energy state, is

$$|\psi_1\rangle = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$$

The probability of finding the particle in the region $0 < x < \frac{L}{4}$ is

$$P = \frac{2}{L}\int_0^{L/4}\sin^2\frac{\pi x}{L}dx = \frac{1}{L}\int_0^{L/4}\left(1-\cos\frac{2\pi x}{L}\right)dx = \frac{1}{L}\times\left[\frac{L}{4}-\frac{L}{2\pi}\right] = \left(\frac{1}{4}-\frac{1}{2\pi}\right)$$

Correct option is (a)

85. The energy of the electron in n^{th} state of the cubic box is given by

$$E_{n_x n_y n_z} = \frac{\pi^2\hbar^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

The energy of the lowest state $\left(n_x = 1, n_y = 1, n_z = 1\right)$

$$E_{111} = \frac{3\hbar^2\pi^2}{2ma^2} = \varepsilon_0$$

Correct option is (d)

86. The energy of the third excited state is $\varepsilon_3 = \frac{11\hbar^2\pi^2}{2ma^2}$ i.e. $n_x^2 + n_y^2 + n_z^2 = 11$

Possible combination of $\left(n_x, n_y, n_z\right)$ are (3, 1, 1), (1, 3, 1), (1, 1, 3)

So, the degeneracy (including spin) of the level $\varepsilon_3 = 3\times 2 = 6$

Correct option is (c)

87. The partition function is $Z = \sum_i e^{-\beta\varepsilon_i} = e^{+\beta\varepsilon_0} + e^0 + e^{-\beta\varepsilon_0}$

$$= 1 + 2\cosh(\beta\varepsilon_0)$$

$$= 1 + 2\cosh(2) \qquad \left[\because \beta\varepsilon_0 = 2\right]$$

CAREER ENDEAVOUR

Required probability $= \dfrac{e^{-0\cdot\beta}}{Z} = \dfrac{1}{1+2\cosh(2)} = \left[1+2\cosh(2)\right]^{-1}$

Correct option is (d).

88. The free energy is

$F = -N k_B T \ \ell n\, Z$

$= -N k_B T \ \ell n \left[1+e^{-\beta\varepsilon_0} + e^{+\beta\varepsilon_0}\right]$

$= -N k_B T \ \ell n \left[1+e^{-x} + e^{x}\right] \qquad \left[\because x = \beta\varepsilon_0\right]$

At high temperature, $x \ll 1 \Rightarrow e^{\pm x} \simeq 1 \pm x$

So, $F = -N k_B T \ \ell n \left[1+1-x+1+x\right]$

$= -N k_B T \ \ell n\, 3$

Correct option is (d).

89. ${}^{41}_{20}\text{Ca} \rightarrow$ Z = 20 $\rightarrow$ $(^1s_{1/2})^2\ (^1p_{3/2})^4\ (^1p_{1/2})^2\ (^1d_{5/2})^6\ (^2s_{1/2})^2\ (^1d_{3/2})^4$

N = 21 $\rightarrow$ $(^1s_{1/2})^2\ (^1p_{3/2})^4\ (^1p_{1/2})^2\ (^1d_{5/2})^6\ (^2s_{1/2})^2\ (^1d_{3/2})^4\ (^1f_{7/2})^1$

$\therefore$ Last proton $\rightarrow$ $(^1d_{3/2})^4$ and last neutron $\rightarrow$ $(^1f_{7/2})^1$

Correct option is (d)

90. N = 21, last neutron $(^1f_{7/2})^1$

$\therefore$ J = $\dfrac{7}{2}$, l = 3 (for f)

$\therefore$ Parity = $(-1)^3 = (-1)^3 = -1$ (odd)

Angular momentum and parity = $\dfrac{7^-}{2}$

Correct option is (a)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

1. $f(x) = 4x^3 \quad (1 \le x \le 3)$

The average value of $f(x) = \frac{1}{(3-1)}\int_1^3 4x^3 dx = \frac{1}{2}\int_1^3 4x^3 dx = \frac{1}{2}\left[\frac{4x^4}{4}\right]_1^3 = \frac{1}{2}\left[3^4 - 1\right] = 40$

Correct option is (c)

2. Unit normal vector to the curve : $\phi = x^3y^2 + xy = 17$ at point (2, 0) is

$$\hat{n} = \left[\vec{\nabla}\phi / \left|\vec{\nabla}\phi\right|\right]_{(2,0)} = \left.\frac{\left(3x^2y^2 + y\right)\hat{i} + \left(2x^3y + x\right)\hat{j}}{\left|\vec{\nabla}\phi\right|}\right|_{(2,0)} = \frac{2\hat{j}}{2} = \hat{j}$$

Correct option is (d)

3. $f(z) = \frac{1}{z+3}$ has a simple pole at z = –3

$$\int_C \frac{dz}{z+3} = 2\pi i\left[\text{Res. } f(z=-3)\right] = 2\pi i\left[(z+3)\frac{1}{(z+3)}\right] = 2\pi i$$

Correct option is (c)

4. Lets λ_1, λ_2 and λ_3 are the eigenvalue of the matrix

Determinant of the matrix = Product of the eigenvalues $\Rightarrow 36 = \lambda_1\lambda_2\lambda_3$

Given : $\lambda_1 = 2, \lambda_2 = 3$

So, $\lambda_3 = 6$

Correct option is (b)

5. For central field potential is r dependent only i.e. $V = V(r)$

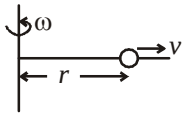
$$\therefore \quad L = \frac{1}{2}m\left(\dot{r}^2 + r^2\,\dot{\theta}^2\right) - V(r)$$

Therefore, energy and p_θ (angular momentum are conserved) since angular momentum is conserved, motion must be confined in a plane.

Hence correct answer is (c)

6. Coriolis force $= \left|2m\vec{v} \times \vec{\omega}\right|$

$= 2mv\omega$



Hence correct answer is (d)

7. None of the given options are correct.

8. Energy is not conserved in such process

Hence correct answer is (b)

9. $\oint B \cdot d\ell = \mu_0\left[I_1 + I_2 + I_3\right] = \mu_0\left[I + I - 2I\right] = 0$

$B = 0$

Correct option is (a)

CAREER ENDEAVOUR

10. $\oint E \cdot dS = \frac{4}{3}\frac{\pi r^3 \rho}{\varepsilon_0}$

$\Rightarrow \quad E \cdot 4\pi r^2 = \frac{4\pi r^3 \rho}{3\varepsilon_0} \Rightarrow E = \frac{r\rho}{3\varepsilon_0}$

Correct option is (a)

11. Super position of two linear orthogonal polarized light of equal amplitude give circular polarized light. Wave propagating towards positive z-direction.

Correct option is (b)

12. For a point source charge particle both the electric and magnetic field will be vary as

$E \propto \frac{1}{r}$ and $B \propto \frac{1}{r}$

Correct option is (b)

13. $\hat{p} \cos(kx) = \cos(-kx) = \cos kx$ (even parity)

$\hat{p} \tanh(kx) = \hat{p}\left(\frac{e^{kx} - e^{-kx}}{e^{kx} + e^{-kx}}\right) = \left(\frac{e^{-kx} - e^{+kx}}{e^{-xk} + e^{+xk}}\right) = -\tanh(kx)$ (odd parity)

Correct option is (d)

14. $\left[L_z, Y_{\ell m}(\theta, \phi)\right]\psi(\theta, \phi)$

$= -i\hbar\frac{\partial}{\partial\phi}Y_{\ell m}(\theta, \phi)\psi(\theta, \phi) + i\hbar Y_{\ell m}(\theta, \phi)\frac{\partial\psi}{\partial\phi}$

$= -i\hbar\frac{\partial\psi}{\partial\phi}Y_{\ell m}(\theta, \phi) - i\hbar\frac{\partial Y_{\ell m}}{\partial\phi}\psi(\theta, \phi) + i\hbar Y_{\ell m}(\theta, \phi)\frac{\partial\psi}{\partial\phi}$

$= -i\hbar(im)Y_{\ell m}(\theta, \phi)\psi(\theta, \phi) \quad \left[\because Y_{\ell m}(\theta, \phi) \propto e^{im\phi}\right]$

$\therefore \quad \left[L_z, Y_{\ell m}(\theta, \phi)\right] = -i\hbar(im)Y_{\ell m}(\theta, \phi) = \hbar m Y_{\ell m}(\theta, \phi)$

Correct option is (c)

15. $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$

According to normalization condition, $\langle\psi|\psi\rangle = 1$

$\Rightarrow |c_1|^2 + |c_2|^2 = 1 \qquad (\because \langle\alpha_n|\alpha_m\rangle = \delta_{mn})$

Correct option is (d)

16. Force on the charge $-q = -qE$

$\Rightarrow \quad -\frac{dV}{dx} = -qE \Rightarrow V = qEx$

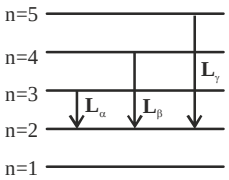
$\hat{H}$ = kinetic energy + potential energy

CAREER ENDEAVOUR

$$= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$$

Correct option is (c)

17. If vacancy is created in L shell, then electron from upper shells will have the transition to L shell (n = 2) giving L series. Transition $n = 3 \rightarrow n = 2 (L_\alpha)$



$n = 4 \rightarrow n = 2 (L_\beta)$ etc.

Hence, correct option is (a).

18. Selection rule for electric dipole transition : $\Delta l = \pm 1$

i.e., parity must change during transition and $\Delta m_l = 0, \pm 1$, $m_l = 0 \not\leftrightarrow m_l = 0$ if $\Delta l = 0$

Hence, correct option is (c).

19. Given : $pV = \frac{1}{3}U$

From first law of thermodynamics, $dQ = dU + pdV$, where $dQ = 0$ for an adiabatic process.

$\Rightarrow \quad dU + pdV = 0$

$\Rightarrow \quad d(3pV) + pdV = 0 \Rightarrow \quad 4pdV + 3V\,dp = 0 \Rightarrow \quad \frac{4dV}{V} + \frac{3dp}{p} = 0$

$\Rightarrow \quad \frac{4}{3}\ell nV + \ell n\, p = \ell n\, c$

where c is a constant of integration.

$\Rightarrow \quad pV^{4/3} = \text{constant.}$

Correct option is (c).

CAREER ENDEAVOUR

20. Number of accessible states for N non-interacting particles of spin s is $\Omega = (2s+1)^N$.

So for spin $\frac{1}{2}$ N-particles, $\Omega = 2^N$.

Correct option is (a).

21. If u is the total internal energy density then pressure is given by

$$p = \frac{2}{3}u = \frac{2}{3}\frac{U}{V} \qquad (U = uV, \text{ the total internal energy})$$

Correct option is (b)

22. **Given :** $g(E) \propto \sqrt{E} \Rightarrow g(E) = A\sqrt{E}$, where A is a constant of proportionality.

The average energy per particle at $T = 0$ K is

$$\langle E\rangle = \frac{\int_0^{E_F} E\, f(E)\, g(E)\, dE}{\int_0^{E_F} f(E)\, g(E)\, dE}$$

At $T = 0$

$$f(E) = \begin{cases} 1 & E < E_F \\ 0 & E > E_F \end{cases}$$

where $f(E) = 1$ at $T = 0$ K

$$\therefore \quad \langle E\rangle = \frac{A\int_0^{E_F} E\sqrt{E}\, dE}{A\int_0^{E_F} \sqrt{E}\, dE} = \frac{\int_0^{E_F} E^{3/2} dE}{\int_0^{E_F} E^{1/2} dE} = \frac{3}{5}E_F$$

Correct option is (d).

23. $\overrightarrow{OP} = \left[a, a, \frac{a}{2}\right] = \left[11\frac{1}{2}\right] = [221].$

Here, $\left(a, a, \frac{a}{2}\right)$ are the intercepts along x, y and z-axis.

Hence, correct option is (a).

24. Since, Argon has 18 atomic number. Therefore, its atomic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6$. Na is strong electropositive and Cl is strong electronegative. Si is a semiconductor with covalent bonding.

Hence, correct option is (c).

25. The β -intensity anti-parallel to the nuclear spin directions is same as that along the nuclear spin directions. This observation gives the evidence for the non-conservation of parity in β -decay.
Correct option is (a)

26. Since, surface energy term $E_s \propto R^2 \propto A^{2/3}$ also its sign is negative.
Correct option is (a)

27. (a) $\mu^- \longrightarrow e^- + \nu_\mu + \overline{\nu}_e$

Charge (Q) : $-1 \rightarrow -1+0+0 \Rightarrow \Delta Q = 0$

Electron Lepton No : $0 \rightarrow 1+1+0-1 \Rightarrow \Delta L_e = 0$

Muonie Lepton No : $+1 \rightarrow 0+1+0 \Rightarrow \Delta L_\mu$

Baryon number (B) : $0 \rightarrow 0+0+0 \Rightarrow \Delta B = 0$ (Allowed)

(b) $\pi^+ \longrightarrow \mu^+ + \nu_\mu$

Charge (Q) : $1 \rightarrow 1+0 \Rightarrow \Delta Q = 0$

Munonic Lepton number (L_μ) : $0 \rightarrow -1+1 \Rightarrow \Delta L_\mu = 0$

Baryon number (B) : $0 \rightarrow 0+0 \Rightarrow \Delta B = 0$ (Allowed)

(c) $\pi^+ \longrightarrow e^+ + \nu_e$

CAREER ENDEAVOUR

Charge (Q) : $1 \to 1+0 \Rightarrow \Delta Q = 0$

Electronic Lepton number (L_e) : $0 \to -1+1 \Rightarrow \Delta L_e = 0$

Baryon number (B) : $0 \to 0+0 \Rightarrow \Delta B = 0$ (Allowed)

(d) $\mu^- \longrightarrow e^+ + e^- + e^-$

Charge (Q) : $-1 \to +1-1-1 \Rightarrow \Delta Q = 0$

Electronic Lepton No : $0 \to -1+1+1 \Rightarrow \Delta L_e \neq 0$

Muonic Lepton No: $+1 \to 0+0+0 \Rightarrow \Delta L_\mu \neq 0$

Lepton number (L) : $+1 \to -1+1+1 \Rightarrow \Delta L = 0$

Baryon number (B) : $0 \to 0+0+0 \Rightarrow \Delta B = 0$

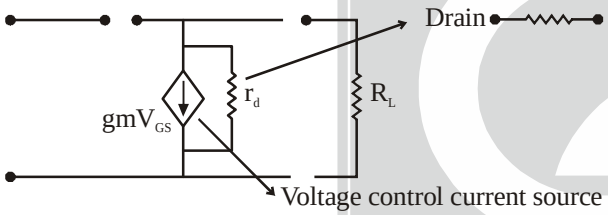
Therefore, this is forbidden.

Correct option is (d)

28. Nuclear forces are short reange, charge independent, velocity dependent and spin dependent.

Correct option is (d)

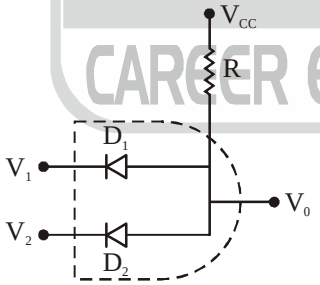
29.



$\Rightarrow$ AC equivalent figure of FET

Correct option is (a)

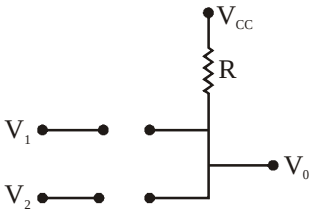
30. Redraw this circuit



Pull up resistor is R

Case (1)

D_1, D_2 OFF



Case - (2)

D_1, D_2 ON

Case - (3)

either D_1, D_2 is ON

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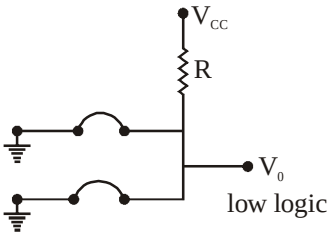
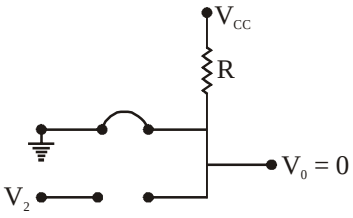




Truth Table

V_1	V_2	D_1	D_2	V_0
0	0	ON	ON	0
0	1	ON	OFF	0
1	0	OFF	ON	0
1	1	OFF	OFF	1

Logic AND Gate

Correct option is (d)

31. $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$ $\quad \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2y & 3z \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(0) = 0$

$\vec{\nabla}\times\vec{F} = 0 \;\Rightarrow \vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right) = 0$

Correct option is (a)

32. $e^z = -3 \Rightarrow e^z = e^{i(2n+1)\pi}\cdot 3$

$\Rightarrow \qquad z = \ln 3 + i(2n+1)\pi \quad (n = 0, \pm 1, \pm 2, ...)$

Correct option is (b)

33. $L\left[f(t)\right] = \hat{f}(s) = \int_0^\infty e^{-st} f(t)\,dt$

$L\left[f(at)\right] = \int_0^\infty e^{-st} f(at)\,dt = \int_0^\infty e^{-s\cdot\frac{z}{a}} f(z)\frac{dz}{a}$ (putting, $at = z \Rightarrow a\cdot dt = dz$]

$= \frac{1}{a}\int_0^\infty e^{-(s/a)z}\cdot f(z)\,dz = \frac{1}{a}\hat{f}(s/a)$

Correct option is (b)

34. $u_4 = \frac{3}{2}u_3$ i.e. u_3, u_4 are linearly dependent

Correct option is (d)

35. z = 0 is a removable singular point of $f(z) = \frac{\sin z}{z}$ as

CAREER ENDEAVOUR

$$\frac{\sin z}{z}=\frac{1}{z}\left[z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}-+.....\right]=1-\frac{z^2}{3!}+\frac{z^4}{5!}+\cdots.$$ (no negative powers of z)

Correct option is (b)

36. Sum of the eigenvalues = Trace of the matrix = 0

Only option (a) satisfies this.

Correct option is (a)

37. $r=a\theta,\quad \theta=\omega t \quad\Rightarrow\quad r=a\omega t$

radial acceleration $a_r=\ddot{r}-r\dot{\theta}^2=0-a\omega t\omega^2=-a\omega^2 t\neq$ constant

Transverse acceleration $a_\theta=2\dot{r}\,\dot{\theta}+r\,\ddot{\theta}=2a\omega\omega+0=2a\omega^2$

$a_\theta\neq 0$, $\therefore$ force is not radial, therefore angular momentum is not conserved.

Kinetic energy $=\frac{1}{2}mv^2=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)=\frac{1}{2}m\left(a^2\omega^2+a^2\omega^2t^2\cdot\omega^2\right)=\frac{1}{2}ma\omega^2\left(1+\omega^2t^2\right)$

it is time dependent therefore not conserved momentum cannot be conserved because force is not zero.

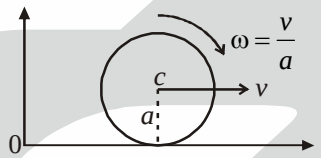
Radial momentum $p_r=mv_r=m\dot{r}=ma\omega$ it is conserved.

Hence correct answer is (d)

38. $L_0=Mva+I_c\omega,\ v=\omega a$

$=M\omega a^2+Ma^2\omega$

$=2M\omega a^2$



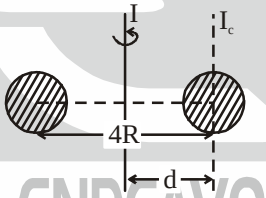
Hence correct answer is (c)

39. Use parallel axes theorem

$I=2\left(I_c+Md^2\right)$

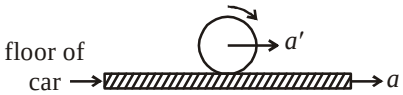
$=2\left[\frac{2}{5}MR^2+M\left(2R\right)^2\right]$

$=2\left[\frac{2}{5}+4\right]MR^2=\frac{44}{5}MR^2$



Hence correct answer is (c)

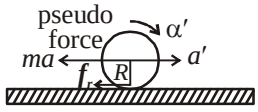
40.



let us observe from frame of car equation of motion for translation

$-ma-f_r=ma'$

$a'=-\left(a+\frac{f_r}{m}\right)$ ... (i)



equation of motion for rotation

$f_r R=I\alpha'=\frac{2}{5}MR^2\alpha'$

$\alpha'=\frac{5f_r}{2MR}$ ... (ii)

CAREER ENDEAVOUR

for rolling $a' = \alpha' R$

$$-\left(a + \frac{f_r}{m}\right) = \frac{5}{2}\frac{f_r}{m}$$

$$-a = \frac{7}{2}\frac{f_r}{m} \quad \Rightarrow \quad f_r = -\frac{2ma}{7}$$

from equation (i)

$$a' = -\left(a - \frac{2ma}{7m}\right) = -\frac{5a}{7}$$

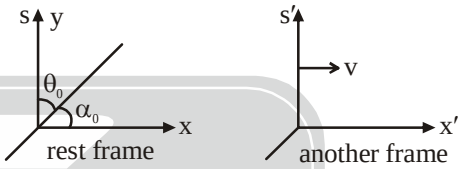
acceleration of sphere with respect to inertial frame

$$= a + a' = a - \frac{5a}{7} = \frac{2a}{7}$$

Hence correct answer is (b)

41. Let α be angle with x axis as seen from s' frame

$$\tan\alpha = \frac{\tan\alpha_0}{\sqrt{1 - v^2/c^2}} = \gamma \tan\alpha_0$$



If θ be angle with y axis as seen from s' then

$$\theta = 90 - \alpha \text{ or } \alpha = 90 - \theta, \text{ and } \alpha_0 = 90 - \theta_0$$

$$\cot\theta = \gamma \cot\theta_0$$

$$\tan\theta = \frac{1}{\gamma}\tan\theta_0$$

$$\theta = \tan^{-1}\frac{1}{\gamma}\tan\theta_0$$

Hence correct answer is (c)

42. $V(x) = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\mu v^2$

Lagrangian $L = T - V = \frac{1}{2}mv^2 - \frac{1}{2}m\omega^2 x^2 - \frac{1}{2}m\mu v^2$

$$p = \frac{\partial L}{\partial v} = mv - m\mu v = mv(1 - \mu)$$

Hence correct answer is (d)

43. Radiation power from accelerated charge particle dependency on charge and acceleration as

$$P \propto q^2 a^2$$

where, q is the charge of the particle and a is the acceleration of particle.

In case one particle is moving with constant speed. So, $p_a = 0$

In case of two and three particles are moving with same (centripital) acceleration. So, $p_b = p_c$

Correct option is (b)

CAREER ENDEAVOUR

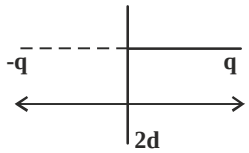
44. We know Laplace equation,

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} \Rightarrow \rho = -\varepsilon_0\nabla^2\phi = -6\phi_0\varepsilon_0$$

Correct option is (b)

45. The work will be done to bringing a charge particle from infinite to a position at a distance 'd' in front of a semiinfinite grounded metal surface is

$$W = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d}$$



46. $$\frac{E_r}{E_i} = \frac{(n_1 - n_2)}{(n_1 + n_2)} = \frac{1-1.5}{2.5} = \frac{-0.5}{2.5} = -\frac{1}{5} \Rightarrow E_r = -\frac{2}{5}$$

$$\frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2} = \frac{2}{2.5} = \frac{4}{5} \Rightarrow E_{ot} = \frac{8}{5}$$

Correct option is (c)

47. Lorent Gauge condition,

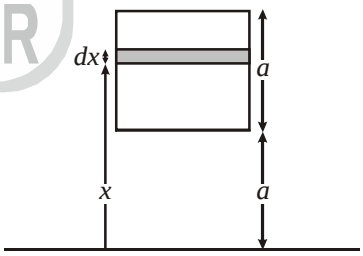
$$\vec{\nabla}\cdot\vec{A} + \mu\varepsilon_0\frac{\partial\phi}{\partial t} = 0$$

$$\Rightarrow \quad -\frac{\mu_0}{4\pi}\frac{Q}{r^2} + \mu_0\varepsilon_0\frac{\partial\phi}{\partial t} = 0$$

$$\Rightarrow \quad \phi = \frac{Qt}{4\pi\varepsilon_0 r^2}$$

Correct option is (d)

48. $d\phi = B\cdot a dx$



$$\Rightarrow \quad d\phi = \frac{\mu_0 I}{2\pi x} a dx$$

$$\Rightarrow \quad d\phi = \frac{a\mu_0 I_0\cos\omega t}{2\pi}\frac{dx}{x} \Rightarrow \phi = \frac{a\mu_0 I_0\cos\omega t}{2\pi}\int_a^{2a}\frac{dx}{x}$$

$$\Rightarrow \quad \phi = \frac{a\mu_0 I_0\cos\omega t}{2\pi}\ln\left[\frac{2a}{a}\right] = \frac{a\mu_0 I_0\cos\omega t}{2\pi}\ln 2$$

$$\Rightarrow \quad emf = -\frac{d\phi}{dt} = +\frac{\omega a\mu_0 I_0\sin\omega t}{R}\ln 2$$

So, current, $I = \frac{emf}{R} = \frac{\mu_0}{2\pi}\frac{aI_0\omega\sin\omega t}{R}\ln 2$

Correct option is (b)

CAREER ENDEAVOUR

49. The de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times150\times1.6\times10^{-19}}} \approx 10^{-8}$$

Correct option is (a)

50. 1-D, time independent, Schrodinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In region-I, $V(x) = 0$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\Rightarrow \quad \psi_1 = A\sin k_1 x + B\cos k_1 x \,(\text{oscillating nature}) \quad \left(\text{where, } k_1 = \sqrt{\frac{2mE}{\hbar^2}}\right)$$

In region-II, $V(x) = V_0 > E$

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi = 0$$

$$\Rightarrow \quad \psi_{II} = A e^{-k_2 x} + Be^{k_2 x} \quad \left(\text{where } k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}\right)$$

For the existance of bound state,

At $x \to \infty$, $\psi_{II}(0) = 0 \Rightarrow B = 0$

So, $\psi_{II} = A e^{-kx}$ (decaying nature)

Correct option is (c)

51.
$$\langle z \rangle = \langle r\cos\theta \rangle = \frac{|A|^2 \int_0^\infty \int_0^{2\pi} \int_0^{\pi} r\cos\theta\, e^{-\frac{2r}{a_0}} r^2 \sin\theta d\theta d\phi dr}{|A|^2 \int_0^{2\pi} \int_0^{2\pi} \int_0^{\pi} e^{-\frac{2r}{a_0}} r^2 \sin\theta dr d\theta d\phi}$$

$$= \frac{\int_0^\infty r^3 e^{-\frac{2r}{a_0}} dr \int_0^{\pi} \sin\theta\cos\theta d\theta \int_0^{2\pi} d\phi}{\int_0^\infty r^2 e^{-\frac{2r}{a_0}} dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi} = 0$$

Correct option is (d)

CAREER ENDEAVOUR

52. The degeneracy of n^{th} state of 3D harmonic oscillator is $\frac{1}{2}\times(n+1)(n+2)$

For $n = 2$ level, $g_2 = \frac{1}{2}\times 3\times 4 = 6$

Correct option is (b)

53. $$S_xS_yS_z = \frac{\hbar^3}{8}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \frac{i\hbar^3}{8}I$$

$$\langle S_xS_yS_z\rangle = \langle\psi|S_xS_yS_z|\psi\rangle = \frac{i\hbar^3}{8}\langle\psi|I|\psi\rangle = \frac{i\hbar^3}{8}$$

Correct option is (a)

54. The time-energy uncertainty principle

$$\Delta E\Delta t = \hbar \Rightarrow h\Delta\nu\Delta t = \hbar$$

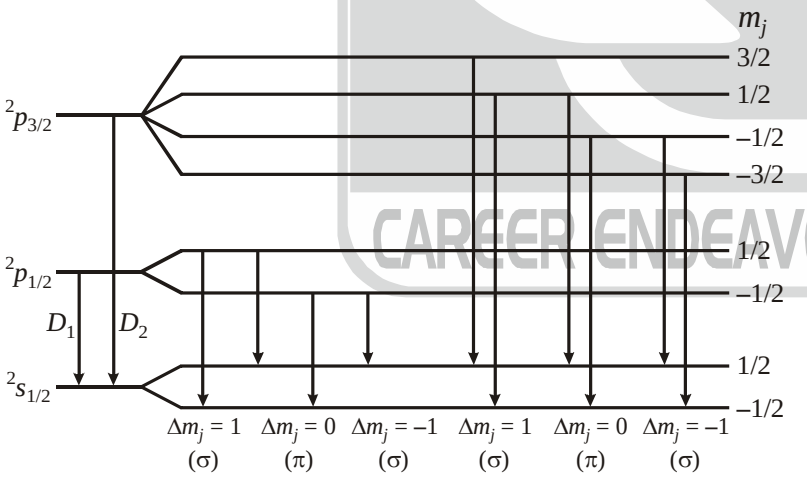
$$\Rightarrow \quad \Delta\nu\Delta t = \frac{1}{2\pi}$$

$$\Rightarrow \quad \Delta\nu = \frac{1}{2\pi\Delta t}$$

$$\therefore \quad \frac{\Delta\nu}{\nu} = \frac{\lambda}{2\pi\Delta tc} = \frac{600\times10^{-9}}{8\times10^{-9}\times3\times10^{8}\times2\pi} \simeq 10^{-8}$$

Correct option is (c)

55.



For sodium doublet lines, the total number of transitions from $^2p_{3/2}$ and $^2p_{1/2}$ levels to $^2s_{1/2}$ level is 10.

Hence, correct option is (d).

56. Here, in the given three level system of atoms, the level with energy E_2 has largest atoms N_2 amongest all three levels, which confirms that this is a metastable state. Similarly, E_1 and E_2 represents the ground and excited state repectively. Thus, the laser emission will be possible only between E_2 to E_1 levels.

Hence, correct option is (b).

57. The rotational Raman shift is given by $\left|\Delta\nu\right|_{\text{Raman}}^{\gamma} = 4B\left|J+\frac{3}{2}\right| = 6B, 10B, 14B, \ldots.$ where $J = 0, 1, 2, \ldots.$ and $B = \frac{h}{8\pi^2 Ic}$. Thus, it is confirmed that the typical Raman shift depends only on I.

Hence, correct option is (c).

58. The energy of rotational level is given by

$$E_J = BJ\left(J+1\right)$$

$$\Delta E = E_J - E_{J-1} = BJ\left(J+1\right) - B\left(J-1\right)\left(J-1+1\right) = B\left[J\left(J+1\right) - J\left(J-1\right)\right] = BJ\left(J+1-J+1\right) = 2BJ$$

$$\Rightarrow \quad \Delta E \propto J$$

Also, vibrational energy is $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$

Vibrational spacing,

$\Delta E = E_n - E\left(n-1\right) = \hbar\omega$ i.e. constant (independent of n)

Correct option is (a)

59. The typical wavelengths emitted by diatomic molecules in purely vibrational transitions lie in infrared region $(1\,\mu\text{m} - 10^2\,\mu\text{m})$. In purely rotational transitions, the typical wavelength lie in microwave region $(10^3\,\mu\text{m} - 10^4\,\mu\text{m})$.

Hence, correct option is (c).

60. Out of above options, only (b) $\Gamma_{\ell_1 s_2} \cdot \Gamma_{\ell_2 s_2} > \Gamma_{\ell_1 \ell_2} \cdot \Gamma_{s_1 s_2}$ is valid.

Hence, correct option is (b).

61. Given : $p(x)\,dx = ae^{-ax}dx$,

Probability that x lies between x_1 and x_2 $(x_2 > x_1)$ is

$$\int_{x_1}^{x_2} p\left(x\right)dx \Big/ \int_{0}^{\infty} p\left(x\right)dx = \int_{x_1}^{x_2} a\cdot e^{-ax}dx \Big/ \int_{0}^{\infty} a\cdot e^{-ax}dx = \left(e^{-ax_1} - e^{-ax_2}\right)$$

Correct option is (a).

62. Given : $Z_1 = \frac{kT}{\hbar\omega}$

The partition for N such independent oscillator is $Z_N = \left(Z_1\right)^N = \left(\frac{kT}{\hbar\omega}\right)^N$

The free energy is $F = -kT\ \ell n\ Z_N = -NkT\ \ell n\left(\frac{kT}{\hbar\omega}\right) = NkT\ \ell n\left(\frac{\hbar\omega}{kT}\right).$

Correct option is (c).

CAREER ENDEAVOUR

63. The possible configurations are

0	ε
••	
	••
•	•

The partition function hence is

$$z = e^{-0/kT} + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT} = 1 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$$

Correct option is (b).

64. Since the random walker takes steps to left or rightwith equal probability,

Probability of taking step to right = $\frac{1}{2}$

And probability of taking step to left = $\frac{1}{2}$

The random walker comes back to origin after N even step if he takes $\frac{N}{2}$ steps to the right and $\frac{N}{2}$ steps to the left.

Therefore, required probability = $^{N}C_{N/2}\left(\frac{1}{2}\right)^{N/2}\left(\frac{1}{2}\right)^{N/2} = \frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^{N}$

Correct option is (a)

65. The density of state in 3-D is

$$g(p)\delta p = \frac{4\pi p^2}{h^3}\delta p,$$

where $p = \sqrt{2mE}$ for free particle

$$\Rightarrow \quad \delta p = \frac{\sqrt{2m}}{2\sqrt{E}}\delta E$$

$$\therefore \quad g(E)\delta E = \frac{4\pi \cdot 2mE}{h^3}\frac{1}{2}\sqrt{\frac{2m}{E}}\delta E = \frac{4\sqrt{2}\pi}{h^3}m^{3/2}E^{1/2}\delta E$$

Since electron spin degeneracy is 2 so we should multiply g(E) by 2.

Therefore, $g(E)\delta E = \frac{8\pi \cdot 2mE}{h^3}\frac{1}{2}\sqrt{\frac{2m}{E}}\delta E$

For *N*-identical particles,

$$g(E)\delta E = \frac{8\sqrt{2}\pi}{h^3}m^{3/2}NE^{1/2}\delta E$$

$$\Rightarrow \quad g(E)\delta E \propto N\sqrt{E}\,\delta E$$

Correct option is (c).

CAREER ENDEAVOUR

66. $F(M) = F_0 + 2(T - T_C)M^2 + M^4$ ... (i)

For minima, $\frac{\partial F(M)}{\partial M} = 0 + 2(T - T_C)\cdot 2M + 4M^3 = 0$

$\Rightarrow \quad M\left((T - T_C) + M^2\right) = 0$

$\Rightarrow \quad M = 0$ or $M^2 = (T_C - T) \Rightarrow M = \pm\sqrt{(T_C - T)}$ ... (ii)

and $\frac{\partial^2 F(M)}{\partial M^2} = 0 + 4(T - T_C) + 12M^2 = 4(T - T_C) + 12M^2$

For, $M = 0$,

$$\frac{\partial^2 F(M)}{\partial M^2} = 4(T - T_C) > 0 \ \left(\text{for } T > T_C\right) \Rightarrow \frac{\partial^2 F(M)}{\partial M^2} > 0$$

Therefore, $F(M)$ have a minima at $M = 0$ for $T > T_C$

For $M = \pm\sqrt{(T_C - T)}$,

$$\frac{\partial^2 F(M)}{\partial M^2} = 4(T - T_C) + 12M^2 = 4(T - T_C) + 12(T_C - T)$$

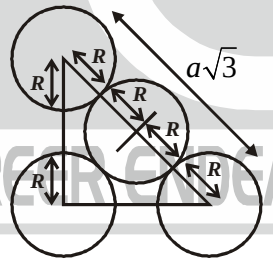
$$= -8T + 8T_C < 0 \quad (\text{for } T > T_C)$$

$\therefore$ $F(M)$ have a maxima at $M = \pm\sqrt{(T - T_C)}$ for $T > T_C$

$\therefore$ Function $F(M)$ has only one minima for $T > T_C$ at $M = 0$.

Hence, correct option is (b).

67. For body centered cubic (bcc) structure, the atoms will lie at corners as well as body centers of cube.



From the figure, $4R = a\sqrt{3} \Rightarrow R = \frac{a\sqrt{3}}{4} \Rightarrow a = \frac{4R}{\sqrt{3}}$.

Hence, correct option is (a).

68. Conductivity, $\sigma = \frac{J}{E} = \frac{nev_d}{E} = \frac{3\times10^{20}\times1.6\times10^{-19}\times100}{200} = 2.4\times10 = 24\ \Omega^{-1} - \text{m}^{-1}$.

Hence, correct option is (a).

69. The density of state of free electrons moving in a solid is proportional to square root of its energy,

$dn \propto \sqrt{E}$ ... (i)

For $E_1 = 0.1\,\text{eV}$, $dn_1 = 2.15\times10^{21}\ \text{eV}^{-1}\text{cm}^{-3}$

For $E_2 = 0.4\,\text{eV}$, $dn_2 = ?$

From equation (i), we have $\dfrac{dn_1}{dn_2} = \sqrt{\dfrac{E_1}{E_2}}$

$$\Rightarrow \quad \frac{2.15\times10^{21}}{dn_2} = \sqrt{\frac{0.1}{0.4}} = \frac{1}{2} \quad \Rightarrow \quad dn_2 = 2\times2.15\times10^{21}\ \text{eV}^{-1}\text{cm}^{-3}$$

$$\Rightarrow \quad \boxed{dn_2 = 4.3\times10^{21}\ \text{eV}^{-1}\text{cm}^{-3}}.$$

Hence, correct option is (d).

70. The concentration of electron in the conduction band

$$n = N_c\, e^{-(E_c - E_F)/k_BT} \qquad \text{... (1)}$$

Similarly, for hole $p = N_v\, e^{-(E_F - E_V)/k_BT}$ ... (2)

By multiplying equation (1) and (2)

$$np = N_cN_v\, e^{-(E_c - E_v)/k_BT}$$

$np = n_i^2$ by mass action law.

The intrinsic carrier concentration

$$n_i = \sqrt{(N_c\, N_v)}\exp\left(-\frac{E_g}{2k_BT}\right)$$

$$= 1.04\times10^{19}\times\exp\left[-\frac{0.66\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times300}\right] = 3\times10^{13}\ \text{cm}^{-3}.$$

Hence, correct option is (b).

71. The statement "It is paramagnetic below T_C" is not correct as $B = 0 \Rightarrow \mu_0(H + M) = 0 \Rightarrow H = -M$ $\Rightarrow \chi = -1$, which implies that superconductor is diamagnetic below T_C.

Hence, correct option is (d).

72. $$Q = k_\alpha\left(\frac{A}{A-4}\right) \Rightarrow 5.26 = k_\alpha\left(\frac{240}{236}\right) \Rightarrow k_\alpha = \frac{236}{240}\times5.26 = 5.17\ MeV$$

Correct option is (b)

73. If T be the threshold temperature of the reaction then,

$$\frac{3}{2}kT = \frac{4e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad T = \frac{2}{3}\cdot\frac{4\times1.44\times10^{-15}\times1.6\times10^{-13}}{1.38\times10^{-23}\times2.4\times1.44\times10^{-5}}$$

$$= 1.28\times10^{10}\,K$$

$$\left[\begin{array}{l} \dfrac{e^2}{4\pi\varepsilon_0} = 1.44\times10^{-15}\times1.6\times10^{-13}\ Jm \\ r = 2r_0A^{1/3} = 2\times1.2\times10^{-15}(3)^{1/3}\ m \\ k = 1.38\times10^{-23}\ kgm^2\sec^{-2}K^{-1} \end{array}\right]$$

Correct option is (a)

CAREER ENDEAVOUR

74. ${}_8O^{15}$ has Z = 8, N = 7 $\rightarrow$ $(^1s_{1/2})^2$ $(^1p_{3/2})^4$ $(^1p_{1/2})^1$

$\therefore$ J = $\frac{1}{2}$

Parity = $(-1)^1 = -1$ (odd)

Angular momentum and parity = $\frac{1^-}{2}$

Correct option is (d)

75. The activity A of a sample decays exponentially with time, $A_t = A_0 e^{-\lambda t}$

$\Rightarrow \log A_t = \log A_0 - \lambda t(2.303)$

It will be a straight line (figure (a)) between $\log A_t$ and t with a negative slope.

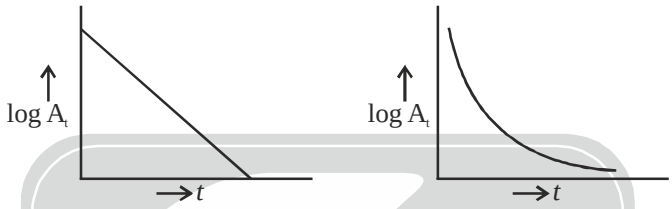


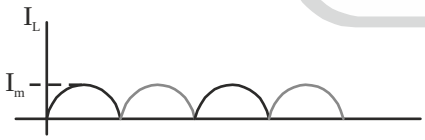
Figure (a) Figure (b)

But for complex decay i.e. for mixture of two half lives, we get the graph between $\log A_t$ vs. t as shown in figure (b).

Correct option is (c)

76. Given circuit consist of two diode its a full wave rectifier

Number of Diode	Rectifier
1	HWR half wave rectifier
2	FWR full wave rectifier
4	BWR Bridge wave



$R_L = 1k\Omega$

$V_{2_{max}} = 10V$

$I_{L_{max}} = 10\ mA$

$I_{DC} = I_{average} = \frac{2\, I_{max}}{\pi} = \frac{20}{\pi} mA$ is average value

Correct option is (a)

CAREER ENDEAVOUR

77. $f = B(A+B) + A(\bar{B}+A)$

$f = AB + B + A\bar{B} + A$

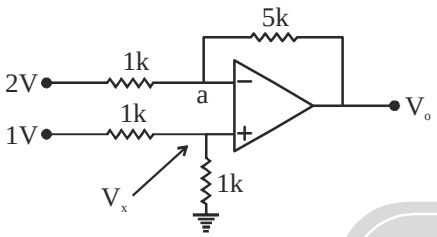
$f = A + B$

1 OR- gate needed

Note: $1 + A = 1 \quad \Rightarrow \quad 1 + \bar{B} = 1$ Axioms

Correct option is (c)

78.



$V_x = \frac{1\times1}{1+1} = \frac{1}{2}$ Volt = 0.5 Volt

As ideal op-amp has infinite input resistance. So, there will be no current pass through op-amp i.e. $V_a = V_x$ due to virtual short.

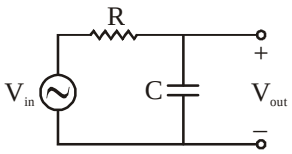
KCL at (a)

$$\frac{0.5-2}{1} + \frac{0.5-V_0}{5} = 0$$

$$\frac{-3}{2} = \frac{V_0 - 0.5}{5} \quad \Rightarrow \quad V_0 = -7 \text{ Volt}$$

Correct option is (a)

79.



By applying voltage divider rule.

$$V_0(s) = \frac{V_i(s) \times \frac{1}{CS}}{R + \frac{1}{CS}}$$

$\frac{V_0(s)}{V_i(s)} = \frac{1}{1+RCS}$ its a low pass filter OR integrator.

Note: If R and C are interchanged circuit will high pass filter OR differenciator.

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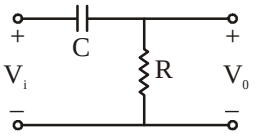


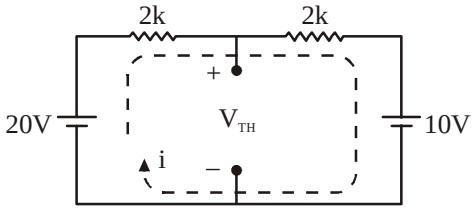
$$\frac{V_0(s)}{V_i(s)} = \frac{RCS}{1+RCS}$$

Correct option is (d)

80. Calculation of Thevenin votlage across $R_L = 1k\Omega$

$\Rightarrow$ Thevenin voltage is also referred as open circuit voltage

Open the load resistance of the circuit for the calculation of V_{Th} and R_{Th}.



$$i = \frac{20-10}{4} = 2.5\ mA$$

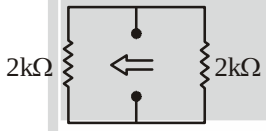
$\Rightarrow$ Apply KVL

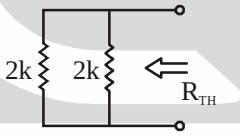
$20 = 2 \times i + V_{TH}$

$v_{TH} = 15$ Volt

$\Rightarrow$ For R_{TH} (Thevenin resistance) calculation

Condition $\begin{cases} \text{Voltage source is short circuited} \\ \text{Current source is open circuited} \end{cases}$





$$\frac{2\times 2}{2+2} = R_{TH} = 1k\Omega$$

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Option (a) is correct

81. **(a, b)**. Here option (a) is correct

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

Let the trial solution be $y = C.e^{mx}$

$\Rightarrow \; e^{mx}\left(m^2 - 2m + 1\right) = 0$

$\Rightarrow \; (m-1)^2 = 0 \qquad \left(\text{since, } e^{mx} \neq 0\right)$

$\Rightarrow \qquad m = 1, 1$

The solution will be $y = (c_1 + c_2 x)e^{mx} = (c_1 + c_2 x)e^{x} = c_1\left(e^{x}\right) + c_2\left(x \cdot e^{x}\right)$

Correct option is (a) and (b)

82. (a) We have the Lagrangian,

$$L = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right). - \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right) + m\omega_0^2\mu x_1 x_2$$

The equation of motion are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = 0$$

$$\Rightarrow \quad m\ddot{x}_1 + m\omega_0^2 x_1 - m\omega_0^2\mu x_2 = 0$$

$$\Rightarrow \quad \ddot{x}_1 + \omega_0^2 x_1 - \omega_0^2\,\mu\, x_2 = 0$$

and $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) - \dfrac{\partial L}{\partial x_2} = 0$

$$\Rightarrow \quad m\ddot{x}_2 + m\omega_0^2 x_2 - m\omega_0^2\mu x_1 = 0$$

$$\Rightarrow \quad \ddot{x}_2 + \omega_0^2\, x_2 - \omega_0^2\,\mu\, x_1 = 0$$

Correct option is ( )

82. (b) $\hat{T} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}, \hat{V} = \begin{pmatrix} m\omega_0^2 & -m\omega_0^2\mu \\ -m\omega_0^2\mu & m\omega_0^2 \end{pmatrix}$

$$\left|\hat{V} - \omega^2 T\right| = 0$$

$$\Rightarrow \quad \begin{vmatrix} m\omega_0^2 - m^2\omega & -m\omega_0^2\mu \\ -m\omega_0^2\mu & m\omega_0^2 - \omega^2 m \end{vmatrix} = 0$$

$$\Rightarrow \quad \left(m\omega_0^2 - \omega^2 m\right)^2 - m^2\omega_0^4\mu^2 = 0$$

$$\Rightarrow \quad \left(m\omega_0^2 - \omega^2 m\right) = \pm m\omega_0^2\mu$$

$$\Rightarrow \quad \omega^2 = \left(\omega_0^2 \pm \omega_0^2\mu\right)$$

$$\Rightarrow \quad \omega = \omega_0\sqrt{1+\mu}, \omega_0\sqrt{1-\mu}$$

Correct option is (b)

83. (a) Surface current density = $\sigma v = \sigma\,\omega\, R$

(b) We know, $\vec{B} = \left(\vec{\nabla} \times \vec{A}\right)$

$$\Rightarrow \quad \int \vec{B}\cdot d\vec{S} = \int\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S}$$

$$\Rightarrow \quad \mu_0 I\pi r^2 = \oint \vec{A}\cdot d\vec{\ell} = A\cdot 2\pi r$$

$$\Rightarrow \quad A = \frac{1}{2}\mu_0 Ir = \frac{1}{2}\mu_0 R\sigma\omega r$$

Correct option is (c)

CAREER ENDEAVOUR

84. (a) The scattered wave is not spherically symmetric, its amplitude depends on the direction (θ, ϕ) along which is scattered. So, scattered wave is given by

$$\psi_{sca}(r) = Af(\theta, \phi)\frac{e^{i\vec{k}\cdot\vec{r}}}{r}$$

Correct option is (a)

(b) The differential scattering cross-section $\sigma(\theta)$ in C.M. frame is given by $\sigma(\theta) = |f(\theta)|^2$.

Correct option is (c)

85. (a) We have, $\rho = 11.35\ \text{g/cm}^3$ and atomic weight $M = 207.2$ amu

Now, the volume of the lead can be calculated by

$$\text{Volume}\ (V) = \frac{\text{mass}\ (M)}{\text{density}\ (\rho)} = \frac{207.2\ \text{amu}}{11.35\ \text{g/cm}^3}$$

There are 6.023×10^{23} atoms present in 207.2 amu of lead and the volume is $\frac{207.2}{11.35}\ \frac{\text{amu}}{\text{g/cm}^3}$

Therefore, number of atoms per $\text{cm}^3 = \frac{6.023\times10^{23}\times11.35}{207.2} = 0.3299\times10^{23}\ \text{atom/cm}^3$

$$= 3.3\times10^{22}\ \text{atom/cm}^3$$

Hence, correct option is (b).

(b) Number of atoms per cm^3 $(n) = 3.3\times10^{22}$ atoms/cm^3

Energy of vancancy formation $(E_V) = 0.55$ eV and temperature $(T) = 500$ K

The number of vacancies per cm^3 $= n_V = n\exp\left(-\frac{E_V}{k_BT}\right) = 3.3\times10^{22}\exp\left(\frac{-0.55}{8.62\times10^{-5}\times500}\right)$

$$= 3.3\times10^{22}\exp(-12.75) = 9.577\times10^{16}\ \text{vacancies/cm}^3$$

Hence, correct option is (d).

CAREER ENDEAVOUR

OBJECTIVE QUESTION

1. Let λ_1, λ_2 and λ_3 are eigen values of 3×3 matrix.

Trace of the matrix = sum of eigen values of the matrix

Therefore, $\lambda_1+\lambda_2+\lambda_3=2$

$\Rightarrow 1+2+\lambda_3=2 \Rightarrow \lambda_3=-1$

Correct option is (a)

2. According to Stokes Theorem,

$$\oint_C \vec{A}\cdot d\vec{\ell} = \iint_S \left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S}$$

Since, $\vec{A}$ is uniform. So, $\left(\vec{\nabla}\times\vec{A}\right)=0$

Therefore, $\oint_C \vec{A}\cdot d\vec{\ell}=0$

Correct option is (a)

3. Hamiltonian $H=\dfrac{\left(\vec{p}-\dfrac{q}{c}\vec{A}\right)^2}{2m}+q\phi$ is correct $\left[\because\ \vec{p}\,'=\left(\vec{p}-\dfrac{q\vec{A}}{c}\right)\right.$ for electromagnetic fields $\left.\right]$

Hence correct answer is (b)

4. For positive energy eccentricity becomes greater than one. Therefore path is hyperbolic

Hence correct answer is (d)

5. $E_x=E'_x \qquad B_x=B'_x$

$E_y=\gamma\left(E'_y+vB'_z\right) \qquad B_y=\gamma\left(B'_y-vE'_z/c^2\right)$

$E_z=\gamma\left(E'_z-vB'_y\right) \qquad B_z=\gamma\left(B'_z+vE'_y/c^2\right)$

Now, $E^2-c^2B^2$

$\Rightarrow E_x^2+E_y^2+E_z^2-c^2\left(B_x^2+B_y^2+B_z^2\right)$

$\Rightarrow E'^2_x-c^2B'^2_x+\gamma^2\left(E'^2_y+E'^2_z-B'^2_y c^2-B'^2_z c^2\right)$

$\Rightarrow E'^2_x-c^2B'^2_x+\gamma^2[E'^2_y+2E'_yB'_zv+v^2B'^2_z+E'^2_z-2E'_zvB'_y+v^2B'^2_y$

$-c^2B'^2_y+2B'_yE'_zv-\dfrac{v^2E'^2_z}{c^2}-B'^2_zc^2-2B'_zE'_yv-\dfrac{v^2E'^2_y}{c^2}]$

$\Rightarrow E'^2_x-c^2B'^2_x+\gamma^2\left(1-v^2/c^2\right)E'^2_y+\gamma^2\left(1-v^2/c^2\right)E'^2_z+\gamma^2\left(v^2-c^2\right)B'^2_z+\gamma^2\left(v^2-b^2\right)B'^2_y$

$\Rightarrow E'^2_x+E'^2_y+E'^2_z-c^2\left(B'^2_x+B'^2_y+B'^2_z\right)$

$\Rightarrow E'^2-c^2B'^2$

Correct option is (b)

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6. We know that $V(r) = \int_r^\infty \vec{E} \cdot d\vec{r} = \int_r^R \vec{E}_{in} \cdot d\vec{r} + \int_R^\infty \vec{E}_{out} \cdot d\vec{r}$

$\therefore$ $\vec{E}_{out}$ depends on total charge. Therefore, $V(r)$ also depends on total charge.

Correct option is (b)

7. Since, particle is moving along +*x* direction. The normalized wave function wtihin length *L* is

$$\psi = \frac{1}{\sqrt{L}} e^{i\frac{p}{\hbar}x}$$

[Since, wave function is just a plane wave $e^{i\vec{k}\cdot\vec{x}}$, where $\vec{k} = \frac{\vec{p}}{\hbar}$ normalised over length L]

Correct option is (d)

8. $\sigma_x\sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i\sigma_z$

Correct option is (c)

9. Given : $F = -\frac{a}{3}VT^{-4}$

The chemical potential is $\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T} = 0$

Correct option is (a).

10. The particles are distinguishable in Maxwell-Boltzmann statistics and hence their wave functions do not overlap. So, the state of the combined two particle system, will be

$\psi_{ns}(r_1, r_2) = \phi_n(r_1)\,\phi_s(r_2)$

Correct option is (d).

11. For radiation (X-ray) $E = eV = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{eV} \Rightarrow \lambda \propto \frac{1}{V}$.

Hence, correct option is (c).

12. The principal series of spectral lines of lithium is obtained by transition between *nP* to *2S* transitions (for $n > 2$).

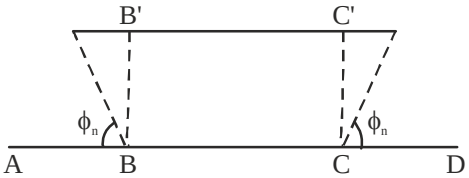
Hence, correct option is (c).

13. Silicon is an indirect band gap semiconductor.

Hence, correct option is (c).

14. $\cos\varphi_n = \left(\frac{N}{2}\right)$, where *N* is an integer

And the number of possible fold axis, $n = \left(\frac{360^o}{\varphi_n}\right)$



Since, $1 < \cos\varphi_n < 1$

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So, possible φ_n are 360°, 180°, 120°, 90°, 60°

Corresponding the number of fold axis are 1, 2, 3, 4, 6

So, five-fold axis is not possible.

Correct option is (d)

15. **Correct option is (c)**

16. Since, Thorium series is 4n series.

Uranium series is (4n + 2) series.
Actinium series is (4n + 3) series and Neptunium series is (4n + 1) series.

$\because$ ^{209}Bi have mass number (A) = 209. $\left(\because\ 209 = 4\times 52 + 1\right)$

Therefore, it belongs to Neptunium series.

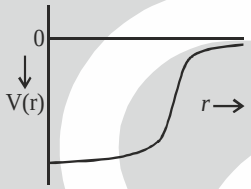
Correct option is (b)

17. Since average binding energy per nucleon i.e. B/A for all the nuclei $\approx$ 8 MeV. (i.e. of order 10 MeV)

Correct option is (d)

18. Interaction potential can be given as

$$V(r) = -\frac{a}{r} + br$$

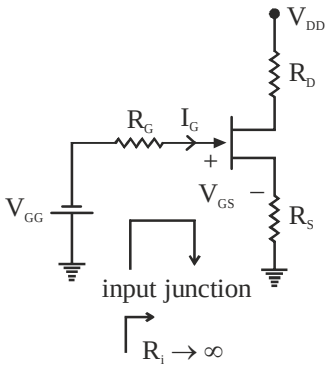


where, $-\frac{a}{r} \rightarrow$ because nuclear forces are short range and decreases with decreasing distance,

$br \rightarrow$ because nuclear forces will be of repulsion type at long range.

Correct option is (c)

19.



Input resistance $R_i = \left|\frac{V_{GS}}{I_G}\right|$

The larger input resistance of JFET is due to

(1) Reverse biasing gate to source junction

(2) Neglegible gate current I_G(nA)

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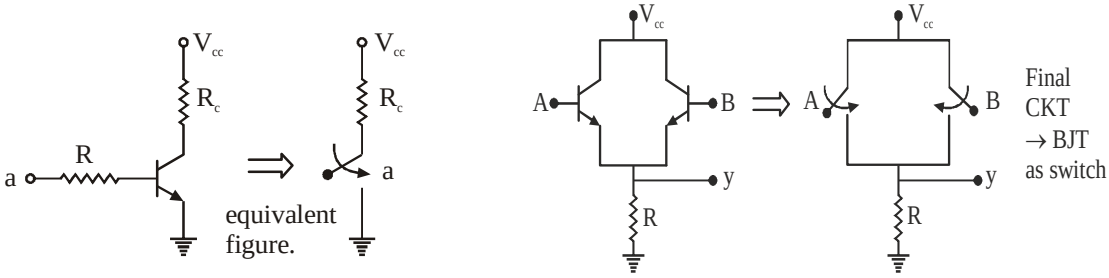
$R_{i_{FET}} \to \infty$

- Input junction of FET is always reverse bias
- $I_g \to (nA)$ practically

$R_{i_{\text{Practically}}} \left(10^6 - 10^8\right)\Omega$

So, **correct option is (b)**

20. BJT as Digital switch.



Truth Table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Then, Y = A + B. So, given circuit is OR gate.

Note: Switch closed current will flow y will have output $y = 0$, when both switches are open.

Correct option is (a)

21. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$

Correct option is (a)

22. $\oint_s \frac{\vec{r} \cdot d\vec{s}}{r^3} = \int_V \vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) dV$ (using divergence theorem)

Since the closed surface encloses the origin i.e. $\vec{r} = 0$

So, $\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) = 4\pi\, \delta^3(\vec{r})$

So, $\int_V \vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) dV = 4\pi \int_V \delta^3(\vec{r})\, dV = 4\pi$

Correct option is (c)

23. $f(z) = \frac{e^{2z}}{(z+1)^4}$ has a pole of order 4 at z = –1.

Residue of $f(z)$ at z = –1

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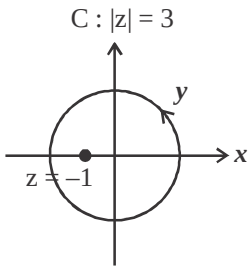
$$= \frac{1}{(4-1)!}\frac{d^3}{dz^3}\left[(z+1)^4 \frac{e^{2z}}{(z+1)^4}\right]_{z=-1}$$

$$= \frac{1}{3!}\left[8e^{2z}\right]_{z=-1} = \frac{4}{3}e^{-2}$$



So, $I = 2\pi i\left(\frac{4}{3}e^{-2}\right) = \frac{8\pi i}{3}e^{-2}$

Correct option is (a)

24. k^{th} Fourier component of $\delta(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\delta(x)e^{-ikx}dx = \frac{1}{\sqrt{2\pi}}$

Correct option is (c)

25. We know that, $\tau = \vec{\mu}\times\vec{B}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mu_x & \mu_y & \mu_z \\ 0 & 0 & B_0 \end{vmatrix} = \hat{i}(\mu_y B_0) - \hat{j}(\mu_x B_0) + 0\hat{k}$$

$\tau_x = \mu_y B_0, \tau_y = -\mu_x B_0, \tau_z = 0$

We know, torque = rate of change of angular momentum,

$$\frac{d}{dt}\left(L_x\hat{i} + L_y\hat{j} + L_z\hat{k}\right) = \tau_x\hat{i} + \tau_y\hat{j} + \tau_z\hat{k}$$

$$\frac{d}{dt}(L_x) = \mu_y B_0 \;;\; \frac{d}{dt}(L_y) = -\mu_x B_0$$

$$\mu = \gamma L;\; L = \frac{\mu}{\gamma}$$

$$\frac{d(\mu_x)}{dt} = \gamma\mu_y B_0;\; \frac{d(\mu_y)}{dt} = -\gamma\mu_x B_0$$

$$\frac{d^2\mu_x}{dt} = \gamma B_0(-\gamma\mu_x B_0) \Rightarrow \frac{d^2\mu_x}{dt^2} = -\gamma^2 B_0^2\mu_x \Rightarrow \frac{d^2\mu_x}{dt^2} + \gamma^2 B_0^2\mu_x = 0$$

Correct option is (c)

26. $V(r) = \alpha r^2 = \alpha r^n \Rightarrow n = 2$

We know from Virial theorem

If $V = \alpha r^n$ then $\langle T\rangle = \frac{n}{2}\langle V\rangle$

$$\langle T\rangle = \frac{2}{2}\langle V\rangle \quad \therefore \langle T\rangle = \langle V\rangle$$

Hence correct answer is (a)

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27. $\vec{r} = 5\hat{i} - 2t^2\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = -4t\hat{j}$$

$$\vec{p} = m\vec{v} = 2\left(-4t\hat{j}\right) = -8t\hat{j}$$

$$\vec{L} = \vec{r} \times \vec{p} = \left(5\hat{i} - 2t^2\hat{j}\right) \times \left(-8t\hat{j}\right)$$

$$= -40t\hat{k} = -40 \times 2\hat{k} = -80\hat{k}$$

Hence correct answer is (b)

28. $I_{xy} = -\sum m_i x_i y_i$

$$= -\left[m(1)(1) + m(-1)(-1) + 2m(-1)(1) + 2m(1)(-1)\right]$$

$$= -\left[-2m\right] = 2m$$

Hence correct answer is (c)

29. For canonical transformation

Poisson Bracket $[Q, P]_{q,p} = 1$

If $Q = p, P = -q$

$$[Q, P]_{q,p} = \frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q} = 0 - (1) \times (-1) = 1$$

It $Q = p, P = q$

$$[Q, P]_{q,p} = \frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q} = 0 - (1) \times (1) = -1$$

$\therefore$ only (i) is canonical transformation

Hence correct answer is (b)

30. $m = \frac{2m_0}{\sqrt{3}}$

$$\frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{2m_0}{\sqrt{3}} \Rightarrow \sqrt{1 - v^2/c^2} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad v = \frac{c}{2}$$

momentum $p = mv = \frac{2m_0}{\sqrt{3}} \cdot \frac{c}{2} = \frac{m_0 c}{\sqrt{3}}$

Hence correct answer is (d)

31. The total charge , $Q = \sum q_i = (q + q - 2q) = 0$

So, the dipole moment of this charge distribution does not depend on choice of origin

Therefore, net dipole moment,

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$$\vec{p} = \sum q_i \vec{r}_i = q\left(-a\hat{j} + a\hat{k}\right) + q\left(a\hat{j} + a\hat{k}\right) - 2q\left(-a\hat{k}\right)$$

$$= -qa\hat{j} + qa\hat{k} + qa\hat{j} + qa\hat{k} + 2aq\hat{k} = 4aq\hat{k}$$

Correct option is (a)

32. According to Ampere's law,

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int \vec{J} \cdot d\vec{s}$$

$$\Rightarrow \quad B \cdot 2\pi d = \mu_0 J_0 \int_0^d \int_0^{2\pi} r dr d\theta$$

$$\Rightarrow \quad B \cdot 2\pi d = \mu_0 J_0 \frac{d^3}{3} 2\pi$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 J_0 d^2}{3} \hat{\phi}$$

Correct option is (c)

33. $\vec{A} = -y\hat{i} + 2x\hat{j}$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & 2x & 0 \end{vmatrix} = (2+1)\hat{k} = 3\hat{k}$$

Correct option is (b)

34. We know the boundary condition, $D_2 \cdot \hat{n} - D_1 \cdot \hat{n} = \sigma$

$D_2 \cdot \hat{n} = D_1 \cdot \hat{n}$ (since surface is charge free)

Or, $D_2 \cos\theta_2 = D_1 \cos\theta_1 \quad \Rightarrow \varepsilon_2 E_2 \cos\theta_2 = \varepsilon_1 E_1 \cos\theta_1$ ... (i)

And $E_{1\tau} = E_{2\tau}$

$\Rightarrow \quad E_2 \sin\theta_2 = E_1 \sin\theta_1$ ... (ii)

Therefore, (ii) divided by (i), we get

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

Correct option is (a)

35. The Maxwell's equations are

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}; \ \vec{\nabla} \cdot \vec{B} = 0; \ \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

For steady state current flow $\frac{\partial \vec{E}}{\partial t} = 0$

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$\therefore \quad \nabla \times B = \mu_0 J$ and since J is time independent $\therefore \ \dfrac{\partial B}{\partial t} = 0$

Therefore, the Maxwell's equation for time independent charge density and current density are

$\vec{\nabla}.\vec{E} = \rho / \varepsilon_0$
$\vec{\nabla}.\vec{B} = 0$
$\vec{\nabla} \times \vec{E} = 0$
$\vec{\nabla} \times \vec{B} = \mu_0 \hat{J}$

Correct option is (b)

36. Observer will see linear polarized light when he will look from *x*-direction. But if observer looks from *z*-axis then he will see electric field vary as a circular polarization light with time.

Correct option is (c)

37. Probability current density,

$$|J| = \left|\frac{i\hbar}{2m}\left(\psi \nabla \psi^* - \psi^* \nabla \psi\right)\right| = \left|\frac{i\hbar}{2m}\left(|A|^2(-ik) - (ik)|A|^2\right)\right| = |A|^2 \frac{\hbar k}{m}$$

Correct option is (a)

38. $$\psi(x) = \frac{3}{\sqrt{14}}\psi_0(x) - \frac{2}{\sqrt{14}}\psi_1(x) + \frac{1}{\sqrt{14}}\psi_2(x) = C_0\psi_0(x) + C_1\psi_1(x) + C_2\psi_2(x)$$

The probability of finding the oscillator in ground state, is

$$P_0 = \left|\langle \psi_0 | \psi \rangle\right|^2 = |C_0|^2 = \frac{9}{14}$$

Correct option is (c)

39. $$\psi(x, t=0) = \frac{2}{\sqrt{5}}\psi_0(x) - \frac{1}{\sqrt{5}}\psi_1(x)$$

Therefore, wave function at time *t* is

$$\psi(x,t) = \frac{2}{\sqrt{5}}\psi_0(x)\, e^{-\frac{iE_0 t}{\hbar}} - \frac{1}{\sqrt{5}} e^{-\frac{iE_1 t}{\hbar}}$$

Correct option is (d)

40. $$[L_x, y] = \left[\left(yp_z - zp_y\right), y\right] = [y, y]\, p_z + y[p_z, y] - [z, y]\, p_y - z\left[p_y, y\right]$$

$$= 0 + 0 - 0 - z(-i\hbar) = i\hbar z$$

$\therefore \quad [L_x, y] = i\hbar z$

Correct option is (d)

41. The probability of finding the electron at $r = 0$ is zero for 1s, 2s, 2p

Hence, (a) , (c) and (d) are correct.

42.

A		B		A	B
N, P T_1	+	N, P T_2	$\Rightarrow$		

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The final temperature would be $T_f = \frac{T_1 + T_2}{2}$ as both the vessels have all same parameters.

Now change in entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = C_v \int_{T_1}^{T_f} \frac{dT}{T} + C_v \int_{T_2}^{T_f} \frac{dT}{T} = C_v \ell n \left(\frac{T_f^2}{T_1 T_2} \right)$$

where $C_v = \frac{3}{2} N k_B$ (for monoatomic gas).

$$\Rightarrow \quad \Delta S = \frac{3}{2} N k_B \left[\ell n \frac{T_f^2}{T_1 T_2} \right]$$

$$= \frac{3}{2} N k_B \left[\ell n \left\{ \frac{(T_1 + T_2)^2}{4 T_1 T_2} \right\} \right]$$

Correct option is (c).

43. Since the gas is expanded adiabatically into vaccum, it is a case of free expansion. Hence, internal energy does not change

$$\Rightarrow \quad E_1 = E_2 \quad \Rightarrow \quad \frac{3}{2} nRT_1 - \frac{a}{V_1} = \frac{3}{2} nRT_2 - \frac{a}{V_2}$$

$$\Rightarrow \quad \frac{3}{2} nRT_2 = \frac{3}{2} nRT_1 + a \left(\frac{1}{V_2} - \frac{1}{V_2} \right) \quad \Rightarrow \quad T_2 = T_1 + \frac{2a}{3nR} \left(\frac{1}{V_2} - \frac{1}{V_2} \right)$$

So for one mole, $T_2 = T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_2} \right)$

Correct option is (c).

44. The Hamiltonian for one-dimension harmonic oscillator is

$$H = \frac{p_x^2}{2m} + \frac{1}{2} kx^2$$

Since the Hamiltonian has two quadratic terms and as per as law of equipartition of energy, each contributes $\frac{1}{2} k_B T$.

So, mean internal energy = $2 \times \frac{1}{2} k_B T = k_B T$

Correct option is (b).

45. Number of ways of choosing n-atoms out of N-atoms $= {}^{N}C_n$

and number of ways for putting n-atoms in N'-sites $= {}^{N'}C_n$.

Therefore, total number of microstates, $\Omega = {}^{N}C_n \times {}^{N'}C_n = \frac{N!}{n!(N-n)!} \frac{N'!}{n!(N'-n)!}$

Correct option is (d).

CAREER ENDEAVOUR

46. The energy of a ion is

$$E = -\vec{\mu}\cdot\vec{H} = \begin{cases} \mu H & \text{for spin} - \frac{1}{2} \text{ ion} \\ -\mu H & \text{for spin} + \frac{1}{2} \text{ ion} \end{cases}$$

So the partition of an ion is $Z_1 = e^{-\frac{\mu H}{k_B T}} + e^{\frac{\mu H}{k_B T}} = 2\cosh\left(\frac{\mu H}{k_B T}\right)$

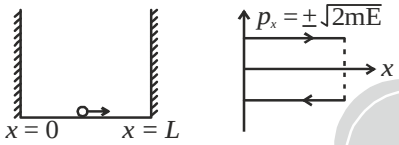
The partition for N such non-interacting ions is $Z_N = (Z_1)^N = \left[2\cosh\left(\frac{\mu H}{k_B T}\right)\right]^N$

The Helmholtz free energy is

$$F = -k_B T\ \ell n\ Z_N = -N\,k_B T\ \ell n\left[2\cosh\left(\frac{\mu H}{k_B T}\right)\right]$$

Correct option is (b).

47.



There is no force on the particle except due to collision.
Therefore momentum of particle remains constant and changes its direction due to collision.
Hence correct answer is (a)

48. Microwave region $\left(10^{-4}\,m - 10^{-2}\,m\right)$ of the electromagnetic spectrum corresponds to the pure rotational spectra.

Energy of the level (in terms of wave number)

$$E(J) = B\,J(J+1),\ J = 0, 1, 2, \ldots$$

Selection rules

$$\Delta J = \pm 1,$$

Therefore, wave number of emitted or absorbed radiation is

$$\nu = 2B(J+1)$$

Putting $J = 0, 1, 2, 3$

$\nu = 2B, 4B, 6B$ is the position of spectral lines.

Separation between spectral lines

$$\Delta\nu = 4B - 2B = 2B = \text{constant}$$

Given:

$$2B = 0.7143\,cm^{-1} = 0.7143\times10^2\,m^{-1}$$

$$\Rightarrow\quad B = \frac{0.7143}{2}\times10^2\ m^{-1} = 0.3571\times10^2 m^{-1}$$

Since, $B = \dfrac{h}{8\pi^2 Ic}$

$$\Rightarrow \text{moment of inertia } I = \frac{h}{8\pi^2 BC} = \frac{6.63\times10^{-34}}{8\times(3.14)^2\times0.3571\times10^2\times3\times10^8} = 7.8\times10^{-46}$$

Hence correct option is (d)

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49. For He-Ne laser the laser transition takes place in Ne only.

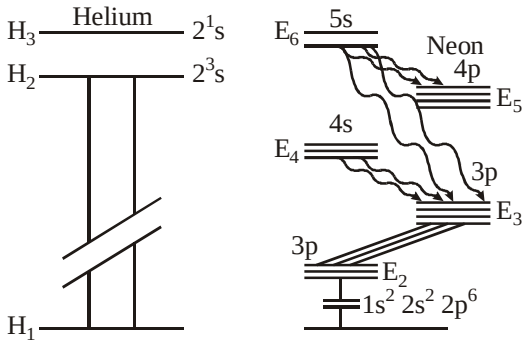


Figure : (Relevant energy level of the He-Ne laser)

According to above diagram, the electronic transition $6s \rightarrow 5p$ is not responsible for LASER action in He-Ne laser.

Hence, correct option is (a).

50. For $n = 2$ level, the allowed values of n, n_1, n_2 and m_ℓ are given in following table :

n	n_2	n_1	$(n_2 - n_1)$	$2(n_2 - n_1)$	$m_\ell = [n - n_2 - n_1 - 1]$
2	1	1	0	0	–1
	1	0	1	2	0
	0	1	–1	–2	0
	0	0	0	0	1

The Stark shift of hydrogen $n = 2$ energy level is shown below :

$$n = 2 \begin{cases} n_2 - n_1 & n(n_2 - n_1) & & m_\ell \\ 1 & 2 & \text{———} & 0 \\ 0 & 0 & \text{———} & -1, +1 \\ -1 & -2 & \text{———} & 0 \end{cases}$$

From above figure, it is clear that two levels corresponding to $m_\ell = -1$ and $m_\ell = +1$ falls together i.e. application of electric field partially lifts the degeneracy of $n = 2$ level and splits $n = 2$ into three levels.

Hence, correct option is (b).

51. The selection rule for the quantum number F in hyperfine transitions : $\Delta F = 0, \pm 1$ but $F = 0 \not\leftrightarrow F = 0$.

Hence, correct option is (c).

CAREER ENDEAVOUR

52.

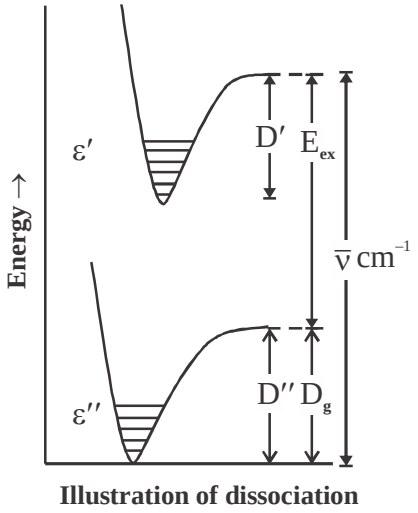


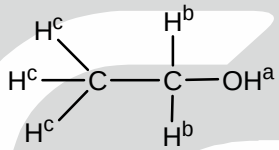
Illustration of dissociation

From the above figure $\bar{\nu}\text{cm}^{-1} = D_g + E_{ex}\text{cm}^{-1} \Rightarrow D_g = \left(\bar{\nu} - E_{ex}\right)\text{cm}^{-1}$

Therefore, dissociation energy of the homonuclear molecule will be $(\bar{\nu} - E_{ex.})$.

Hence, correct option is (c).

53. In the NMR spectroscopy of CH_3CH_2OH three bunches are observed



Three type of proton exist in this molecule so multiplicity (2NI + 1) for every proton

For–CH_3 : $3H^c \rightarrow (2NI+1)$

where, N is number of near by proton (H)

I = spin of proton (H)

$\left(2\times 2\times \frac{1}{2}+1\right) = (3)$ triplet

For –CH_2 group : $2H^b \rightarrow (2NI+1)\left(2\times 3\times \frac{1}{2}+1\right) = 4$ (quartet)

For –OH group : $^{a}H \rightarrow$ alcoholic hydrogen broad signal

So, Due to –CH_2 group quartet are obtain four time are observed

Hence, correct option is (d).

54. Effective mass $(m^*) = \dfrac{\hbar^2}{\left(\dfrac{d^2E}{dk^2}\right)}$ ... (i)

Here, $E(\vec{k}) = Ak^2 + Bk^4$

$\dfrac{dE}{dk} = 2Ak + 4Bk^3$; $\dfrac{d^2E}{dk^2} = 2A + 12Bk^2$

At $k = k_0$, $\left.\dfrac{d^2E}{dk^2}\right|_{k=k_0} = 2A + 12Bk_0^2$ ... (ii)

CAREER ENDEAVOUR

From equation (ii), equation (i) implies $m^* = \dfrac{\hbar^2}{(2A + 12Bk_0^2)}$.

Hence, correct option is (c).

55. The first BZ is a square of side $\left(\dfrac{2\pi}{a}\right)$ and second BZ is a square of side $\left(\dfrac{4\pi}{a}\right)$.

$\therefore$ Area of first BZ $= \left(\dfrac{2\pi}{a}\right)^2 = \dfrac{4\pi^2}{a^2}$ ... (i)

and area of second BZ = $4 \times \dfrac{1}{2} \times \left(\dfrac{2\pi}{a}\right) \cdot \left(\dfrac{\pi}{a}\right) = \dfrac{4\pi^2}{a^2}$ ... (ii)

Therefore, area of second BZ = area of first BZ.
Hence, correct option is (d).

56. The highest occupied energy band of the crystal is half filled.
Hence, correct option is (c).

57. The plasma frequency (ω_p) is directly proportional to density of free electrons

i.e., $\omega_p \propto \sqrt{N}$ ... (i)

Initially the electron density, $N_1 = 10^{28}$ electrons/m^3

Finally the electron density, $N_2 = 10^{26}$ electrons/m^3 and initial plasma frequency $(\omega_p)_1 = 5.7 \times 10^{15}$ Hz.

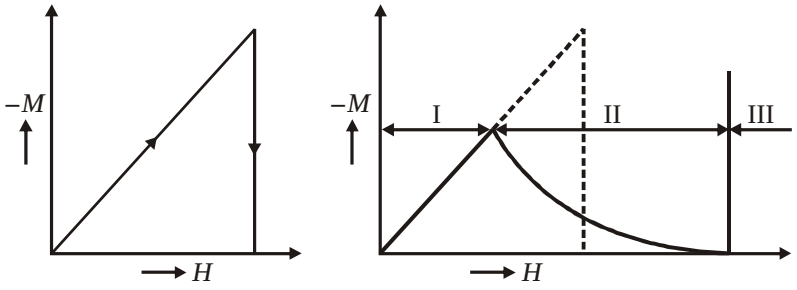
Let the final plasma frequency $(\omega_p)_2 = x$

$$\therefore \quad \frac{(\omega_p)_1}{(\omega_p)_2} = \sqrt{\frac{N_1}{N_2}} = \sqrt{\frac{10^{28}}{10^{26}}} \quad \Rightarrow \quad \frac{5.7 \times 10^{15}}{x} = \sqrt{\frac{10^{28}}{10^{26}}}$$

$$\Rightarrow \quad x = 5.7 \times 10^{15} \times \sqrt{\frac{10^{26}}{10^{28}}} = 5.7 \times 10^{15} \times \left(\frac{1}{10}\right) \quad \Rightarrow \quad x = 5.7 \times 10^{14} \text{ Hz}$$

Hence, correct option is (b).

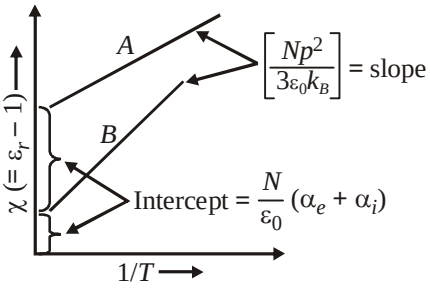
58. Type-I superconductors exhibit complete Meissner effect and are completely diamagnetic. Type-II superconductors exhibit complete Meissner effect upto H_{C_1} only. Above H_{C_1}, the flux begins to penetrate the speciman and for $H = H_{C_2}$, the complete penetration occurs and material becomes completely normal conductor. In the region between the field H_{C_1} and H_{C_2}, the Meissner effect is not strictly followed.



Type-I superconductor **Type-II superconductors**

Therefore, the statement in option (b) is not true.
Hence, correct option is (b).

59. The susceptibility (χ) is given by $\chi = (\varepsilon_r - 1) = \dfrac{N}{\varepsilon_0}(\alpha_e + \alpha_i) + \left(\dfrac{Np^2}{3\varepsilon_0 k_B T}\right)$



Since, slope of B > slope of A, therefore, B is more polar.
Since intercept of A is larger than B, therefore ionic polarizability of B is lesser in comparison to A.
Hence, correct option is (d).

60. The experimentally measured spin g-factors of proton and neutron indicate that both proton and neutron are not elementary point particles.
Correct option is (b)

61. ${}_{-1}e^0 + {}_{25}Mn^{54}_{29} \rightarrow {}_{24}Cr^{54}_{30} + ...$

For the conservation of lepton number in the above reaction (equation), we need to add a lepton member on the right side. i.e. either electron or electron neutrino or muon or muon neutrino.

$${}_{-1}e^0 + {}_{25}Mn^{54}_{29} \rightarrow {}_{24}Cr^{54}_{30} + \nu_e$$

Since, on left side there is an electron so we need a neutrino on right side.
Correct option is (a)

62. For (a) $\bar{\nu} + p \rightarrow n + e^+$

Charge $(Q): 0 + 1 \rightarrow 0 + 1, \Delta Q = 0$

$L: -1 + 0 \rightarrow 0 - 1, \Delta L = 0$

$B: 0 + 1 \rightarrow +1 + 0, \Delta B = 0$ (allowed)

For (b) $\pi^- \rightarrow e^- + \pi^0 + \nu_e$

Charge $(Q): -1 \rightarrow -1 + 0 + 0 \Rightarrow \Delta Q = 0$

Electronic Lepton No : $0 \rightarrow +1 + 0 + 1 \Rightarrow \Delta L_e \neq 0$ (forbidden)

For (c) $\pi^- + p \rightarrow n + K^+ + K^-$

Charge $(Q): -1 + (+1) \rightarrow 0 + 1 - 1, \Delta Q = 0$

$L: 0 + 0 \rightarrow 0 + 0 + 0 \Rightarrow \Delta L = 0$

$B: 0 + 1 \rightarrow 1 + 0 + 0 \Rightarrow \Delta B = 0$ (allowed)

For (d) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

Charge $(Q): -1 \rightarrow -1 + 0 + 0 \Rightarrow \Delta Q = 0$

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Electronic Lepton No : $+1 \rightarrow 0+0+1 \quad \Delta L_e = 0$

Muonic Lepton No : $+1 \rightarrow 0+0+1 \quad \Delta L_\mu = 0$

$B: 0 \rightarrow 0+0+0 \Rightarrow \Delta B = 0$ (allowed)

Correct option is (b)

63. Deutron is a single two nucleon (one p and one n) bound system which is found in nature. The important experimental determinal properties about deutron are given below:

(i) The B.E. of deutron/nucleon is very small compare to other nuclei i.e. it is a weakly system.

(ii) The ground state spin of deutron $I_d = 1$ (iii) The ground state parity of deutron = even (+)

(iv) The quadrupole momento of deutron $(Q_d) \neq 0$

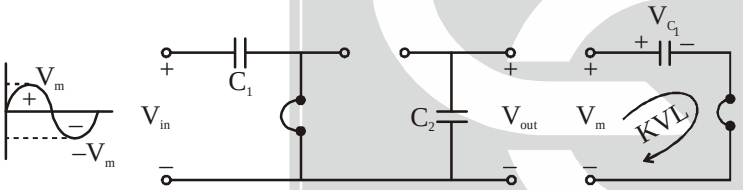
(v) The magnetic moment of deutron is slightly different from the sum of intrinsic mag. moments of neutron + proton i.e. $(\mu_n + \mu_p) - \mu_d \rightarrow 0$

These factor represents that the ground state of Deutron is a mixture of $^3S_{(L=0)}$ & $^3D_{(L=2)}$ states in which ^3S contribution is 96% and ^3D contribution is only 4%.

Correct option is (b)

64. Considering positive haf cycle

$D_1(\text{ON})$ and $D_2(\text{OFF})$

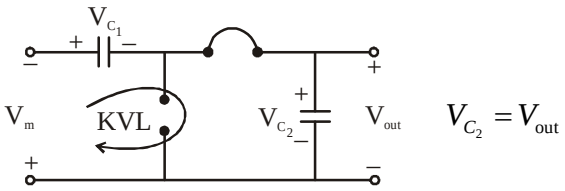


$-V_m + V_{C_1} = 0$

$V_{C_1} = V_m$

Considering (–)ve half cycle.

$D_1(\text{OFF}),\ D_2(\text{ON})$



$V_{C_2} = V_{\text{out}}$

$V_m + V_{C_1} + V_{C_2} = 0$

$V_m + V_m + V_{C_2} = 0$

$V_{C_2} = V_{\text{out}} = -2V_m$

$V_{\text{out}} = 2V_m$

Note : This circuit is also called voltage doubler circuit

Consider magnitude. So, change V_{out} (Polarity)

Correct option is (a)

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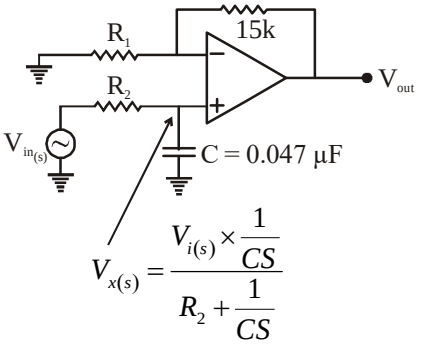
65. Given $f_{\text{cutoff}} = 2KHz$

$f_H = 2KHz$ [LPF]

Gain = 1.5



$$V_{x(s)} = \frac{V_{i(s)} \times \frac{1}{CS}}{R_2 + \frac{1}{CS}}$$

$$V_{x(s)} = \frac{V_{i(s)}}{1 + R_2 CS}$$

Virtual ground concept at inverting input

$$V_{\text{out}} = \left[1 + \frac{15}{R_1}\right] \cdot \frac{V_{i(s)}}{1 + R_2 CS}; \quad \frac{V_{\text{out}(s)}}{V_{i(s)}} = \frac{1 + \frac{15}{R_1}}{1 + R_2 CS} = \frac{\text{Gain}}{1 + R_2 CS}$$

$$\text{Gain } 1.5 = 1 + \frac{15}{R_1} \quad \Rightarrow \quad R_1 = 30\ k\Omega$$

equating $\frac{\text{Gain}}{1 + R_2 C\, j\omega} = \frac{\text{Gain}}{1 + j 2\pi f R_2 C} = \frac{A_0}{1 + j \frac{f}{f_H}}$; $f_H = \frac{1}{2\pi R_2 C}$ f_H; upper-3dB frequency

$$2 \times 10^3 = \frac{1}{2\pi R_2 \times 0.047 \times 10^{-6}}$$

$R_2 = 1.7\ k\Omega$; $R_1 = 30k$ and $R_2 = 1.7\ k\Omega$

Correct option is (d)

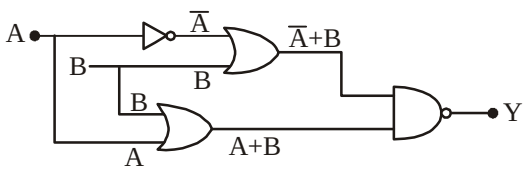
66. Consider the differential equation, $\frac{d^2 v}{dt^2} - 2\frac{dv}{dt} + v_1 = 0$

This is a second order differential equation.
If we can integrate it two times, we can easily get the solution of the above equation.
(Since, because it is involving second order derivatives)
So, two op-amp integrator must be there.

Since, there will be two solution of differential equation $v(t)$ and v_1. So, we also need an adder to add the outputs.

Correct option is (a)

67. Output Y = ?



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$$Y = \overline{(A+B)(\bar{A}+B)} \quad \Rightarrow \quad y = \overline{AB + \bar{A}B + B}$$

$$Y = \overline{B(1+A+\bar{A})} \quad \Rightarrow \quad Y = \bar{B}$$

Note: $1 + a = 1 \quad \Rightarrow \quad 1 + a + \bar{a} = 1$ (by boolean rule)

Correct option is (d)

68. 1 volt analog output for $\underbrace{010100}_{\text{digital input}}$ → 20 in decimal

for $000001 \rightarrow \frac{1}{20}$ Volt step size/least count.

for $\underbrace{111\ 111}_{2^6 - 1 = 63} \quad \Rightarrow \quad \frac{63 \times 1}{20}$ Volt

(max) input $\underbrace{111\ 111}$ output is 3.15 Volt

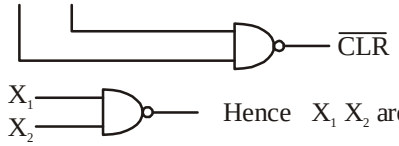
Correct option is (c)

69. CLOCK signal is applied to Flip-Flop "A", so it becomes "LSB" of Flip-Flop.

(i) Here clock is applied at negative edge trigger and output is taken at Q so it is up-counter.
(ii) It is MOD 12 counter and here Q_A is LSB so output must be 1100 (DCBA) to reset the counter.

Truth table of given counter :

D	C	B	A	
0	0	0	0	Counter will count till 11 and 12 output will be discarded.
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	



Hence X_1 X_2 are C and D

It is clear from truth table, that as soon as 1100 appears, input to the NAND gate becomes 11 which reset the flip-flop with clear = 1

So, X_1 and X_2 should be connected $Q_C = 1$ and $Q_D = 1$ i.e. C and D.

Correct option is (d)

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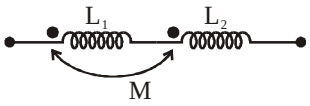
70. Hartely oscillator, 

$L_{\text{equivalent}} = L_1 + L_2 + 2M$ (Same behaviour)

$$f_0 = \frac{1}{2\pi\sqrt{L_{eq.}C}} \qquad L_{eq} = L_{\text{equivalent}}$$

↑
Put value

$$f_0 = \frac{1}{2\pi\sqrt{(L_1 + L_2 + 2M)\cdot C}}\,Hz$$

71. Given : perturbed Hamiltonian

$$H = \begin{pmatrix} E_1 & A \\ A^* & E_2 \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} 0 & A \\ A^* & 0 \end{pmatrix} = H_0 + H'$$

Therefore, the first order correction in E_1 is

$$\langle \phi_1 | H' | \phi_1 \rangle = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & A \\ A^* & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ A^* \end{pmatrix} = 0$$

Correct option is (d)

72. The second order correction in E_1, is

$$\frac{\left|\langle \phi_2 | H' | \phi_1 \rangle^2\right|}{E_1 - E_2} = \frac{\left|\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A \\ A^* & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|^2}{E_1 - E_2} = \frac{\left|\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ A^* \end{pmatrix}\right|^2}{E_1 - E_2} = \frac{|A|^2}{E_1 - E_2}$$

Correct option is (d)

73. The first order correction to the eigenfunction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is

$$\frac{\langle \phi_2 | H' | \phi_1 \rangle}{E_1 - E_2} | \phi_2 \rangle = \frac{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A \\ A^* & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{E_1 - E_2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ A^* \end{pmatrix}}{(E_1 - E_2)} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{A^*}{E_1 - E_2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \dfrac{A^*}{E_1 - E_2} \end{pmatrix}$$

Correct option is (a)

74. Sum of the eigenvalues = Trace of the matrix = 5

$$\Rightarrow \quad \lambda_1 + \lambda_2 + \lambda_3 = 5$$

$$\Rightarrow \quad 5 + \lambda_2 + \lambda_3 = 5 \Rightarrow \lambda_2 + \lambda_3 = 0$$

So, $\lambda_2 = 1$ and $\lambda_2 = -1$

Correct option is (c)

75. Eigenvalue equation : $(A - \lambda I) X = 0$

$$\Rightarrow \quad \begin{bmatrix} 2-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

CAREER ENDEAVOUR

Putting $\lambda = 5$, we get

$$\Rightarrow \quad \begin{bmatrix} -3 & 3 & 0 \\ 3 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \quad -3x_1 + 3x_2 = 0 \text{ and } -4x_3 = 0 \Rightarrow x_1 = x_2 \text{ and } x_3 = 0$$

Assuming, $x_1 = x_2 = k$, $x_3 = 0$

$$X = \begin{bmatrix} k \\ k \\ 0 \end{bmatrix},$$

putting normalization condition $X^{\dagger}X = 1 \Rightarrow k = \frac{1}{\sqrt{2}}$

So, $X = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Correct option is (d)

76. $(hkl) = (002)$, $\theta = 60°$, $\lambda = 1.54$ Å

From equation, $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{a}{\sqrt{0^2 + 0^2 + 2^2}} = \frac{a}{\sqrt{4}} = \frac{a}{2}$

From Bragg's law, $2d \sin\theta = n\lambda$

$\Rightarrow \quad 2 \times \frac{a}{2} \times \sin 60^{\circ} = 1 \times 1.54$ Å (Taking first order reflections, $n = 1$)

$\Rightarrow \quad a = \frac{1 \times 1.54 \times 2}{\sqrt{3}} = 1.78$ Å (No option matches the result)

By taking second order reflections i.e., $n = 2$: $a = \frac{2 \times 1.54 \times 2}{\sqrt{3}} = 3.556$ Å $\cong 3.56$ Å

Hence, correct option is (c).

77. Density of the crystal $(\rho) = \frac{nM}{Na^3} = \frac{2 \times 50.94}{6.023 \times 10^{26} \times (3.56 \times 10^{-10})^3}$ kg/m^3

$= 0.3749 \times 10^4$ kg/m^3 $= 3.75 \times 10^3$ kg/m^3

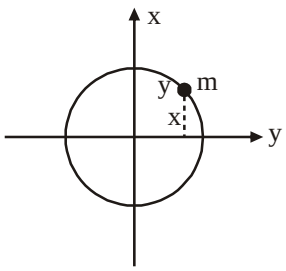
Hence, correct option is (a).

78. $L = T - V$

$L = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2\right) - mgx$

$x = A\cos\theta, \quad y = A\sin\theta$

$\dot{x} = -A\sin\theta\,\dot{\theta}, \quad \dot{y} = A\cos\theta\,\dot{\theta}$



CAREER ENDEAVOUR

$$\therefore \quad L = \frac{1}{2} m\left(A^2 \sin^2\theta\,\dot{\theta}^2 + A^2\cos^2\theta\,\dot{\theta}^2\right) - mg\,A\cos\theta$$

$$L = \frac{1}{2} m\,A^2\dot{\theta}^2 - mg\,A\cos\theta$$

Hence correct answer is (a)

79. Equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt}\left(mA^2\,\dot{\theta}\right) - mg\,A\sin\theta = 0$$

$$m\,A^2\,\ddot{\theta} - mg\,A\sin\theta = 0$$

$$\ddot{\theta} - \frac{g}{A}\sin\theta = 0$$

Hence correct answer is (d)

80. $$\rho = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k r^2\right) = -4kr$$

Correct option is (b)

81. $$\oint E\cdot dS = \int \frac{\rho\cdot dV}{\varepsilon_0}$$

$$\Rightarrow \qquad E\cdot 4\pi d^2 = -\int_0^d \frac{4kr}{\varepsilon_0} r^2\sin\theta\, d\phi\, d\theta\, dr \quad \Rightarrow \vec{E} = -\frac{kd^2}{\varepsilon_0}\hat{r}$$

Correct option is (a)

82. The number of final states of electrons corresponding to momenta between p and $(p + dp)$ is proportional to $p^2 dp$.

Correct option is (c)

83. The number of emitted electrons with momentum (p) and energy (E) in the allowed approximation is proportional to $p\left(E_0 - E\right)$.

Correct option is (b)

84. The density of state is

$$g(p)\,dp = \frac{4\pi p^2 dp V}{h^3},$$

where $E = pc$ for photons $\Rightarrow \quad dp = \dfrac{dE}{c}$

$$\therefore \quad g(E)\,dE = g\left(p\right)dp = \frac{4\pi V}{h^3}\frac{E^3}{c^2}\frac{dE}{c} = \frac{4\pi V}{h^3 c^3}E^2 dE$$

$$\Rightarrow \quad g(E) = \frac{4\pi V}{h^3 c^3}E^2$$

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Since photons have two planes of polarization, we should multiply the density of state by the factor of two.

Therefore, $g(E) = \dfrac{8\pi V}{h^3 c^3} E^2$

Correct option is (a).

85. The energy density is

$g(E)\,dE = \dfrac{8\pi V}{h^3 c^3} E^2 dE$, where $E = \hbar\omega$ for photons

$$\therefore \quad g(\omega)\,d\omega = g\left(E\right)dE = \frac{V\omega^2}{\pi^2 c^3}\,d\omega \qquad \left[\because\ \hbar = \frac{h}{2\pi}\right]$$

Also, for bosons (e.g., photons) $f(\omega) = \dfrac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}$

The average number of photons are

$$\langle N\rangle = \int_0^\infty f(E)\,g(E)\,dE = \int_0^\infty f(\omega)\,g(\omega)\,d\omega = \frac{V}{\pi^2 c^3}\int_0^\infty \frac{\omega^2 d\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1}$$

$$= \frac{V}{\pi^2 c^3}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^\infty \frac{x^2}{e^x - 1}\,dx \qquad \left[\because\ x = \frac{\hbar\omega}{k_B T}\right]$$

Clearly, $\langle N\rangle \propto T^3$

Correct option is (c).

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.20 : Carry ONE mark each.

1. Eigenvalues of an anti-hermitian matrix are either zero or purely imaginary
Correct option is (d)

2. When fire engine radially outward, its velocity is not perpendicular to radial direction. Therefore new bound orbit cannot be circle. So new bound orbit must be ellipse.
Hence correct answer is (c)

3. $P_0 = 150W$; $P_i = 1.5W$

$$\text{Power Gain} = \frac{150}{1.5} = 100$$

$$\text{Gain in dB} = 10\log_{10} 100 = 20 \text{ dB}$$

Note: For power we take 10 log P

4. Since, total charge is zero. So, monopole moment is zero.

Also, quadrupole moment, $\varphi_{ij} = \sum \left(3x_{ik}x_{jk} - r_{ij}\delta_{ij}\right) q_k = 0$

So, electrostatic potential will be due to charge distribution of dipole moment.

5. When displacement current reduces to zero this says that the current in the circuit is will be zero. So, total energy will stored entirely in its electric field.

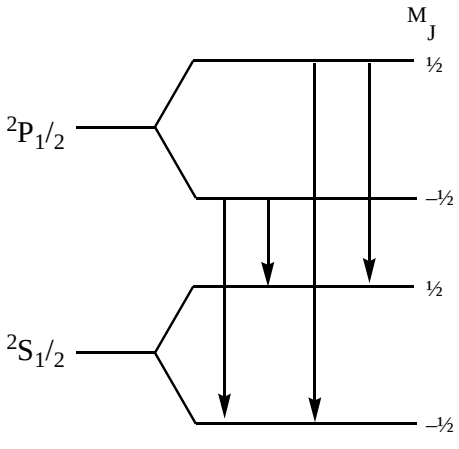
6. P : Frank-Hertz experiment was performed to identify energy levels in atoms.
Q : Hartree-Fock method is used for the wave functions of atoms.
R : Stern-Gerlach experiment was performed to check the space quantisation of "spin angular momentum of atoms".
S : Frank-Condon principle was given for the electronic excitation of molecules.
Hence, correct option is (a).

7. $\psi(x) = e^{-iax+b} = e^b\, e^{-iax} = A\, e^{-iax}$

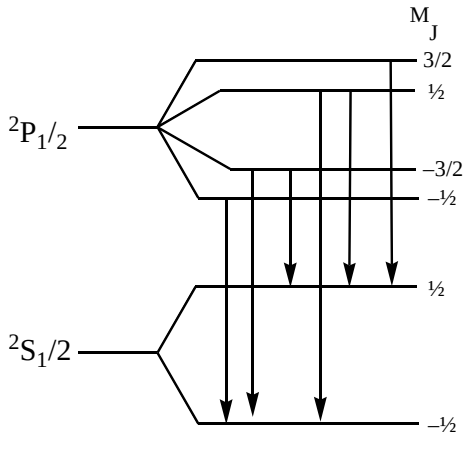
This is wavefunction of free partile. So, $V(x) = 0$

Correct option is (c)

8. In weak magnetic field (Anomalous Zeeman effect), selection rules are $\Delta M_J = 0, \pm 1$, $\left(M_J = 0 \not\leftrightarrow 0, \text{ if } \Delta J = 0\right)$. Thus, D_1 and D_2 will give 4 and 6 transitions.



4 transitions



6 transitions

Correct option is (a)

9. In a He-Ne laser (Four level laser), the population inversion is achieved by electric discharge He atoms help in achieving a population inversion in the Ne atoms. Thus, the laser transition takes place in Ne only.
Hence, correct option is (b).

10. If we take classical approach, the gas molecules are taken to be distinguishable. Hence,

$$Z_N = (Z_\alpha)^N$$

But if take semi-classical approach, the gas molecules are taken to be indistinguishable.

Therefore, $Z_N = \dfrac{(Z_\alpha)^N}{N!}$

Correct option is (a) or (b).

11. According to Meissner effect, the superconductor expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field and it behaves like a perfect diamagnet
Hence, correct option is (d).

12. Since, potential is time independent, therefore the probability of finding the particle in one well will be time independent.
Correct option is (b)

13. The quantum statistics applies when the de-Broglie wavelength of the particles (or associated with the particles) is comparable to the size of the particles. Under this condition, their wavefunctions start overlapping.
Correct option is (a).

14. Electron configuration of oxygen, $O_8 : 1s^2 2s^2 2p^4$

So, it has two free electron in the outer shell.

So, it will be attracted towards the higher field because it is paramagnetic.

Correct option is (d)

15. Since number of neutron increases in comparision to number of protons with atomic number i.e. (N–Z) increases with Z. For the radioactive elements like $_{92}U^{235}$ and $_{92}U^{238}$, (*N-Z*) values are 51 and 54 respectively. When a uranium nucleus, undergoes fission fragments (Kr and Ba) generally contain many neutrons.
Correct option is (a)

16. $N_E > N_C > N_B$

As per manufacturing of BJT doping of emitter > collector > base.

Correct option is (a)

17. For He ($2p^2$) configuration, we have two equivalent electrons in $2p^2$ state.

Thus, For one electron, $l_1 = 1, \quad s_1 = \dfrac{1}{2}$

For other electron, $l_2 = 1, \quad s_2 = \dfrac{1}{2}$

Thus, possible values of *s* and *l* are :

$$s = |s_1 - s_2|,\ |s_1 - s_2| + 1, \ldots, (s_1 + s_2) = \left(\frac{1}{2} - \frac{1}{2}\right),\ \left(\frac{1}{2} + \frac{1}{2}\right) = 0, 1$$

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Multiplicity $(2s+1) = 1, 3$ and

$l = |l_1 - l_2|,\ |l_1 - l_2| + 1, ..., (l_1 + l_2) = |1-1|,\ |1-1|+1, ..., (1+1) = 0, 1, 2$ (S, P, D, states)

Thus, in total we have six terms, three singlet and three triplet terms.

All these terms are even because the configuration $2p^2$ is even $(\Sigma l = 1+1 = 2)$. We can write these terms as

$^1S,\ ^1P,\ ^1D,\ ^3S,\ ^3P,\ ^3D$

To take into account spin-orbit interaction, let us combine L and S to form J.

Now, $J = |L - S|, ... (L + S)$.

For single terms, we have

$\begin{cases} S = 0 \\ L = 0 \end{cases}$; $J = 0$; $^1S_0$

$\begin{cases} S = 0 \\ L = 1 \end{cases}$; $J = 1$; $^1P_1$

$\begin{cases} S = 0 \\ L = 2 \end{cases}$; $J = 2$; $^1D_2$

For triplet terms, we have

$\begin{cases} S = 1 \\ L = 0 \end{cases}$; $J = 1$; $^3S_1$

$\begin{cases} S = 1 \\ L = 1 \end{cases}$; $J = 2, 1, 0$; $^3P_{0,1\,2}$

$\begin{cases} S = 1 \\ L = 2 \end{cases}$; $J = 3, 2, 1$; $^3D_{1,2\,3}$

For the equivalent electrons (according Hund's rule), terms $^1P_1,\ ^3S_1,\ ^3D_{1,2,3}$ will not be present and terms $^1S_0,\ ^1D_2,\ ^3P_{0,1,2}$ will be present.

Hence, correct option is (b).

18. For the given potential,

$\psi(x) \neq 0$ for $x > 0$ and $\psi(x) = 0$ for $x < 0$. For the sake of continuity at $x = 0$

$\psi(x)$ should be zero at $x = 0$

The normalized wavefunction of a particle moving under a linear harmonic oscillator potential is given by

$$\psi_n = A\, e^{\frac{-\alpha^2 x^2}{2}} H_n(\alpha x) \qquad (\text{where, } \alpha^2 = \frac{m\omega}{\hbar})$$

$\psi_n(x = 0)$ will be non-zero for $n = 0, 2, 4, 6$ ..... and will be zero 1, 3, 5, 7..... So, possible value of 'n' is odd.

The energy eigenvalues of the particle will be

$$E_n = \left(2n + 1 + \frac{1}{2}\right)\hbar\omega = \left(2n + \frac{3}{2}\right)\hbar\omega$$

Correct option is (c)

19. **Correct option is (c)**

20. Strangeness is conserved in strong and electromagnetic both interactions.

Correct option is (d)

Q.21 – Q.75 : Carry TWO marks each.

21. $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$; eigenvalue equation : $|A - \lambda I| = 0$

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$$\Rightarrow \quad \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0 \;\Rightarrow (\lambda-2)(\lambda-5)-4=0 \;\Rightarrow \lambda^2-7\lambda+6=0 \;\Rightarrow\; \lambda = 6, 1$$

Now, $(A-\lambda I)X = 0$

$$\Rightarrow \quad \begin{pmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

Putting $\lambda = 6 \;\Rightarrow \begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \;\Rightarrow -x_1 + 4x_2 = 0 \;\Rightarrow x_1 = 4x_2$

So, $\quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

Putting $\lambda = 1 \Rightarrow \begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \;\Rightarrow 4x_1 + 4x_2 = 0 \;\Rightarrow x_1 = -x_2$

So, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Correct option is (a)

22. $\vec{F} = \dfrac{y^2}{L}\hat{j} + z\hat{k}$

The net flux of F associated with the cube

$$\iint \vec{F}\cdot d\vec{s} = \iiint \left(\vec{\nabla}\cdot\vec{F}\right) dV \quad \text{(using Divergence theorem)}$$

$$= \int_0^L\int_0^L\int_0^L \frac{2y}{L}\,dxdydz + \int_0^L\int_0^L\int_0^L dxdydz = L^3 + L^3 = 2L^3$$

Correct option is (a)

23. $\vec{r} = x\hat{i} + y\hat{j} \;\Rightarrow \vec{\nabla}\cdot\vec{r} = 2$

$$\vec{\nabla}|\vec{r}| = \vec{\nabla}r = 1\cdot\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

Correct option is (c)

24. The rotation matrix corresponding to the anti-clockwise rotation about y-axis by an angle θ will be

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & 0 & -\sin 60^\circ \\ 0 & 1 & 0 \\ \sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \left(1-\sqrt{3}\right) \\ 3 \\ \left(1+\sqrt{3}\right) \end{bmatrix}$$

Correct option is (a)

CAREER ENDEAVOUR

25. $f(z)=\dfrac{1}{z^2+a^2}$ has simple poles at $z=\pm ia$

Correct option is (b)

26. $L^{-1}\left[\dfrac{s+1}{s^2-4}\right]=L^{-1}\left[\dfrac{s}{s^2-2^2}\right]+\dfrac{1}{2}L^{-1}\left[\dfrac{2}{s^2-2^2}\right]=\cosh 2x+\dfrac{1}{2}\sinh 2x$

Correct option is (b)

27. The solution of Legendre differential equation will be

$$y=A\,P_n(x)+BQ_n(x)$$

where $P_n(x)$ is Legendre Polynomial of order 'n' and $Q_n(x)$ is Legendre function of second kind and given by

$$Q_0(x)=\frac{1}{2}\ell n\left(\frac{1+x}{1-x}\right)$$

$$Q_1(x)=\frac{x}{2}\ell n\left(\frac{1+x}{1-x}\right)-1$$

This shows that, $Q_n(x)$ will diverge at $x=\pm 1$.

Correct option is (c)

28. $x\dfrac{dy}{dx}+y=x^4 \Rightarrow \dfrac{dy}{dx}+\dfrac{y}{x}=x^3$

$I.F.=e^{\int dx/x}=e^{\ln x}=x$

Solution will be $xy=\int x^3 x\,dx \Rightarrow xy=\dfrac{x^5}{5}+c$

Putting the condition, at $x=1$, $y=1$, we get $c=\dfrac{4}{5}$

So, $xy=\dfrac{x^5}{5}+\dfrac{4}{5} \Rightarrow y=\dfrac{x^4}{5}+\dfrac{4}{5x}$

Correct option is (d)

29. (P) rest mass $\rightarrow$ (2) Lorentz invariant
(Q) charge $\rightarrow$ (4) conserved and Lorentz invariant
(R) Fourmomentum $\rightarrow$ (1) time like vector
(S) Electromagnetic field $\rightarrow$ (3) tensor of rank 2
Hence correct answer is (c)

30. $M=\dfrac{4}{3}\pi r^5\rho$

$$I_{yy}=\frac{2}{5}\left(\frac{4\pi}{3}r^5\rho\right)-2\times\frac{7}{5}\left(\frac{4\pi}{3}\cdot\left(\frac{r}{2}\right)^5\rho\right)$$

$$=\frac{4\pi}{3\times 5}r^5\rho\left(2-\frac{7}{16}\right)=\frac{4\pi}{15}r^5\rho\frac{25}{16}=\frac{5}{12}\pi r^5\rho$$

Hence correct answer is (b)

CAREER ENDEAVOUR

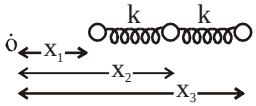
31. L explicitly depends on t so energy is not conserved
z is cyclic coordinate so p_z is conserved
L is invariant under rotation about z axis so L_z is conserved
Hence correct answer is (c)

32. Kinetic energy

$$T = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2\right)$$

Potential energy

$$V = \frac{1}{2}k\left(x_2 - x_1 - \ell\right)^2 + \frac{1}{2}k\left(x_3 - x_2 - \ell\right)$$



33. $H = \frac{p^2}{2m} + pq$

Hamilton's equation $\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} + q$

$$p = m\left(\dot{q} - q\right)$$

Lagrangian $L = p\dot{q} - H$

$$= p\dot{q} - \frac{p^2}{2m} - pq = p\left(\dot{q} - q\right) - \frac{p^2}{2m}$$

Putting the value of p from above

$$L = m\left(\dot{q} - q\right)^2 - \frac{m\left(\dot{q} - q\right)^2}{2}$$

$$L = \frac{1}{2}m\left(\dot{q} - q\right)^2$$

Hence correct answer is (b)

34. $\oint \vec{B} \cdot d\vec{\ell} = \mu_r \mu_0 IN \Rightarrow B = \frac{\mu_r \mu_0 IN}{2\pi r}$

Correct option is (c)

35. We have, $I = 150\ Wm^{-2}$

We know, $I = \langle u \rangle c = \frac{1}{2}\varepsilon_0 E_0^2 c$

Therefore, the average power $= I \times \pi R^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times \left(60\right)^2 \times \pi \times \left(2\right)^2 \times 3 \times 10^8 = 120$

Correct option is (c)

36. $\bar{\nabla} \cdot \bar{A} = \left(\omega - 3zy\right)\hat{i} - 3xy\hat{j} - 3xy\hat{k} \neq 0$ does not follow coulomb gauge

$\bar{\nabla} \cdot \bar{A} + \frac{1}{C^2}\frac{\partial \varphi}{\partial t} = 4 - \frac{4}{C} \neq 0$ does not follow Lorentz gauge

Correct option is (d)

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37. Schrodinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi(x)=E\,\psi(x)$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left(-6a+4a^2x^2\right)x\,e^{-ax^2}+V(x)\,x\,e^{-ax}=\frac{3}{2}\hbar\omega x\,e^{-ax}$$

$$\Rightarrow \quad +\frac{6\hbar^2 a}{2m}-\frac{2a^2\hbar^2x^2}{m}+\frac{1}{2}m\omega^2x^2=\frac{3}{2}\hbar\omega$$

Comparing the coefficient of x^2 from both sides,

$$a=\frac{m\omega}{2\hbar}$$

Correct option is (c)

38. We have $[x,\ p]=i\hbar$

$$\therefore \qquad \left[x^3,\ p\right] \qquad \left(\because\ \left[x^n,\ p\right]=nx^{n-1}[x,\ p]\right)$$

$$=3x^2[x,\ p]=3x^2i\hbar=3i\hbar x^2$$

Correct option is (c)

39. There will be only three bound state if $\pi < R < \frac{3\pi}{2}$

$$\Rightarrow\ \pi^2 < R^2 < \frac{9\pi^2}{4}$$

$$\Rightarrow\ \pi^2 < \frac{mV_0\,a^2}{2\hbar^2} < \frac{9\pi^2}{4}$$

$$\Rightarrow\ \frac{2\hbar^2\pi^2}{m\,a_0^2} < V_0 < \frac{9\pi^2\hbar^2}{2m\,a_0^2}$$

Correct option is (a)

40. $$\psi(r,\theta,\phi)=\frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

Compare the $e^{-\frac{r}{2a_0}}$ with $e^{-\frac{r}{na_0}}$. We can say $n = 2$

Since, there are only one cosine term so $\ell = 1$

Since, there are no term containing ϕ, so $m = 0$

So, the state is 2*P*.

Correct option is (c)

41. $[X,Y]=i\hbar Z,\ [Y,Z]=i\hbar X$ and $[Z,X]=i\hbar Y$

Since, component of total angular moment follow the above relation

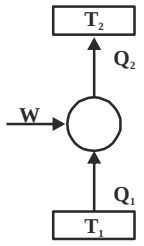
CAREER ENDEAVOUR

$$\left[J_x, J_y\right] = i\hbar J_z,\ \left[J_y, J_z\right] = i\hbar J_x\ \ \left[J_z, J_x\right] = i\hbar J_z$$

So, z operator is behaving like J_z and it has eigen values $\pm m_J\hbar$ where $m_J = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}$.....

Correct option is (c)

42. Given : $T_1 = 0\ ^\circ\text{C} = 273\ \text{K}$, $Q_1 = ?$ and $T_2 = 22\ ^\circ\text{C} = 295\ \text{K}$, $Q_2 = 450\ \text{kJs}^{-1}$



Using $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$, we have

$$\Rightarrow \quad Q_1 = \frac{T_1}{T_2} Q_2 = \frac{273}{295} \times 450 \simeq 417\ \text{kJs}^{-1}$$

Correct option is (d).

43. At triple point, vapour pressure of a solid phase gets equal to that of liquid phase.

$$\Rightarrow \quad 23 - \frac{3863}{T} = 19 - \frac{3063}{T} \ \Rightarrow \quad \frac{800}{T} = 4 \Rightarrow T = 200\ \text{K}$$

Correct option is (d).

44. Given : $F = -\left(\frac{a}{3}\right)VT^4$

The entropy is $S = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{4a}{3}VT^3$

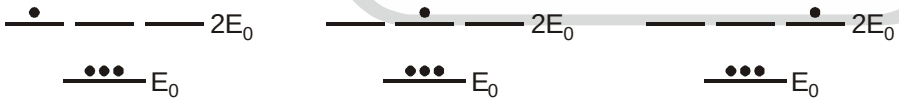
and the pressure is $P = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{a}{3}T^4$

Correct option is (a).

45. Given : Every excited state is 3-fold degenerate.

i.e., E_0 $2E_0$ $3E_0$ and so on.

The possible states where the total energy is $5E_0$ are as follows :



So number of possible states are three.

Correct option is (b).

46. For Helium : $4\ ^3P_1$

For the electric dipole transition, $\Delta\ell = \pm 1$ i.e. the parity must change. Since the parities are determined by $(-1)^\ell$, the transitions corresponding to $\Delta\ell = 0, \pm 2, \pm 4, ...$ are not allowed, because in these cases the parities of the initial and the final eigen function would be the same. The transitions allowed by parity are $\Delta\ell = \pm 1, \pm 3, \pm 5, ...$

The selection rule $\Delta m_\ell = 0, \pm 1$ disallowes the transitions $\Delta\ell = \pm 3, \pm 5, ...$ because then Δm_ℓ may be greater than 1. Hence, the selection rule for ℓ is $\Delta\ell = \pm 1$. *Out of these four options, only option (d) corresponds to* $\Delta\ell = \pm 1$.

Hence, correct option is (d).

CAREER ENDEAVOUR

47. For $^3D_3$ state, $L = 2,\ J = 3,\ 2S+1 = 3 \Rightarrow S = 1$

The angle between the orbital and spin angular momentum can be calculated using formula,

$$\cos\theta = \cos(\vec{L}\cdot\vec{S}) = \frac{J(J+1) - L(L+1) - S(S+1)}{2\sqrt{L(L+1)}\cdot\sqrt{S(S+1)}}$$

$$= \frac{3(3+1) - [2(2+1) + 1(1+1)]}{2\sqrt{2(2+1)}\cdot\sqrt{1(1+1)}}$$

$$= \frac{12 - (6+2)}{2\times\sqrt{6}\cdot\sqrt{2}} = \frac{4}{2\times 2\sqrt{3}} = +\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \cos^{-1}(\vec{L}\cdot\vec{S}) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Hence, correct option is (a).

48. For $Na\left(3\,^2P_{3/2}\right)$ with nuclear spin $I = \frac{3}{2}$, we have, $J = \frac{3}{2},\ 2S+1 = 2 \Rightarrow S = \frac{1}{2}$ and $L = 1$

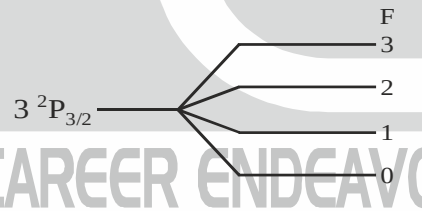
For hyperfine structure,

Total angular momentum F = total angular momentum of electron + total nuclear spin

$$= |J+I|,\ |J+I|-1, ..., |J-I|$$

$$\Rightarrow \vec{F} = \vec{J} + \vec{I} = \left(\frac{3}{2}+\frac{3}{2}\right) \text{ to } \left(\frac{3}{2}-\frac{3}{2}\right) = 3, 2, 1, 0$$

Thus the hyperfine structure of $Na\left(3\,^2P_{1/2}\right)$ with nuclear spin $I = \frac{3}{2}$ has 4 states as shown below.



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Correct option is (d)

49. The allowed rotational energy

$$E_J = BJ(J+1) \qquad \text{... (i)}$$

The wave number $(\bar{\nu})$ of the emitted or absorbed radiation under allowed transition is given by

$$\bar{\nu} = 2B(J+1) \qquad \text{... (ii)}$$

where J is rotational quantum number and $B = \frac{h}{8\pi^2 Ic}$ is rotational constant.

For the reduced mass μ the wave number corresponding to the transition $J = 4 \rightarrow 5$ is given by

$$\bar{\nu} = 2B(4+1) = 10B = 10\left(\frac{h}{8\pi^2 Ic}\right) \Rightarrow \bar{\nu} = \frac{10h}{8\pi^2(\mu r^2)c} \qquad \text{... (iii)}$$

[We take lower value of *J* i.e. *J* = 4, here]

For the reduced mass $\mu' = 1.05\,\mu$ and $J = 4 \rightarrow 5$ transition the wave number is given by

$$\vec{\nu}' = 2B'(4+1) = 10B' = 10\left(\frac{h}{8\pi^2 I' c}\right)$$

$$\Rightarrow \quad \vec{\nu}' = \frac{10\,h}{8\pi^2(\mu' r^2)c} = \frac{10\,h}{8\pi^2(1.05\,\mu r^2)c} \qquad \text{... (iv)}$$

From equations (iii) and (iv), we get

$$\nu - \nu' = \frac{10\,h}{8\pi^2(\mu r^2)c}\left(1 - \frac{1}{1.05}\right) = 10\,B\left(1 - 0.9524\right)$$

$$\Rightarrow \quad \Delta\nu = 10\,B \times 0.0476 = 0.476\,B$$

Hence, correct option is (a).

50. The number of fundamental vibrational modes of CO_2 molecule is three. Out of these, two are infrared active and one is Raman active.

51. The magnetic moment of the nucleus $\vec{\mu} = g\,\frac{eh}{2\pi mp}\,\vec{I}$

The energy absorbed at resonance frequency

$$= -\bar{\mu} \cdot \hat{H}_0 = \frac{gehH_0}{2\pi m_p} m_I \qquad \left[M_I = \pm\frac{1}{2}\right]$$

If ν_0 is the resonance frequency, then

$$h\gamma_0 = \frac{gehH_0}{2\pi m_p}\left(\frac{1}{2} + \frac{1}{2}\right) \quad \Rightarrow \qquad \gamma_0 = \frac{geH_0}{2\pi m_p}$$

Correct option is (c)

52. $V(r) = -\frac{P}{r^6} + \frac{Q}{r^{12}}$,

At $r = r_0$, potential energy will be minimum i.e., $\left.\frac{dV(r)}{dr}\right|_{\text{at } r = r_0} = 0$

$$\Rightarrow \quad -P(-6r_0^{-7}) + Q(-12\,r_0^{-12-1}) = 0$$

$$\Rightarrow \quad +\frac{6P}{r_0^7} + Q\left(-\frac{12}{r_0^{13}}\right) = 0$$

$$\Rightarrow \quad 6P = \frac{12Q}{r_0^6} \quad \Rightarrow \quad r_0 = \left(\frac{2Q}{P}\right)^{1/6}$$

$$\therefore \quad \text{Bond length } (r_0) = \left(\frac{2Q}{P}\right)^{1/6}$$

Hence, correct option is (a).

CAREER ENDEAVOUR

53.

Crystal structure	APF
P : Simple cubic	52%
Q : Body-centered cubic	68%
R : Face-centered cubic	74%
S : Diamond	34%
T : Hexagonal close packed	74%

From above table, it is clear that, the APF of face-centered cubic (R) and hexagonal close packed (T) are same.

Hence, correct option is (d).

54. For face-centered cubic sample, $\theta_1 = 30°,\ \theta_2 = ?$

For first peak, (hkl) = (111)

For second peak, (hkl) = (200)

Therefore, from Bragg's law: $2d \sin\theta = \lambda$

$$\Rightarrow \quad \frac{2a}{\sqrt{h^2 + k^2 + l^2}} \sin\theta = \lambda \qquad \text{... (i)}$$

For first peak ; $\dfrac{2a}{\sqrt{h^2 + k^2 + l^2}} \sin\theta_1 = \lambda \ \Rightarrow\ \lambda = \dfrac{2a \sin\theta}{\sqrt{1^2 + 1^2 + 1^2}}$

$$\Rightarrow \quad \lambda = \frac{2a \sin 30°}{\sqrt{3}} = \frac{a}{\sqrt{3}} \qquad \text{... (ii)}$$

For second peak ; $\dfrac{2a}{\sqrt{2^2 + 0^2 + 0^2}} \sin\theta_2 = \lambda \ \Rightarrow\ \lambda = \dfrac{2a \sin\theta_2}{\sqrt{4}}$

$$\Rightarrow \quad \lambda = a \sin\theta_2 \qquad \text{... (iii)}$$

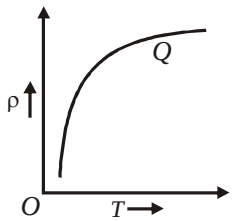
Therefore, from equations (ii) and (iii), we get, $a \sin\theta_2 = \dfrac{a}{\sqrt{3}} \Rightarrow \sin\theta_2 = \dfrac{1}{1.732} = 0.577$

$$\Rightarrow \quad \theta_2 = \sin^{-1}(0.5773) = 35.26°$$

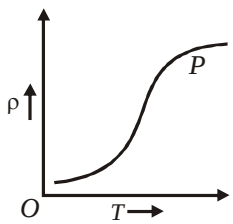
Hence, correct option is (d).

CAREER ENDEAVOUR

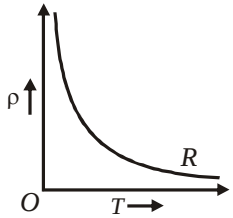
55. For superconductors, $\rho - T$ graph is



For conductors, $\rho - T$ graph is



For semiconductors, $\rho - T$ graph is



Hence, correct option is (b).

56. Conductivity, $\sigma = eN\mu_0 = eN_0\mu e^{-x/L}$

Consider an elementary length *dx*.

Therefore, the resistance of the elementary length

$$dR = \rho\frac{dx}{A} = \frac{1}{\sigma}\frac{dx}{A} = \frac{1}{eN_0\mu A}e^{\frac{x}{L}}dx$$

$$\therefore \qquad R = \frac{1}{eN_0\mu A}\int_0^L e^{\frac{x}{L}}dx = \frac{L}{eN_0\mu A}\left[e^{\frac{x}{L}}\right]_0^L = \frac{L}{eN_0\mu A}\left[e^1 - 1\right]$$

Correct option is (a)

57. Ferromagnetic mixture of Iron (Fe) and Copper (Cu)

Amount (%) of Fe = 75%; % of Cu = 25%

Total number of atoms, $\frac{N}{V} = n = 8\times10^{28}\ m^{-3}$

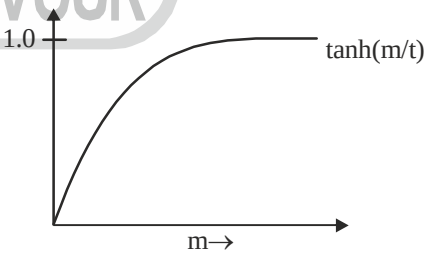
75% of n contributes for saturation magnetisation

So, $n_{eff} = 0.75\times8\times10^{28} = 6\times10^{28}\ m^{-3}$

$$M = N\mu\tanh\left(\frac{\mu\lambda M}{k_BT}\right)$$

CAREER ENDEAVOUR

Graphical solution of this equations is



$$m = \frac{M}{N\mu} \text{ and } t = \frac{k_BT}{N\mu^2\lambda}$$

$$m = \tanh\left(\frac{m}{t}\right) = 1 \ \left(\text{when it gets saturated}\right)$$

$$\frac{M}{N} = \mu;\ \mu = \frac{1.3\times10^6}{6\times10^{28}} = 0.216\times10^{-22}$$

$$\mu = \frac{0.216\times10^{-22}}{9.27\times10^{-24}} \simeq 2.3 \text{ Bohr magneton}$$

Correct option is (b)

58. Given that half life $T_{1/2} = 4 \times 10^8$ years

Total time = 4×10^9 years.

Therefore, number of half life time = $\frac{4\times10^9}{4\times10^8} = 10$

Therefore, remaining number of particles after 10 half life time

$10^3 = N = N_0\left(\frac{1}{2}\right)^{10} = \frac{N_0}{1024} \Rightarrow N_0 = 1024\times10^3$ radioactive nuclei.

Correct option is (d)

59. Since, tritium is radioactive, therefore more energy releases in (deuterium + tritium) fusion.

Correct option is (a).

60. For $_8O^{17}$, Z = 8, N = 9

$9N = (^1s_{1/2})^2\ (^1p_{3/2})^4\ (^1p_{1/2})^2\ (^1d_{5/2})^2$

$\therefore\ J = \frac{5}{2}$, l = 2 (for d)

Therefore, parity = $(-1)^2$ = +1 (even)

Therefore, spin-parity = $\frac{5^+}{2}$

Correct option is (b)

61. Travelling distance = 3×10^{-3} m

The time taken by the particle to travel the given distance

$t = \frac{3\times10^{-3}}{3\times10^8}$ sec = 10^{-11} sec, life time = 10^{-11} sec

Therefore, interaction is weak.

Correct option is (c)

62. For Σ^+ baryon, quark structure is 'uus'

$u\quad u\quad s$

Since, charge $(Q) = +1 = +\frac{2}{3}+\frac{2}{3}-\frac{1}{3} = 1$

Baryon number $(B) = 1 = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$

Strangeness $(S) = -1 = 0+0-1 = -1$

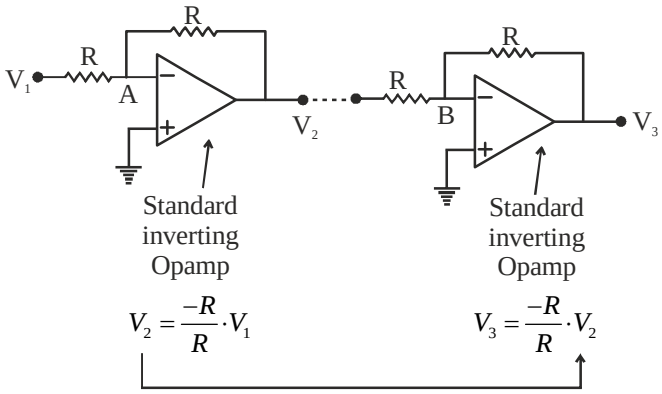
Correct option is (c)

63. Since there is no loss of energy in elastic scattering. Therefore, the initial and final states of the neutron have the same energy.

Correct option is (a)

CAREER ENDEAVOUR

64.



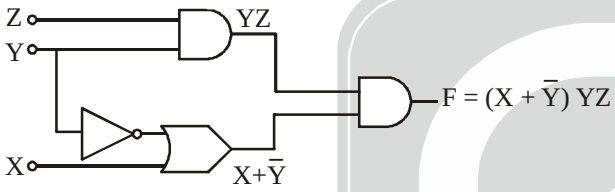
Ideal op-amp has infinite input resistance so there will no current pass through op-amp and V_A and V_B equal to zero due to virtual ground concept.

$V_3 = V_1$

- Output same as input. So, it is work as Buffer amplifier

Correct option is (b)

65.



Output expression $F = \left(X + \bar{Y}\right)YZ$

$= XYZ + Y\bar{Y}Z$

Hence, $F = XYZ \qquad \left(\because\ Y\bar{Y} = 0\right)$

Correct option is (d)

66. Given $Q_1 = 1,\ Q_2 = 0$

CLK	J_1	K_1	J_2	K_2	Q_1	Q_2
0	1	1	1	1	1	0
1	1	1	1	1	0	0
2	1	1	1	1	1	1

For $J = K = 1$ output is complemented (Toggle), and Q_2 will be toggle when Q_1 become from $0 \rightarrow 1$ (as it is clock pulse of Q_2)

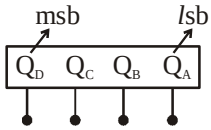
So, after two clock pulse output will be 11

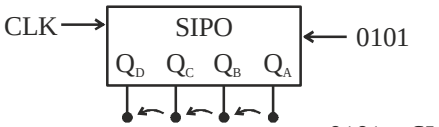
Correct option is (c)

CAREER ENDEAVOUR

67. Serial input is $S_I = 0101$

Parallel output is





Clock				
	1	0	1	0
1 →	0	1	0	0
2 →	**1**	**0**	**0**	**1**
3 →	0	0	1	0
4 →	0	1	0	1

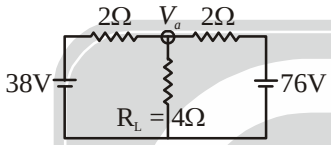
0101 = SI

Serial input "0" first CLK.

Parallel output of 2nd clock = 1 0 0 1

Correct option is (a)

68. To find potential difference (in volts) across R_L.



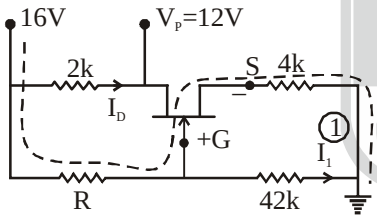
Nodal-Analysis at (a)

$$0 = \frac{V_a - 38}{5} + \frac{V_a}{4} + \frac{V_a - 76}{2}$$

$V_a = 48$ Volt

Correct option is (a)

69.



$$I_D = \frac{16 - 12}{2} = 2 \text{ mA}$$

$V_{GS} = -2$Volt

KVL (1)

$+V_{GS} + I_D 4 = 42\, I_1$

$-2 + I_D \times 4 = 42\, I_1$

Pul $I_D = 2$ mA

$$6 = 42\, I_1 ;\ I_1 = \frac{1}{7} \text{mA}$$

$\Rightarrow$ KVL as per dashed line shown

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$16 = I_1 \times R - 2 + 8$ Put $I_1 = \frac{1}{7}$ mA

$16 = I_1 \times R + 6$

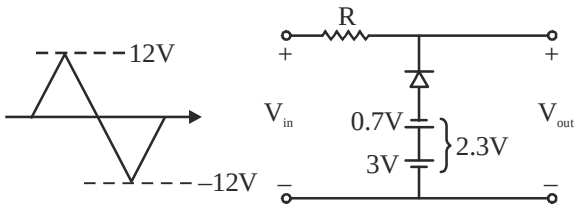
$10 = I_1 \times R$ Put I_1 value

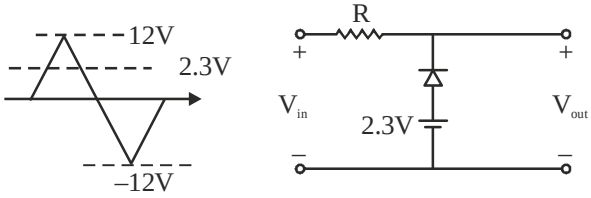
$R = 70\ k\Omega$

Correct option is (d)

70.

Equivalent circuit.

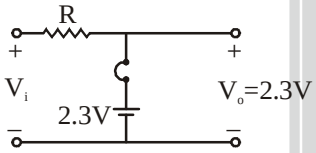




Case - (1)

$0 \le V_{in} < 2.3V$

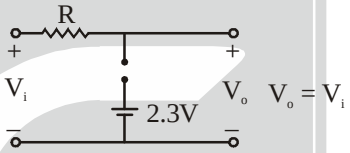
Diode - ON



Case - (2)

$2.3V < V_i < 12V$

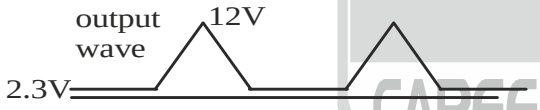
Diode - OFF



Case - (3)

$V_{in} < 0$

Diode - ON



Note: When the diode is upward direction the signal will transmitted above the reference voltage minus cut in voltage of diode (V_r = 0.7) (Plus minus depend on polarity of reference voltage).

Correct option is (c)

71. $\psi(x) = \psi_0 \cos\frac{\pi x}{4L} \qquad (-2L < x < 2L)$

Normalization condition:

$$\therefore \quad \int_{-2L}^{2L} |\psi|^2 \, dx = 1$$

$$\Rightarrow \quad |\psi_0|^2 \int_{-2L}^{2L} \cos^2 \frac{\pi x}{4L} dx = 1 \Rightarrow \frac{|\psi_0|^2}{2} \int_{-2L}^{2L} \left[1 + \cos\frac{\pi x}{2L}\right] dx = 1$$

CAREER ENDEAVOUR

$$\Rightarrow \quad \frac{|\psi_0|^2}{2}4L+\left[\frac{\sin\frac{\pi x}{2L}}{\pi/2L}\right]_{-2L}^{2L}=1 \Rightarrow |\psi_0|^2=\frac{1}{2L}\Rightarrow|\psi_0|=\sqrt{\frac{1}{2L}}$$

Correct option is (c)

72. The energy of the ground state will be

$$E_1=\frac{\hbar^2\pi^2}{2m(4L)^2}=\frac{\hbar^2\pi^2}{32mL^2}$$

Correct option is (d)

73. The expectation value of

$$\langle P^2\rangle=\langle E\rangle 2m=2m\times\frac{\hbar^2\pi^2}{32mL^2}=\frac{\hbar^2\pi^2}{16L^2}$$

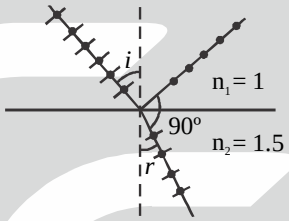
Correct option is (c)

74. When the incident angle i is become (i_p) Brewster angle then the reflected light is completely polarized.

$$\therefore \quad E_{\parallel}^{reflected}=0$$

$$\therefore \quad \cos r-\mu\cos i=0$$

$$\Rightarrow \quad \mu\cos i=\cos r$$



Correct option is (a)

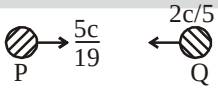
75. We know that

$$R=\left(\frac{n_1-n_2}{n_1+n_2}\right)^2 \text{ for normal incidence}$$

$$=\left(\frac{1-1.5}{1+1.5}\right)^2=\left(\frac{0.5}{2.5}\right)^2=\frac{1}{25}=0.04$$

Correct option is (d)

76. Speed of P in rest frame of Q



$$V_{PQ}=\frac{V_P-V_Q}{1-\frac{V_P\cdot V_Q}{c^2}}=\frac{\frac{5c}{19}+\frac{2c}{5}}{1+\frac{5}{19}\cdot\frac{2}{5}}=\frac{63c}{21\times5}=\frac{3c}{5}$$

Hence correct answer is (c)

77. Energy of P in rest frame of Q is

$$E=\frac{m_0c^2}{\sqrt{1-v_Q^2/c^2}}=\frac{m_0c^2}{\sqrt{1-\left(\frac{3}{5}\right)^2}}=\frac{5m_0c^2}{4}$$

Hence correct answer is (b)

CAREER ENDEAVOUR

78. We know that conductivity,

$$\sigma = n\, e\mu$$

$$\Rightarrow\ \mu = \frac{\sigma}{ne} = \frac{1}{\rho\, ne} = \frac{1}{1.6\times10^{-8}\times5.85\times10^{28}\times1.6\times10^{-19}} = 6.67\times10^{-3}\ \ m^2V^{-1}s^{-1}$$

Correct option is (a)

79. Relaxation time $(\tau) = \dfrac{\mu m_e}{e} = \dfrac{6.67\times10^{-3}\times9.1\times10^{-31}}{1.6\times10^{-19}} = 37.93\times10^{-15} = 3.79\times10^{-14}$ sec.

Hence, correct option is (b).

80. Bound surface charge density

$$\sigma_b = \vec{P}\cdot\hat{n} = \vec{P}\cdot\hat{r} = kr = k\left|\vec{r}\right|$$

Bound volume charge density,

$$\rho = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2kr\right) = -3k$$

Correct option is (b)

81. The electric field at the out side of the sphere

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q_b}{\varepsilon_0} = \frac{4\pi kr^3 - \dfrac{4}{3}\pi r^3 3k}{\varepsilon_0} = 0$$

$$\Rightarrow \qquad \vec{E} = 0 \text{ for } r > R$$

Correct option is (a)

82. The energy is $E_n = \left(n+\dfrac{1}{2}\right)\hbar\omega;\ n = 0,1,2,...$

The partition function of a single oscillator is

$$Z = \sum_n e^{-\frac{E_n}{k_BT}} = \sum_n e^{-\left(n+\frac{1}{2}\right)\frac{\hbar\omega}{k_BT}}$$

$$= e^{-\frac{\hbar\omega}{2k_BT}}\sum_n e^{-n\frac{\hbar\omega}{k_BT}}$$

$$= e^{-\frac{\hbar\omega}{2k_BT}}\left[1+e^{-\frac{\hbar\omega}{k_BT}}+e^{-\frac{2\hbar\omega}{k_BT}}+\cdots\right]$$

$$= e^{-\frac{\hbar\omega}{2k_BT}}\cdot\frac{1}{1-e^{-\frac{\hbar\omega}{k_BT}}}$$

$$= \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}},\ \text{where } \beta = \frac{1}{k_BT}$$

Correct option is (a).

CAREER ENDEAVOUR

83. The average number of energy quanta are

$$\langle n\rangle = \frac{\sum_{n=0} n e^{-\beta E}}{\sum_{n=0} e^{-\beta E}} = \frac{\sum_{n=0} n e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega}}{\sum_{n=0} e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega}} = \frac{\sum n e^{-n\beta\hbar\omega}}{\sum e^{-n\beta\hbar\omega}} = \frac{0 + e^{-\beta\hbar\omega} + 2e^{-2\beta\hbar\omega} + 3e^{-3\beta\hbar\omega} + \cdots}{1 + e^{-\beta\hbar\omega} + e^{-2\beta\hbar\omega} + \cdots}$$

$$\Rightarrow \quad \langle n\rangle = e^{-\beta\hbar\omega}\frac{\left[1 + 2e^{-\beta\hbar\omega} + 3e^{-2\beta\hbar\omega} + \cdots\right]}{\left[1 + e^{-\beta\hbar\omega} + e^{-2\beta\hbar\omega} + \cdots\right]}$$

Letting $e^{-\beta\hbar\omega} = x$, then

$$\langle n\rangle = \frac{x\left[1 + 2x + 3x^2 + \cdots\right]}{\left[1 + x + x^2 + \cdots\right]} = \frac{x(1-x)^{-2}}{(1-x)^{-1}}$$

$$= \frac{x}{1-x} = \frac{e^{-\beta\hbar\omega}}{1 - e^{-\beta\hbar\omega}} = \frac{1}{e^{\beta\hbar\omega} - 1}$$

Correct option is (a).

84. Given that, $I = 16\mu A = 16\times10^{-6}\,A$

Charge $\left(q_\alpha\right) = 2e = 2\times1.6\times10^{-19}\,C$

$$\therefore \qquad I = \frac{q_\alpha}{t} = \frac{n(2e)}{t} \qquad (\text{for } t = 1\,\text{sec})$$

$$\Rightarrow \qquad nq_\alpha = I \;\Rightarrow n = \frac{I}{q_\alpha} = \frac{16\times10^{-6}}{2\times1.6\times10^{-19}}$$

$$\Rightarrow \qquad \boxed{n = 0.5\times10^{14}}$$

Correct option is (a)

85. For a reaction, $X + x \rightarrow y + Y \quad or \quad X(x, y)Y$

where *x*, *y* are incident and emitted particles.

The cross-section $\sigma(x, y)$ of the reaction is given by

$$\sigma(x, y) = \frac{I_y}{I_x \times \text{number of nuclei}(x) \text{ per unit area}}$$

where I_x is number of particles incident per second

I_y is number of particles outgoing per second

In reaction, $\underset{\downarrow x}{\alpha} + \underset{\downarrow X}{^{100}Rh} \longrightarrow \underset{\downarrow Y}{^{101}Pd} + \underset{\downarrow y}{3n}$

$I_x = 0.5\times10^{14}$ particles/sec as calculated in Q.84

CAREER ENDEAVOUR

The above formula is for 1 to 1 corresponding i.e. for 1x particle 1y should be formed. In this equation 3n are formed.

At which number of α -particles hitting the target

$$I_x = 0.5\times10^{14} \text{ / sec}$$

At which neutrons are produced = 1.806×10^{8} / sec

Since, 3n are produced

$$\therefore \quad I_y = \frac{1}{3}\times1.806\times10^{8} \text{ / sec} = 0.602\times10^{8} \text{ / sec}$$

Now, to calculate number of Rh nuclei per unit area

100 gm of Rh possesss = 6.022×10^{23} nuclei$\left(\text{Avagadro No.}\right)$

Since, 1gm of Rh possess = 6.022×10^{21} nuclei$\left(\text{Avagadro No.}\right)$

Since, density = 10^4 kg/m^3 = 10^7 gm/m^3

Therefore, 10^7 gm of Rh possess = 6.022×10^{28} nuclei/m^3

Now, 1 area × thickness possess = 6.022×10^{28} nuclei/m^3

1 area × 1μm possess = $6.022\times10^{28} \times 10^{-6}$ nuclei/area

Therefore, number of Rh nuclei per unit area = 6.0222×10^{22}

$$\therefore \quad \sigma(x,y) = \frac{I_y}{I_x \times \text{number of Rh nuclei per unit area}} = \frac{0.602\times10^{8}}{0.5\times10^{14}\times6.022\times10^{22}} = 0.2\times10^{-28}\ m^2$$

$$= 0.2 \text{ barn} \qquad \left(1 \text{ barn} = 10^{-28}\ m^2\right)$$

Correct option is (b)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.20 : Carry ONE mark each.

1. $Tr(EFGH) = Tr(HEFG) = Tr(HFEG)$

Correct option is (a)

2. For unitary matrix, $U^{-1} = U^{\dagger} = \begin{bmatrix} a\,e^{-i\alpha} & c\,e^{-i\beta} \\ b & d \end{bmatrix}$

Correct option is (d)

3. $\vec{F} = 3z\hat{y} + 5y\hat{z}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 3z & 5y \end{vmatrix} = 2\hat{i}$$

Correct option is (b)

4. Rotational Kinetic energy is given as

$$T_R = \frac{1}{2}\vec{\omega} I \vec{\omega}$$

$$\frac{dT_R}{dt} = \frac{1}{2}\left(\frac{d\vec{\omega}}{dt} I\vec{\omega} + \vec{\omega} I \frac{d\vec{\omega}}{dt}\right)$$

$$= \frac{1}{2}\left(\vec{\alpha} I \vec{\omega} + \vec{\omega} I \vec{\alpha}\right) = \frac{1}{2}\left(\vec{\alpha}\cdot\vec{L} + \vec{\omega}\cdot\vec{\tau}\right)$$

where, $\vec{L} = I\vec{\omega} =$ angular momentum

$\vec{\tau} = I\,\vec{\alpha} =$ torque

Correct option is (c)

5. We need two generalised coordinates, one for rotation and the other for translation. However, after imposing the condition for rolling, one coordinate gets eliminated.

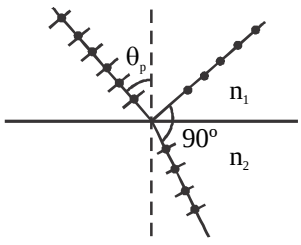
Hence correct answer is (b)

6. The electric field inside the two parallel plate is along $-\hat{z}$ and from right handed rule we can say the magnetic field along $-\hat{\theta}$.

Therefore, poynting vector, $\vec{s} = \vec{E} \times \vec{H}$

Therefpre. direction of poynting $\vec{s} = -\hat{z} \times -\hat{\theta} = -\hat{r}$

Correct option is (b)



7. We know that at $\theta = \theta_p$

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1.5}{1}$$

$$\Rightarrow \quad \theta_p = \tan^{-1}(1.5) = 56^{\circ}$$

Correct option is (c)

CAREER ENDEAVOUR

8. Since, the wave train is propagating along the positive x-axis with constant speed v.

So, wave equation will be $e^{i(kx-\omega t)} = \psi(x,t)$

$$\frac{\partial^2}{\partial x^2} = -k^2 \text{ and } \frac{\partial^2}{\partial t^2} = -i\omega \times -i\omega = -\omega^2$$

Speed = $v = \frac{\omega}{k}$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = -k^2 - \frac{k^2}{\omega^2} \times \left(-\omega^2\right) = 0$$

$$\Rightarrow \quad \left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = 0$$

Correct option is (a)

9. Since, the ground state of hydrogen atom does not depends on the ϕ

$$\left[L_x, L_y\right]\left[\psi_0\right] = i\hbar\hat{L}_z\left|\psi_0\right\rangle = \hbar^2\frac{\partial}{\partial\phi}\left|\psi_0\right\rangle = 0$$

Correct option is (a)

10. The parameters required are

MC : (E, V, N)

CE : (N, V, T)

GC : (V, T, μ)

Correct option is (b).

11. P-T curve is parallel to the temperature axis

$$\Rightarrow \quad \frac{dP}{dT} = 0$$

From Clausius-Clapeyron equation,

$$\frac{dP}{dT} = \frac{L}{T\Delta V} = \frac{\Delta S}{\Delta V}$$

$$\Rightarrow \quad \Delta S = 0$$

Correct option is (d).

12. Radial probability density of finding the electron

$$P(r) = \left|R_{10}\right|^2 r^2 = \frac{4}{a_0^3}e^{-2r/a_0} \cdot r^2$$

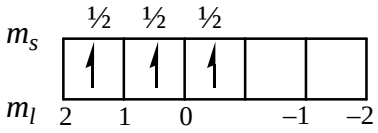
For most probable value of r, $\frac{dP}{dr} = 0 \Rightarrow \frac{4}{a_0^3}e^{-2r/a_0}\left(2r - \frac{2r^2}{a_0}\right) = 0$

$$\Rightarrow \quad 2r - \frac{2r^2}{a_0} = 0 \Rightarrow r = a_0$$

Correct option is (a)

CAREER ENDEAVOUR

13. For Mn^{4+} electronic configuration is $3d^3$. To fill 3 electrons



$$\therefore \quad S = \Sigma m_s = \frac{3}{2};\ L = \Sigma m_\ell = 3$$

$$\therefore \quad J = \left|3 - \frac{3}{2}\right| \text{ to } \left|3 + \frac{3}{2}\right| = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$$

For less than half filled ground state is for $J = \frac{3}{2}$

Therefore, term is ${}^4F_{3/2}$

Correct option is (b)

14. Length 'L' of the active medium is related with the wavelength λ of laser light by

$$L = \frac{n\lambda}{2} \qquad \text{... (i)}$$

Using $c = \nu\lambda \Rightarrow \lambda = \frac{c}{\lambda}$; $L = \frac{nc}{2\nu} \Rightarrow \nu_n = \frac{nc}{2L}$

Frequency width $\Delta\nu = \nu_{n+1} - \nu_n = \frac{c}{2L} \Rightarrow \frac{1}{\Delta\nu} = \frac{2L}{c}$

Coherence length ℓ is given by $\ell = \tau c$, where τ is coherent time

But $\ell = \left|\frac{1}{\Delta\nu}\right| \Rightarrow \ell = \left(\frac{2L}{c}\right)\cdot c \Rightarrow \ell \propto L$

i.e., coherent length is proportional to length of the using medium.

Correct option is (a)

15. The number of atom in the unit cell of B.C.C. = 2. Each atom will give one free electrons. So, the concentration of conduction electrons.

$$= \frac{2}{64\times10^{-24}} = 3.125\times10^{22}\ cm^{-3}$$

Correct option is (b)

16. Susceptibility, $\chi = \frac{c}{(T+T_c)} \Rightarrow \frac{1}{\chi} = (T+T_c)/c$ at $T = -T_c$, $\frac{1}{\chi} \to 0$

Correct option is (b)

17. K^+ has charge (Q) = +1, Strangeness (S) = 1, Baryon number (B) = 0

Since, the Baryon number is zero. Therefore, we need two quarks. One with positive sign and other with negative sign. Also strangeness is 1, we can take one strange quark with negative sign i.e. $\bar{s}$.

Now in order to give it charge (Q) = +1, we should add one up quark u so that its charge become $\left(\frac{2}{3} + \frac{1}{3}\right) = +1$.

CAREER ENDEAVOUR

Therefore, quark structure of K^+ meson is $u\bar{s}$.

Correct option is (c)

18. Since, radius $R \propto A^{1/3}$ and $V = \frac{4}{3}\pi R^3$

For ${}_8O^{16}$, $R_O \propto 16^{1/3}$ ... (i)

and for ${}_{54}Xe^{128}, R_{Xe} \propto 128^{1/3}$ ... (ii)

$$\frac{\text{equation (i)}}{\text{equation (ii)}} \Rightarrow \frac{R_O}{R_{Xe}} = \frac{16^{1/3}}{128^{1/3}} \Rightarrow \frac{R_O^3}{R_{Xe}^3} = \frac{16}{128} = \frac{1}{8}$$

$$\Rightarrow R_{Xe}^3 = R_O^3 \times 8 \Rightarrow \frac{4}{3}\pi R_{Xe}^3 = 8 \times \frac{4}{3}\pi R_O^3$$

$$\Rightarrow V_{Xe} = 8 \times V_O$$

Therefore, $V_{Xe} = 8V \quad (\because V_O = V)$

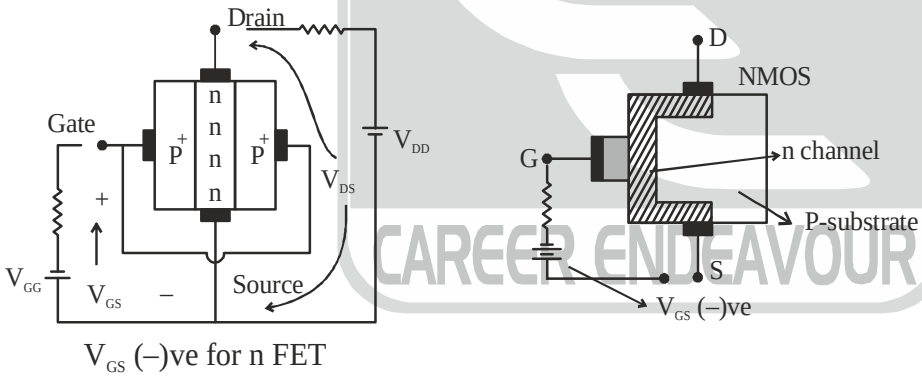
Correct option is (a)

19. As temperature increase the β of the transistor increase and as a result I_C also start to increase. So, Q point will be shift towards saturated region

Correct option is (d)

Q.21 – Q.75 : Carry TWO marks each.

20. n-FET



V_{GS} (–)ve for n FET

Hence, both biased with negative potential.

Correct option is (b)

21. Let the λ is the eigenvalue of the matrix. Therefore, the eigenvalue equation,

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} \cos\theta - \lambda & -\sin\theta \\ \sin\theta & \cos\theta - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (\cos\theta - \lambda)^2 = -\sin^2\theta \Rightarrow \cos\theta - \lambda = \pm i\sin\theta$$

$$\Rightarrow \lambda = (\cos\theta \pm i\sin\theta) = \frac{1}{2}\left(\sqrt{3} \pm i\right) \text{ when } \theta = 30^\circ$$

Correct option is (b)

22. $F\left[\delta\left(x-a\right)\right]=\exp\left[-i2\pi va\right]$

$$\Rightarrow \quad F^{-1}\left[\exp\left(-i2\pi va\right)\right]=\delta\left(x-a\right)$$

$$\Rightarrow \quad F^{-1}\left[\cos\left(2\pi va\right)\right]=\frac{1}{2}F^{-1}\left[\exp\left(i2\pi va\right)+\exp\left(-i2\pi va\right)\right]$$

$$=\frac{1}{2}\left[\delta\left(x+a\right)+\delta\left(x-a\right)\right]$$

Correct option is (d)

23. z = 0 is branch point of $f\left(z\right)$ and will behave like a simple pole.

$$\text{Res}\cdot f\left(z=0\right)=\underset{z\to 0}{Lt}\left(z-0\right)\ln z=\underset{t\to\infty}{Lt}\frac{1}{t}\cdot\ln\left(\frac{1}{t}\right)=-\underset{t\to\infty}{Lt}\frac{\ln t}{t}=0$$

Correct option is (a)

24. $$\int_{-i}^{i}\pi\left(z+1\right)dz=\pi\left[\frac{z^2}{2}+z\right]_{-i}^{i}=\pi\left[\frac{i^2}{2}+i-\frac{i^2}{2}+i\right]=2\pi i$$

Correct option is (b)

25. Consider the equation,

$$\frac{d^2y}{dz^2}+\frac{1}{z}\frac{dy}{dz}+y=0$$

$$\Rightarrow \quad z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}+z^2y=0$$

This is a Bessel differential equation with $v=0$

Here, $\dfrac{a_1\left(z\right)}{a_2\left(z\right)}=\dfrac{\text{coefficient of }\left(\dfrac{dy}{dz}\right)}{\text{coefficient of }\left(\dfrac{d^2y}{dz^2}\right)}=\dfrac{1}{z}$

$$\frac{a_0\left(z\right)}{a_2\left(z\right)}=\frac{\text{coefficient of }\left(y\right)}{\text{coefficient of }\left(\dfrac{d^2y}{dz^2}\right)}=1$$

Since, $\dfrac{a_1\left(z\right)}{a_2\left(z\right)}$ is not defined at $z=0$, therefore $z=0$ is a regular singular point of the differential equation.

The given Bessel differential equation has two linearly independent solutions i.e.
(i) $J_0(z)$ which is defined everywhere and it is entire solution
(ii) $Y_0(z)$ which is not defined at $z=0$ and it is singular (not analytic) at $z=0$
Correct option is (c)

26. $$v'_1=\frac{1}{\sqrt{2}}\left(v_1+v_2\right),\qquad v'_2=\frac{1}{\sqrt{2}}\left(-v_1+v_2\right),\ v'_3=v_3$$

CAREER ENDEAVOUR

Consider, $a_{11}=\frac{1}{\sqrt{2}}, a_{12}=\frac{1}{\sqrt{2}}, a_{13}=0$

$a_{21}=-\frac{1}{\sqrt{2}}, a_{22}=\frac{1}{\sqrt{2}}, a_{23}=0$

$a_{31}=1, a_{32}=0,\ a_{33}=0$

Rotation in rank-2 tensor matrix is given as

$T_{ij}=\sum_{\ell=1}^{3}\sum_{m=1}^{3}a_{i\ell}a_{jm}T_{\ell m}$ and keeping the same rotation we can write

$$T'_{ij}=\sum_{m=1}^{3}\left[a_{i1}a_{jm}T_{1m}+a_{i2}a_{jm}T_{2m}+a_{i3}a_{jm}T_{3m}\right]=\left(a_{i1}\,a_{j1}T_{11}+a_{i1}+a_{j2}T_{12}+.....\right)$$

$$T'_{11}=\begin{bmatrix}a_{11}a_{11}T_{11}+a_{11}a_{12}T_{12}+a_{11}a_{13}T_{13}+a_{12}a_{11}T_{21}+a_{12}a_{12}T_{22}\\+a_{12}a_{13}T_{23}+a_{13}a_{11}T_{31}+a_{13}a_{12}T_{32}+a_{13}a_{13}T_{33}\end{bmatrix}$$

$$T'_{11}=\frac{1}{2}T_{11}+\frac{1}{2}T_{12}+\frac{1}{2}T_{22}+\frac{1}{2}T_{21}$$

$$T'_{11}=\frac{1}{2}\left(T_{11}+T_{22}+T_{12}+T_{21}\right)$$

Correct option is (d)

27. $L=\frac{1}{2}\dot{q}^2+q\,\dot{q}-\frac{1}{2}q^2$

Equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}}\right)-\frac{\partial L}{\partial q}=0;\ \ \frac{d}{dt}\left(\dot{q}+q\right)-\dot{q}+q=0$$

$$\ddot{q}+\dot{q}-\dot{q}+q=0$$

$$\ddot{q}+q=0$$

This is equation of motion of a harmonic oscillator

Hence correct answer is (a)

28. Moment of inertia about given axis is

$$=n^T\,I\,n=\begin{pmatrix}\frac{1}{2}&\frac{\sqrt{3}}{2}&0\end{pmatrix}\begin{pmatrix}8&0&-4\\0&4&0\\-4&0&8\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\\frac{\sqrt{3}}{2}\\0\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{2}&\frac{\sqrt{3}}{2}&0\end{pmatrix}\begin{pmatrix}4\\2\sqrt{3}\\-2\end{pmatrix}=2+3+0=5$$

Hence correct answer is (b)

CAREER ENDEAVOUR

29. Time period of oscillation is

given as $t = 2\pi\sqrt{\frac{I}{mgd}}$

I = moment of inertia about axis of rotation

d = distance between point of suspension and center of mass.

Here $I = 2MR^2, d = R$

$\therefore \quad t = 2\pi\sqrt{\frac{2MR^2}{Mg \cdot R}} = 2\pi\sqrt{\frac{2R}{g}}$

Hence correct answer is (a)

30. Since the force is radial, angular momentum is conserved. If mass fall into the hole, its angular momentum will become zero. Which will violate conservation of angular momentum. Therefore it will not fall into the hole.
Hence correct answer is (d)

31. $A = \frac{p + iq}{\sqrt{2}}$

$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$

$\therefore$ Hamiltonian is $H = \frac{p^2}{2} + \frac{1}{2}q^2$

Poisson bracket $\{A, H\} = \frac{\partial A}{\partial q} \cdot \frac{\partial H}{\partial p} - \frac{\partial A}{\partial p} \cdot \frac{\partial H}{\partial q}$

$$= \frac{p}{\sqrt{2}} \cdot p - \frac{1}{\sqrt{2}} \cdot q = \frac{1}{\sqrt{2}}(ip - q) = \frac{i}{\sqrt{2}}(p + iq) = iA$$

Hence correct answer is (a)

32. We have, $E = E_0(\hat{x} + e^{i\delta}\hat{y})\exp\{i(kz - \omega t)\}$

So, $E_x = E_0, E_y = E_0 e^{i\delta}$

For $\delta = n\pi, n = 0,1,2.....$ it will be plane wave.

So, electric field will be zero $\frac{\omega}{\pi}$ times in one second.

But if $\delta \neq n\pi,\ n = 0,1,2$,

It will not be linear polarized light. E will not be zero.
Correct option is (a)

33. Since electronic permeability of dielectric material ε is greater than the ε_0. So, the field intensity inside the material will be increase.
Correct option is (d)

CAREER ENDEAVOUR

34. Since rod is rotated in the xy-plane the direction of J and A will be along $\hat{\phi}$

Since, $d >> L$, so we can consider rod as magnetic dipole, the vector potential due to magnetic dipole as

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{M}\times\vec{d}}{d^3}$$

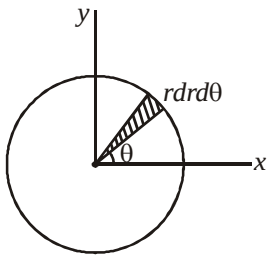
$$\vec{A} \propto \frac{1}{d^2}$$

Correct option is (b)

35. The elementary area on the disc.

$ds = rdrd\theta$

$\therefore$ Total charge on the disc is given by



$$Q = \iint \sigma \cdot ds = \frac{\sigma_0}{a}\int_a^{2\pi} \cos\theta d\theta \int_0^a r^2 dr = 0$$

So, dipole moment independent of choice of origin.

So, the position of any point on the disc is given by

$\bar{r} = r(\cos\theta\hat{x} + \sin\theta\hat{y})$

The dipole moment,

$$\bar{p} = \int r \cdot dq = \frac{\sigma_0}{a}\int_0^{2\pi} \cos^2\theta\, d\theta \int_0^a r^3 dr\hat{x} = \frac{\sigma_0 \pi a^3}{4}\hat{x}$$

Correct option is (b)

36. We know that, from equation of continuity

$$\vec{\nabla}\cdot\vec{J} + \frac{d\rho}{\partial t} = 0$$

$$\Rightarrow \quad \vec{\nabla}\cdot\vec{J} = -\frac{d\rho}{dt}$$

$$\Rightarrow \quad \left(\vec{\nabla}\cdot\vec{E}\right) = -\frac{d\rho}{dt} \quad \left(\because \vec{J} = \sigma\vec{E}\right)$$

$$\Rightarrow \quad \frac{\sigma\rho}{\varepsilon} = -\frac{d\rho}{\partial t} \Rightarrow \int\frac{d\rho}{\rho} = -\int\frac{\sigma}{\varepsilon}dt$$

$$\Rightarrow \quad \ln\rho = -\frac{\sigma}{\varepsilon}t + c$$

$$\Rightarrow \quad \rho(r,t) = \rho(r,0)e^{-\frac{\sigma t}{\varepsilon}}$$

$$\Rightarrow \quad \boxed{\rho(r,t) = \rho(r,0)\,e^{-\frac{\sigma t}{\varepsilon}}}$$

Correct option is (a)

37. **Correct option is (b)**

CAREER ENDEAVOUR

38. Since, Q_1 and Q_2 are compatible variable, therefore, they will commute with each other.

So, $Q_1Q_2|\psi\rangle = Q_2Q_1|\psi\rangle$

Correct option is (d)

39. Wrong question

40. For ground state of hydrogen atom, $\ell = 0 \quad s = \frac{1}{2}$

Therefore, $j = |\ell \pm s| = \frac{1}{2}$

Magnetic moment, $\vec{\mu}_J = \frac{q}{2m} g_J \vec{J} \Rightarrow \langle \vec{\mu}_J \rangle = \frac{qg_J}{2m}\left[\langle J_x\rangle \hat{i} + \langle J_y\rangle \hat{j} + \langle J_z\rangle \hat{k}\right]$

$\langle \vec{\mu}_J \rangle = -\frac{e}{2m} g_J \langle J_z\rangle \hat{k}$ (for ground state of hydrogen atom $\langle J_x\rangle = \langle J_y\rangle = 0$)

$= -\frac{e}{2m} \times 2 \times m_j \hbar \hat{k} = \mp \frac{e\hbar}{2m}\hat{k} = \mp \mu_B \hat{k}$

Therefore, $|\langle \vec{\mu}_J \rangle| = \mu_B$

Correct option is (b)

41. $|\psi_n\rangle = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \quad (0 \le x \le L)$

The probability of finding the particle between 0 to $\frac{L}{2}$ is

$$P = \frac{2}{L}\int_0^{L/2} \sin^2\frac{n\pi x}{L}dx = \frac{1}{L}\int_0^{L/2}\left(1-\cos\frac{2n\pi}{L}x\right)dx$$

$$= \frac{1}{L}[x]_0^{L/2} - \left[\frac{\sin\frac{2n\pi x}{L}}{2n\pi/L}\right]_0^{L/2} = \frac{1}{2}$$

Correct option is (c)

42. At $z = 0$, momentum will be maximum and when z will be maximum then momentum will be zero. Again when z will start decrease momentum will be start to increase but direction will be opposite. And since in the inelastic collision. So, the momentum of the ball will be less than the initial value.

Correct option is (c)

43. The energy of a particle with magnetic moment μ in an external magnetic field B is

$$E = -\vec{\mu}\cdot\vec{B} = \begin{cases} \mu B & \text{for } s = -\frac{1}{2} \text{ particle} \\ -\mu B & \text{for } s = +\frac{1}{2} \text{ particle} \end{cases}$$

The partition function of a particle is

$$Z_1 = e^{-\mu B/k_BT} + e^{\mu B/k_BT} = 2\cosh\left(\frac{\mu B}{k_BT}\right)$$

The partition function for N non-interacting such particles is

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$$Z_N = (Z_1)^N = \left[2\cosh\left(\frac{\mu B}{k_B T}\right)\right]^N$$

The Helmholtz free energy is $F = -k_B T\ \ell n\, Z_N$

$$F = -N\, k_B T\ \ell n\left[2\cosh\left(\frac{\mu B}{k_B T}\right)\right]$$

The magnetization is $\langle m\rangle = -\left(\frac{\partial F}{\partial B}\right)_T = N\, k_B T\, \frac{2\sinh\left(\frac{\mu B}{k_B T}\right)}{2\cosh\left(\frac{\mu B}{k_B T}\right)} \cdot \frac{\mu}{k_B T}$

$$= N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$$

Correct option is (b).

44. The distributions are as follows :

Bosons

1	2	3
•	•	
	•	•
•		•
••		
	••	
		••

$r_B = \frac{P(2)}{P(1)} = \frac{1/6}{2/6} = \frac{1}{2}$

Fermions

1	2	3
•	•	
•		•
	•	•

$r_F = \frac{P(2)}{P(1)} = \frac{0}{2/3} = 0$

Classical particles

1	2	3
A	B	
B	A	
	A	B
	B	A
A		B
B		A
AB		
	AB	
		AB

$r_C = \frac{P(2)}{P(1)} = \frac{1/6}{4/6} = \frac{1}{4}$

Here we have choosen 1-state to be the given energy state.

So the ratio is $r_B : r_F : r_C = \frac{1}{2} : 0 : \frac{1}{4} = 1 : 0 : \frac{1}{2}$

Correct option is (d).

45. The photons are bosons and hence follow Bose-Einstein statistics.
The mean number of photons are

$$\langle n\rangle = \frac{1}{\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) - 1}, \text{ where } \mu = 0 \text{ (for photons) and } \varepsilon = \hbar\omega \text{ (given)}$$

$$\therefore \quad \langle n\rangle = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} = \left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1\right]^{-1}$$

Correct option is (d).

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46.

A		B		AB
N atoms, T, V	+	N atoms, T, V	$\Rightarrow$	$2N$ atoms, $T, 2V$

Since the gases are ideal in nature, then internal energy depends only on the temperature. Furthermore, temperature of the mixture remains T as $T_A = T_B = T$.

i.e., $(\Delta T)_{\text{mixture}} = 0$

$\Rightarrow \quad \Delta U = 0$

From first law of thermodynamics,

$$dQ = dU + pdV$$

$$= pdV \qquad [\because dU = 0]$$

The entropy change of the system is

$$\Delta S = (\Delta S)_1 + (\Delta S)_2$$

$$= \int \frac{dQ_1}{T} + \int \frac{dQ_2}{T}$$

$$= \int \frac{pdV}{T} + \int \frac{pdV}{T}$$

$$= N k_B \int_V^{2V} \frac{dV}{V} + N k_B \int_V^{2V} \frac{dV}{V} \qquad [\because pV = N k_B T]$$

$$= 2 N k_B \ell n\, 2$$

Correct option is (b)

47. The mean energy is

$$\langle E \rangle = \frac{\sum_i E_i\, g(E_i) e^{-E_i/k_B T}}{\sum_i g(E_i) e^{-E_i/k_B T}}$$

$$= \frac{0.1e^{-0/k_B T} + \varepsilon.2.e^{-\varepsilon/k_B T} + 2\varepsilon.4.e^{-2\varepsilon/k_B T}}{1e^{-0/k_B T} + 2e^{-\varepsilon/k_B T} + 4e^{-2\varepsilon/k_B T}}$$

$$= \frac{2\varepsilon e^{-\varepsilon/k_B T} + 8\varepsilon e^{-2\varepsilon/k_B T}}{1 + 2e^{-\varepsilon/k_B T} + 4e^{-2\varepsilon/k_B T}}$$

$$= \varepsilon \left[\frac{2e^{-\varepsilon/k_B T} + 8e^{-2\varepsilon/k_B T}}{1 + 2e^{-\varepsilon/k_B T} + 4e^{-2\varepsilon/k_B T}} \right]$$

Correct option is (b).

48. The wave number of three consecutive lines are 64.275 cm^{-1}, 77.130 cm^{-1} and 89.985 cm^{-1}.

..... 89.985 cm^{-1} } 2 B
..... 77.130 cm^{-1} } 2 B
..... 64.275 cm^{-1} }

Therefore, the difference in wave number

$$\Delta\bar{\nu} = 2B = 89.985 - 77.130$$

$$\Rightarrow \quad 2B = 12.855 \quad \Rightarrow \qquad 2 \times \frac{h}{8\pi^2 I_B c} = 12.855$$

$$\Rightarrow \qquad I_B = \frac{2h}{8\pi^2 c(12.855)} = \frac{\hbar^2}{12.855\, hc}\ \text{gm cm}^2$$

$I_A = 0$ ($\because$ r = 0, from the bond axis passing through the center of mass).

Hence, correct option is (b).

49. The wave number of the rotational Raman lines are given by

$$\bar{\nu} = \nu_e \pm 4B\left(J + \frac{3}{2}\right);\ J = 0,1,2,3,....$$

$$= \nu_e \pm 6B,\ 10B,\ 14B,\ 18B,....$$

Stoke's lines is corresponding of (–) values

i.e., $\bar{\nu}_{\text{stokes}} = \nu_e - 10B,\ \nu_e - 14B,\ \nu_e - 18B,...$

Correct option is (a)

50. For vibrational spectroscopy with anharmonicity the selection rule is $\Delta\nu = \pm 1, \pm 2, \pm 3,....$ The anharmonicity leads to multiple absorption lines. The intensity of hot band lines are stronger than the fundamental absorption is incorrect statement as fundamental lines are most intense.

Hence, correct option is (c).

51. The CO_2 molecule do not have permanent dipole moment while COS have permanent dipole moment. Thus CO_2 molecule will not show absorption lines.

Hence, correct option is (d).

52. Length of the resonator must satisfy relation $L = \frac{n\lambda}{2}$, where λ is wavelength of laser light, n is number of mode. Velocity of light in a medium of refractive index μ is $\nu = \frac{c}{\mu} = \nu\lambda, \Rightarrow \lambda = \frac{c}{\mu\nu}$ where ν is frequency of light

$$\Rightarrow \quad L = \frac{n\lambda}{2} \Rightarrow L = \frac{nc}{2\mu\nu} \quad \Rightarrow \quad \nu = \frac{nc}{2\mu L}.\ \text{For } n = 1 \text{ frequency } \nu = \frac{c}{2\mu L}$$

If length L is kept constant but refractive index μ is changed from $\mu \to \mu + \Delta\mu$, frequency becomes

$$\nu' = \frac{c}{2(\mu + \Delta\mu)L}$$

Change in frequencies

$$\Delta\nu = \nu - \nu' = \frac{c}{2\mu L} - \frac{c}{2(\mu + \Delta\mu)L} = \frac{c}{2L}\left[\frac{1}{\mu} - \frac{1}{\mu + \Delta\mu}\right] = \frac{c}{2L}\left[\frac{\Delta u}{\mu(\mu + \Delta\mu)}\right]$$

$$= \frac{c}{2L}\left[\frac{\Delta\mu}{\mu(\mu + \Delta\mu)}\right]$$

Correct option is (c)

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53. Volume of the bcc lattice, $V = \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

$$= \frac{a}{2}\left(\hat{x}+\hat{y}-\hat{z}\right)\cdot\left[\frac{a}{2}\left(-\hat{x}+\hat{y}+\hat{z}\right)\times\frac{a}{2}\left(\hat{x}-\hat{y}+\hat{z}\right)\right]$$

$$= \left(\frac{a^3}{8}\right)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \frac{a^3}{8}\left[1(1+1)-1(-1-1)-1(1-1)\right]$$

$$= \frac{a^3}{8}\left[2+2\right] = \frac{a^3}{2}$$

$\therefore$ Primitive translation vectors of reciprocal lattice are

$$\vec{A} = \frac{2\pi\left(\vec{b}\times\vec{c}\right)}{a\cdot\left(\vec{b}\times\vec{c}\right)} = \frac{2\pi\left[\frac{a}{2}\left(-\hat{x}+\hat{y}+\hat{z}\right)\times\frac{a}{2}\left(\hat{x}-\hat{y}+\hat{z}\right)\right]}{\left(\frac{a^3}{2}\right)}$$

$$= \frac{2\pi}{\left(\frac{a^3}{2}\right)}\cdot\left(\frac{a^2}{4}\right)\left[-\hat{x}\times\hat{x}+\hat{x}\times\hat{y}-\hat{x}\times\hat{z}+\hat{y}\times\hat{x}-\hat{y}\times\hat{y}+\hat{y}\times\hat{z}+\hat{z}\times\hat{x}-\hat{z}\times\hat{y}+\hat{z}\times\hat{z}\right]$$

$$= \frac{2\pi}{2a}\left[\hat{z}+\hat{y}-\hat{z}+\hat{x}+\hat{y}+\hat{x}\right] = \frac{\pi}{a}\left[2\left(\hat{x}+\hat{y}\right)\right] = \frac{2\pi}{a}\left(\hat{x}+\hat{y}\right)$$

Similarly, $\vec{B} = \frac{2\pi}{a}\left(\hat{y}+\hat{z}\right)$ and $\vec{C} = \frac{2\pi}{a}\left(\hat{z}+\hat{x}\right)$

Hence, correct option is (d).

54. **For bcc structure:** (0, 0, 0) and $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ are atomic positions

$\therefore \quad S_{hkl} = f\left[1+e^{i\pi(h+k+l)}\right]$ ... (i)

For odd value of $(h+k+l)$ above term will be zero. For, even value of $(h+k+l)$, $S_{hkl} = 2f$.

Therefore, for bcc structure, reflections like (100), (111), (210) etc. are missing whereas the diffraction lines corresponding to (110), (200), (222) etc. are present.

For fcc structure: An fcc unit cell has four identical atoms. One of the atoms is contributed by corners and may arbitrarily be assigned coordinates (0, 0, 0), whereas other three are contributed by face-centers and have the coordinates $\left(\frac{1}{2},0,\frac{1}{2}\right)$, $\left(\frac{1}{2},\frac{1}{2},0\right)$ and $\left(0,\frac{1}{2},\frac{1}{2}\right)$.

$\therefore \quad S_{hkl} = f\left[1+e^{\pi i(h+l)}+e^{\pi i(h+k)}+e^{\pi i(k+l)}\right]$ ... (ii)

From above equation (ii), it is clear that structure factor is non-zero only if h, k and l are all even or all odd and has a value equal to 4. Hence reflections of the type (111), (200), (222) etc. are present, whereas those of the type (100), (110), (211) etc. are absent for an fcc crystal.

From the given options : (a) $\left.\begin{matrix}\text{bcc}: & (200); & (110); & (222) \\ \text{fcc}: & (111); & (311); & (400)\end{matrix}\right\}$ is correct.

Hence, correct option is (a).

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55. According to Dulong-Petit law : $C_V = 3R = \text{constant}$ ... (i)

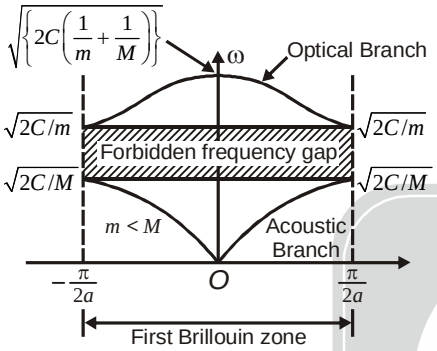
According to Einstein model : $C_V = 3Nk_B\left(\frac{\theta_E}{T}\right)^2 e^{-\theta_E/T} \propto \frac{e^{-a/T}}{T^2}$ ... (ii)

According to Debye model : $C_V = \frac{12}{5}\pi^4 Nk_B\left(\frac{T}{\theta_D}\right)^3 \propto \left(\frac{T}{A}\right)^3$ ... (iii)

Hence, correct option is (c).

56. Maximum frequency of aquostic branch = $\sqrt{\frac{2C}{M}}$

It depends on mass of heavier atoms, not on the mass of lighter atoms



Dispersion curves for linear diatomic lattice showing acoustical and optical modes.

From above figure, it is clear that ω_{optical} is larger than $\omega_{\text{acoustical}}$.

Hence, correct option is (b).

57. The kinetic energy of a free electron at a corner and center of the first Brillouin zone in two-dimensional square lattice is given by

$$\therefore \quad E_{\text{corner}} = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right) = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2\right] = \frac{\hbar^2}{2m}\left(\frac{2\pi^2}{a^2}\right)$$

and $E_{\text{center}} = \frac{\hbar^2}{2m}\left(k_x^2\right) = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 = \frac{\hbar^2}{2m}\left(\frac{\pi^2}{a^2}\right)$

The ratio of kinetic energies = b

$$\Rightarrow \quad \frac{E_{\text{corner}}}{E_{\text{center}}} = b \quad \Rightarrow \quad \frac{\frac{\hbar^2}{2m}\left(\frac{2\pi^2}{a^2}\right)}{\frac{\hbar^2}{2m}\left(\frac{\pi^2}{a^2}\right)} = b$$

$$\Rightarrow \quad b = 2$$

Hence, correct option is (b).

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58. We know that the electron density in the conduction band

$$n = \frac{1}{4}\left(\frac{2m_e k_B T}{\pi\hbar^2}\right)^{3/2} e^{(E_F - E_C)/k_B T}$$

And the density of hole at the valence band

$$p = \frac{1}{4}\left(\frac{2m_h k_B T}{\pi\hbar^2}\right)^{3/2} e^{(E_V - E_F)/k_B T}$$

For an intrinsic semiconductor the numbers of electrons in the conduction band is equal to the number of holes in the valence band only when an electron make a transition to the condition band.

$\therefore \quad n = p$

$$\Rightarrow \quad \frac{1}{4}\left(\frac{2m_e k_B T}{\pi\hbar^2}\right)^{3/2} e^{E_F - E_C/k_B T} = \frac{1}{4}\left(\frac{2m_h k_B T}{\pi\hbar^2}\right)^{3/2} e^{E_v - E_F/k_B T}$$

$$\Rightarrow \quad \left(\frac{m_e}{m_h}\right)^{3/2} = e^{\frac{E_v + E_C - 2E_F}{k_B T}}$$

Taking log both side we get

$$\frac{3}{2}\ell n\left(\frac{m_e}{m_h}\right) = \frac{E_v + E_C - 2E_F}{k_B T} \quad \Rightarrow \quad E_F = \frac{E_v + E_c}{2} + \frac{3}{4}k_B T \ell n\left(\frac{m_h}{m_e}\right)$$

Correct option is (c)

59. For (a) the reaction $K^+K^- \longrightarrow pp^-$ can proceed irrespective of the kinetic energies of K^+ and K^-, this statement is not correct.

For (b) $K^+K^- \longrightarrow pp^-$

Baryon number (B): $0+0 \rightarrow 1-1 \Rightarrow \Delta B = 0$ (allowed). This statement is not correct.

For (c) $K^+K^- \longrightarrow 2\gamma$;

Strangeness (S) : $1-1 \rightarrow 0+0 \Rightarrow \Delta S = 0$ (allowed). This statement is not correct.

(d) The decay $K^0 \longrightarrow \pi\pi^-$; proceeds via weak interaction. This statement is correct.

Correct option is (d)

60. For (a), ${}_{10}Ne^{20}$ nucleus is stable ($\because$ A = 20 is a magic number).
For (b), a spheroidal nucleus shows an electric quadrupole moment.
For (c), ${}_8O^{16}$
For Z = N = even i.e. even-even nuclei, nuclear spin J = 0 and not ½.
Correct option is (c)

61. Since, ${}_4Be^9$ has Z = 4, N = 9 – 4 = 5
Therefore, for N = 5, $(^1s_{1/2})^2$ $(^1p_{3/2})^3$

and nuclear spin $J = \frac{3}{2}$

Correct option is (a)

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62. For mirror nucleus $_6C^{11}$ and $_5B^{11}$

Mass difference between mirror nuclei is

$$M(A,Z)-M(A,Z-1)=M_p-M_n+a_3A^{2/3}$$

For $_6C^{11}$ and $_5B^{11}$ mass difference is $\frac{\Delta MeV}{c^2}$

$$\Rightarrow \quad \Delta M_p-M_n+a_3(11)^{2/3} \Rightarrow \Delta=-1.3+a_3(11)^{2/3}$$

$$\Rightarrow \quad a_3=[\Delta+1.3]/(11)^{2/3}$$

Now for $_9F^{17}$ and $_8O^{17}$

Mass difference = $M_p-M_n+a_3(17)^{2/3}$

Putting value of a_3.

Therefore, mass difference $=-1.3+[\Delta+1.3]\left(\frac{17}{11}\right)^{2/3}=-1.3+1.336\Delta+1.73=(1.336\Delta+0.437)\frac{\text{MeV}}{c^2}$

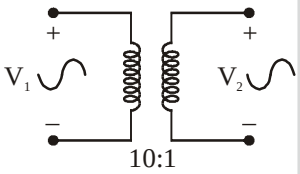
Correct option is (b)

63. Since, proton is charged particle. Therefore, it costs more energy to add extra proton in nucleus in order to minimize the Coulomb repulsion.

Correct option is (d)

64.

65.

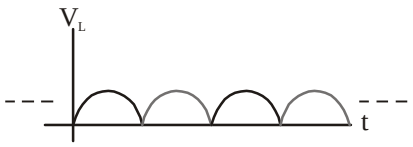


We know transfermer ratio $\frac{V_1}{V_2}=\frac{N_1}{N_2}$

$$\frac{V_1}{V_2}=\frac{10}{1} \qquad \Rightarrow \quad V_{2_{rms}}=\frac{220}{10}$$

$$V_{2_{rms}}=V_{2_{rms}}=22$$

For full wave rectifier output will be



DC voltage, $V_{DC}=\frac{2V_m}{\pi}=\frac{2\times22}{\pi}=\frac{44}{\pi}$

Correct option is (d)

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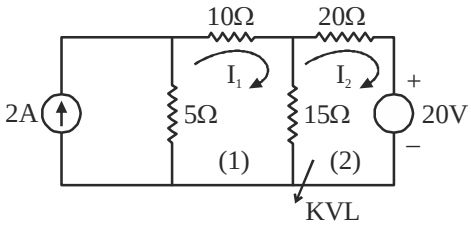
66.



Applying KVL at Loop (2)

$$20I_2 + 20 + 15(I_2 - I_1) = 0$$

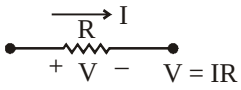
$$-15I_1 + 35I_2 = 20 \quad -15I_1 + 35I_2 = 20$$

Applying KVL at Loop (1)

$$10I_1 + 15(I_1 - I_2) + 5(I_1 - 2) = 0$$

$$30I_1 - 15I_2 = 10$$

Note:-



Correct option is (a)

67. Given $Y = P + \bar{P}Q$

$$Y = (P + \bar{P})(P + Q)$$

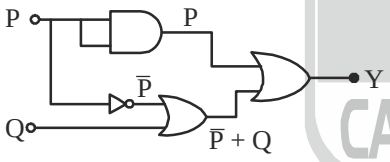
$Y = P + Q$ {A + BC = (A + B)(A+C), by distribution theorem}

{ $A + \bar{A} = 1$ by boolean theorem}

Check the output of given option i.e. $Y = P + Q$

(a)

(b)



$Y = P + \bar{P} + Q$

$Y = 1 + Q$

$Y = 1$

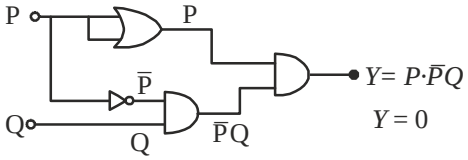
Note:- $x + \bar{x} = 1$

$P + \bar{P} = 1$ axioms/property

(c)

$Y = P + Q$

(d)



$Y = 0$

Correct option is (c)

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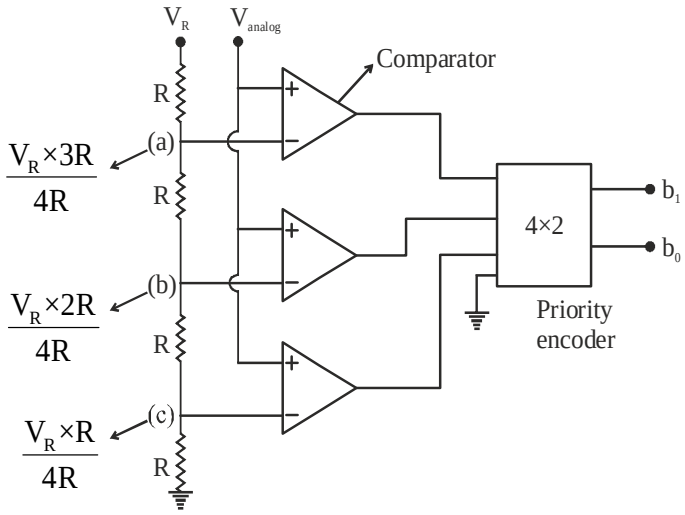
68. 2-bit flash "ADC"



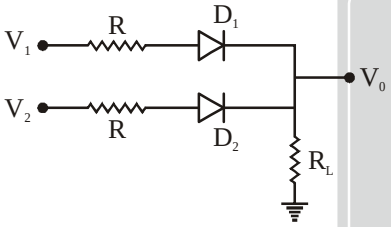
- Number of comparator $= 2^n - 1 = 2^2 - 1 = 3$

Note : (i) Number of resistor $= 2^n$

(ii) Reference voltage are mentioned at point (a), (b) and (c) (using voltage division rule)

Correct option is (a)

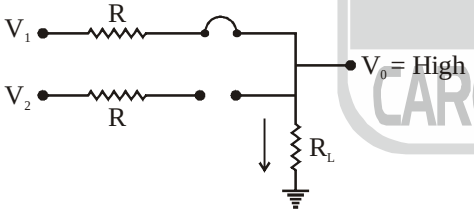
69. [Analysing CKT for Positive Logic]



Case - (1)

V_1 High

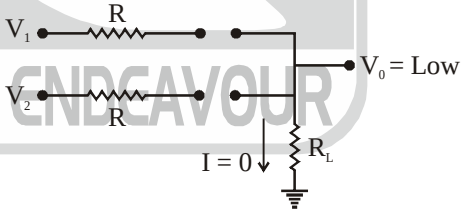
V_2 Low



Case - (2)

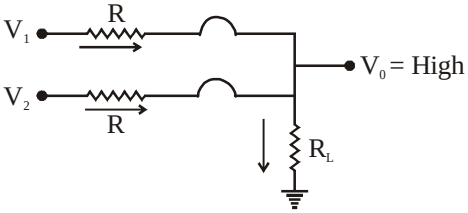
V_1 = low

V_2 = low



Case - (3) :

V_1 and V_2 High



Truth Table

V_1	V_2	D_1	D_2	V_0
0	0	OFF	OFF	0
0	1	OFF	ON	1
1	0	ON	OFF	1
1	1	ON	ON	1

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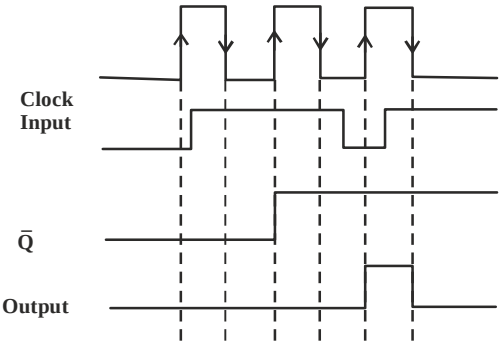
Note: CKT is (+)ve logic-OR operation or , (–)ve logic AND operation.
Duality convert (+)ve logic to (–)ve logic and vice versa.
Correct option is (d)

70.



71. The schrodinger equation between $0 < x < L$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V_0\psi = E\psi$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2}-\frac{2m}{\hbar}(V_0-E)\psi = 0 \Rightarrow \frac{d^2\psi}{dx^2}-k^2\psi = 0 \text{, where } k=\sqrt{\frac{2m}{\hbar^2}(V_0-E)}$$

$$\Rightarrow \quad \psi = A\,e^{kx}+B\,e^{-kx}$$

Correct option is (a)

72. Transmission coefficient, $T=\dfrac{|\text{transmitted current density}|}{|\text{Incident current density}|} \Rightarrow T=\dfrac{\text{transmitted current density}}{S_0}$

$$\Rightarrow \quad \text{transmitted current} = TS_0$$

Correct option is (b)

73. According to energy conservation

R + T = 1

$$\Rightarrow \quad \frac{|A_R|^2}{|A_0|^2}+\frac{|A_T|^2}{|A_0|^2}=1 \Rightarrow \left|\frac{A_B}{A_0}\right|^2=1-T \Rightarrow \frac{A_B}{A_0}=\sqrt{1-T}$$

Correct option is (b)

74. Since, charge remain outer surface of a conductor.
So, $E = 0$ for $0 < r < a$ (since there are no charge for $r < a$)
Also, $E = 0$ for $a < r < b$ (since there are no charge for $a < r < b$)

$E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ for $b < r < c$ (due to charge on the outer surface of the inner sphere)

And $E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$ for $r > d$ (Q charge for outer sphere and another Q induce charge will be appear at the outer surface)

Correct option is (d)

CAREER ENDEAVOUR

75. Since, both surface have equal surface charge density

$$\therefore \quad \frac{Q}{4\pi b^2} = \frac{2Q}{4\pi d^2} \Rightarrow d^2 = 2b^2 \Rightarrow d = \sqrt{2}\, b \text{ and } c > a$$

Correct option is (c)

76. β-de cay of a free neutron

$${}_Z X^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}_e$$

which is written in terms of n and p as

$$n \rightarrow P + e + \bar{\nu}$$

For anti-neutrino to have maximum energy, electron should produce at rest, as P or nucleus Y will have almost negligible energy as they are heavier. Energy released Q is always distributed among electron and $\bar{\nu}_e$ as their kinetic energies. For maximum anti-neutrino energy electron should be produces at rest.

Correct option is (d)

77. $n \rightarrow P + e + \bar{\nu}$

$$\text{Q value} = \left[m_n - \left(m_p + m_e + m_{\bar{\nu}_e}\right)\right]c^2 = \left[939.57 - \left(938.27 + 0.51 + 0\right)\right]c^2$$

$$= 0.79\ MeV \simeq 0.8\ MeV$$

This whole energy goes to $\bar{\nu}_e$ -for maximum energy.

Correct option is (b)

78. The density of state for 2-D electron gas is

$$g(p)\,dp = \frac{2\pi p \cdot A dp}{h^2}, \text{ where } A = L^2 \text{ and } p = \sqrt{2mE}$$

$$\therefore \quad g(E)\,dE = \frac{2\pi L^2}{h^2}\sqrt{2mE}\,\frac{\sqrt{2m}}{2\sqrt{E}}\,dE$$

$$= \frac{2m\pi L^2}{h^2}\,dE$$

Since electron spin degeneracy is 2 so we should multiply density of states by a factor of 2.

$$g(E)\,dE = \frac{4\pi m L^2}{h^2}\,dE$$

Correct option is (a).

79. Total number of particles is

$$N = \int_0^\infty f(E)\,g(E)\,dE, \text{ where } f(E) = \begin{cases} 1, & E \le E_F \\ 0, & E > E_F \end{cases} \text{ at } 0K$$

$$= \int_0^{E_F} \frac{4\pi L^2 m}{h^2}\,dE = \frac{4\pi L^2 m}{h^2} E_F$$

$$\Rightarrow \quad E_F = \frac{h^2 N}{4\pi L^2 m}$$

Also the ground state energy is

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$$E_0 = \int_0^{E_F} E\, f(E)\, g(E)\, dE = \int_0^{E_F} \frac{4\pi L^2 m}{h^2} E\, dE$$

$$= \frac{4\pi L^2 m}{h^2} \frac{E_F^2}{2} = \frac{N}{E_F} \cdot \frac{E_F^2}{2} = \frac{N}{2} E_F$$

Correct option is (b).

80. According to question

$$(Rate)_{spaceship} = \frac{3}{5}(Rate)_{earth} \quad \text{i.e. } (time)_{spaceship} = \frac{5}{3}(time)_{earth}$$

using time dilation formula we get

$$\frac{(time)_{earth}}{\sqrt{1 - v^2/c^2}} = \frac{5}{3}(time)_{earth}$$

$$\therefore \quad 1 - v^2/c^2 = \left(\frac{3}{5}\right)^2 \qquad \therefore \quad v = \frac{4}{5}c$$

Hence correct answer is (a)

81. $$(time)_{Aakashganga} = \frac{13}{5}(time)_{earth}$$

$$\frac{(time)_{earth}}{\sqrt{1 - v^{12}/c^2}} = \frac{13}{5}(time)_{earth}$$

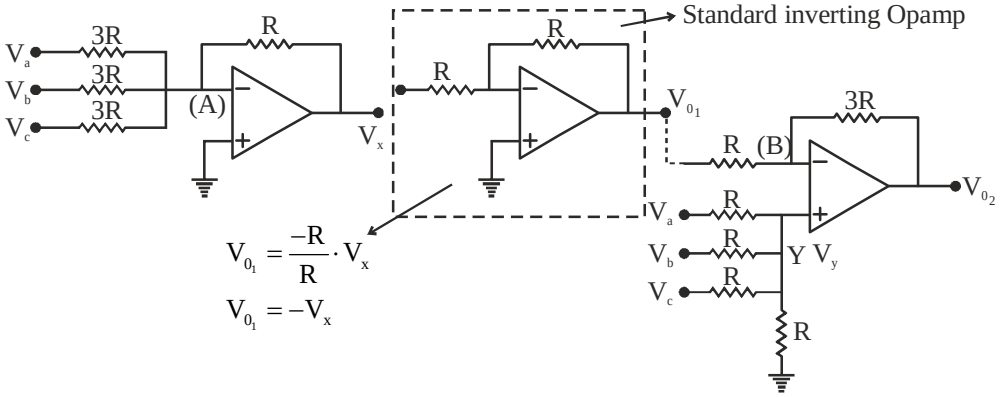
$$\therefore \quad \frac{1}{\sqrt{1 - \frac{v^{12}}{c^2}}} = \frac{13}{5} \qquad \therefore \quad v' = \frac{12}{13}c$$

Therefore, Speed of Aakashganga relative to Suryashakti is

$$V_{as} = \frac{v' - v}{1 - \frac{v' \cdot v}{c^2}} = \frac{\frac{12}{13}c - \frac{4}{5}c}{1 - \frac{48}{65}} = \frac{8c}{17}$$

Hence correct answer is (c)

82.



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As ideal op-amp has infinite input resistance, so there will no current pass through the op-amp and V_A will be virtual ground i.e. $V_A = 0$

(i) Applying KCL at node (A)

$$\frac{0-V_a}{3R}+\frac{0-V_b}{3R}+\frac{0-V_c}{3R}+\frac{0-V_x}{R}=0$$

$$V_x=-\frac{1}{3}\left[V_a+V_b+V_c\right]$$

$$V_{0_1}=-V_x=\frac{V_a+V_b+V_c}{3}$$

Correct option is (c)

83. (ii) Applying KCL at node (B)

(iii) As ideal op-amp has infinite input resistance. So, V_B & V_y will be virtual short i.e. $V_B = V_y$

$$\frac{V_y-V_{0_1}}{R}+\frac{V_y-V_{0_2}}{3R}=0$$

$$V_y\left[\frac{1}{R}+\frac{1}{3R}\right]-\frac{V_{0_1}}{R}=\frac{V_{0_2}}{3R}$$

$$\frac{4}{3R}\cdot V_y-\frac{V_{0_1}}{R}=\frac{V_{0_2}}{3R}$$

Put value of V_y and V_{0_1}

$$\frac{V_a+V_b+V_c}{3}-\left(\frac{V_a+V_b+V_c}{3}\right)=\frac{V_{0_2}}{3}$$

$$V_{0_2}=0$$

For calculation of V_y, applying KCL at node (Y)

$$\frac{V_y}{R}+\frac{V_y-V_a}{R}+\frac{V_y-V_b}{R}+\frac{V_y-V_c}{R}=0$$

$$4V_y=\frac{V_a+V_b+V_c}{1}$$

Correct option is (d)

84. We have, $f(x)=a_0+a_1x+a_2x^2$ and $q(x)=b_0+b_1x+b_2x^2$

$$\langle f(x)g(x)\rangle=a_0b_0+a_1b_1+a_2b_2$$

85. Let $R(x)=1+x=1+x+0\cdot x^2$

If $f(x)$ is the subspace of V that is orthogonal to $R(x)$ then $\langle R(x)\cdot f(x)\rangle=0$

Only option (a) follow the above condition

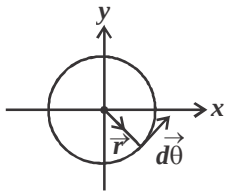
Correct option is (a)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.20 : Carry ONE mark each.

1. $\left|\int_C \vec{r}\times d\vec{\theta}\right| = \int_C r d\theta \sin 90^\circ = r\int_0^{2\pi} d\theta = 2\pi r$



Correct option is (a)

2. Let any two points on the region A and B
We know that,

$$\text{Potential difference } \int_{V_B}^{V_A} dV = -\int_B^A \vec{E}\cdot d\vec{r} \Rightarrow \Delta V_{AB} = \vec{E}\cdot\overrightarrow{AB} \neq 0$$

Correct option is (c)

3. $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta\right)$

coordinate ϕ is cyclic, threfore canonical momentum $p_\phi = \frac{\partial L}{\partial \dot{\phi}}$ is conserved.

Hence correct answer is (c)

4. Let consider an elementary surface is
Therefore, the magnetic flux pass through the elementary area dS is

$$d\varphi = B\cdot dS$$

$\therefore$ Induced emf $= -\frac{\partial\varphi}{dt} = -\int\frac{\partial\vec{B}}{\partial t}dS$

Correct option is (a)

5. Eigenvalue equation : $\begin{vmatrix} -\lambda & i \\ i & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - i^2 = 0 \Rightarrow \lambda = \pm i$

Correct option is (b)

6. $\det(\sigma_i) = -1$

Correct option is (d)

7. For bound state $\psi \to 0$ as $x \to \infty$

$$-\frac{d^2}{dx^2}\left[A_0 e^{-2x}\right] = +2\frac{d}{dx}A_0 e^{-2x} = -4\left[A_0\, e^{-2x}\right]$$

Correct option is (c)

8. Nuclear force is charge independent.
Yukawa potential is given by

$$V(r) = -V_0\frac{e^{-ar}}{r}$$

where, $V_0 = \alpha z$, α is fine structure constant

z is atomic number, a is screening parameter

The range of the nuclear force is of the order of two fermi

According to Yukawa Mesons Theorem, mesons are exchanged during nucleons interaction.

Wrong statement is (b)

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9. For canonical transforma $PB\{Q, P\}_{q,p} = 1$

i.e. $\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q} = 1$

(a) $\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q} = \alpha\cdot\frac{1}{\alpha} - 0 = 1$

(b) $\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q} = \alpha\cdot\alpha - \beta\cdot\beta = \alpha^2 - \beta^2 = 1$

(c) $\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q} = 0 - 1.1 = -1$

(d) $\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q} = 0 - 1\times(-1) = 1$

Therefore, transformation given in option (c) is not a canonical transformation.
Hence correct answer is (c)

10. Given $CMRR = 100\ dB$

$\rho = CMRR$

$20\log_{10}\rho = 100\ dB = \rho_{dB} \Rightarrow$ V_0 for differential input $\frac{2V}{200\mu V}$

A_d (differential gain) = 10000

$A_{d_{(dB)}} = 20\log_{10}10000$

$\rho = \frac{A_d}{A_c}$

A_d = differential gain, A_c = Common mode gain

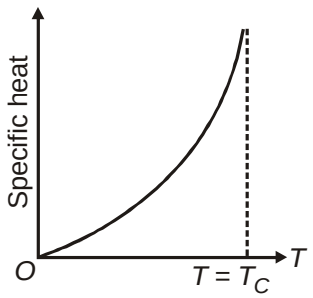
$\rho_{dB} = A_{d_{(dB)}} - A_{c_{(dB)}}$

$100 = 80 - A_{c_{dB}} \Rightarrow -20_{dB} = A_{c_{dB}}$

$-20 = 20\log_{10} A_c \Rightarrow A_c = 0.1$

Correct option is (b)

11. For ferromagnetic substance, the specific heat is maximum at T_c. Thus correct graph of specific heat versus T is expressed as : $C_V = -2\lambda\ M_s\left(\frac{\partial M_s}{\partial T}\right)$, where ; $M_s = N\mu\tan h\left(\frac{\mu_0 h\lambda M_s}{k_B T}\right)$



Thus, option (d) is not correctly shown.
Hence, correct option is (d).

12. $k_1 \frac{dx}{dt} + k_2 x = k_3 \Rightarrow \frac{dx}{dt} + \frac{k_2}{k_1} x = \frac{k_3}{k_1}$

$$I.F. = e^{\int \frac{k_2}{k_1} \cdot dt} = e^{(k_2/k_1)t}$$

Therefore, the solution will be

$$x \cdot e^{(k_2/k_1)t} = \int \frac{k_3}{k_1} \exp\left(\frac{k_2}{k_1} t\right) dt$$

$$\Rightarrow \quad x \cdot e^{(k_2/k_1)t} = \frac{k_3}{k_2} \exp\left(\frac{k_2}{k_1} t\right) + c$$

$$\Rightarrow \quad x = \frac{k_3}{k_2} + c \cdot \exp\left(-\frac{k_2}{k_1} t\right)$$

Putting the condition, at $t = 0$, $x = 0$, we get $C = -\frac{k_3}{k_2}$

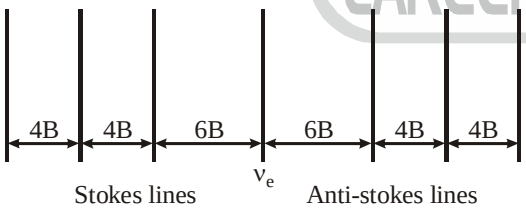
Therefore, $x = \frac{k_3}{k_2}\left[1 - \exp\left(-\frac{k_2}{k_1} t\right)\right]$

Correct option is (a)

13. The first order phase transition involves the concept of latent heat that is needed when a liquid transform into a gas at its critical temperature.
Correct option is (a).

14. P : Doppler broadening gives pressure.
Q : Natural broadening gives life time of the energy level
R : Rotational spectrum gives moment of inertia
S : Total internal reflection gives refractive index
Hence, correct option is (a).

15.



The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant is 12B.
Hence, correct option is (d).

16. Flux gets quantised in the units of $\frac{h}{2e}$

Hence, correct option is (b)

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17. For optical phonon branch, frequency of lattice vibration is given by $\omega = \sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$.

Hence, correct option is (a).

18. Proton is represented as 'uud'.

$u \quad u \quad d$

Since, it has charge $Q = \left(\frac{2}{3}+\frac{2}{3}-\frac{1}{3}\right) = \frac{3}{3} = +1$

and it has Baryon Number $B = \left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right) = +1$

and it has strangeness Number $S = (0+0+0) = 0$

$I_3 = \frac{1}{2} \qquad \because \frac{1}{2}+\frac{1}{2}-\frac{1}{2} = \frac{1}{2}$

$Y = 1 \qquad \because \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$

Correct option is (b)

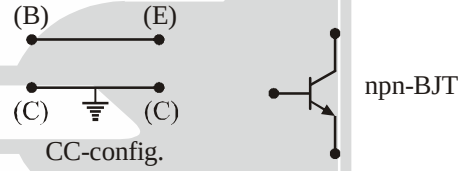
19. From figure

- Input is base
- Output is emitter

A_v; voltage gain of common collector is ≤ 1

$A_{v(\text{ideally})} = 1$

- CC configurationb is also referred as emitter follower.



CC-config.

npn-BJT

Correct option is (d)

20. Since, BE = Δmc^2

= (mass of total protons + mass of total neutrons – mass of nucleus) × c^2

$\Rightarrow BE = (Zm_p + Nm_n - A)\times c^2$

$\Rightarrow \frac{BE}{c^2} = Zm_p + Nm_n - A \Rightarrow A = Zm_p + Nm_n - \frac{BE}{c^2}$

Correct option is (c)

Q.21 – Q.60 : Carry TWO marks each.

21. $\vec{J} = \left(\vec{\nabla}\times\vec{H}\right)$

$= \frac{1}{r}\frac{\partial}{\partial r}\left[10^4\left(\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(\alpha r)\right)\right]\hat{k}$

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$$= \frac{10^4}{r}\left[\frac{1}{\alpha}\cos\alpha r - \frac{1}{\alpha}\cos(\alpha r) + r\sin(\alpha r)\right]\hat{k}$$

$$= 10^4 \sin(\alpha r)\hat{k}$$

$$I = \int \vec{J}\cdot d\vec{s}$$

$$= 10^4 \int_0^a \sin\alpha r \; 2\pi r dr = 10^4 2\pi \int_0^a \sin(\alpha r) r dr$$

$$= 2\pi\times 10^4\left[-\frac{(\cos(\alpha r))\cdot r}{\alpha} + \frac{\sin(\alpha r)}{\alpha^2}\right]_0^a = 2\pi\times 10^4\left[-\frac{\cos\alpha a}{\alpha}\cdot a + \frac{\sin\alpha a}{\alpha^2}\right]$$

$$= 2\pi\times 10^4\left[-\frac{\cos\frac{\pi}{2}}{\alpha}\cdot a + \frac{\sin\frac{\pi}{2}}{\left(\frac{\pi^2}{4a^2}\right)}\right] = \frac{8a^2}{\pi}\times 10^{+4} = \frac{800}{\pi}$$

Correct option is (b)

22. We know that.

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\vec{\nabla}\times\vec{A} = \vec{B}$$

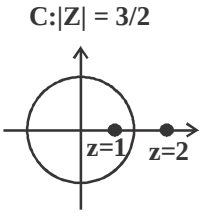
$$\therefore \quad \vec{J} = \frac{1}{\mu_0}\left(\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)\right) = \frac{1}{\mu_0}\left(\vec{\nabla}\times(2x-2y)\hat{k}\right)$$

$$= \frac{1}{\mu_0}2\left(-\hat{i}-\hat{j}\right) = -\frac{2}{\mu_0}\left(\hat{i}+\hat{j}\right)$$

Correct option is (b)

23. $f(z) = \dfrac{e^z}{z^2-3z+2}$ has simple poles at $z = 1$ and $z = 2$

Only $z = 1$ lies within the circle.



$$I = 2\pi i\left[\text{Residue of } f(z) \text{ at } z = 1\right]$$

$$= 2\pi\left[\lim_{z\to 1}(z-1)\frac{e^z}{(z-1)(z-2)}\right] = 2\pi i(-e) = -2\pi ie$$

Correct option is (c)

24. We know the relation between $\vec{E}, \vec{k}, \vec{H}$ is

$$\vec{k}\times\vec{E} = \omega\mu_0\vec{H}$$

$$\vec{H} = \frac{k}{\omega\mu_0}\left(\vec{k}\times\vec{E}\right)$$

We have,

$$\vec{E} = 20\sin\left(10^8 t - kz\right)\hat{j}$$

$$\hat{k} = \hat{z}\,,\ \omega = 10^8$$

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$$\therefore \qquad \vec{H} = \frac{20k}{\omega\mu_0}\left(\hat{z}\times\hat{j}\right)\sin\left(10^8 t - kz\right) = -\frac{20k}{10^8\mu_0}\sin\left(10^8 t - kz\right)\hat{i}$$

Correct option is (c)

25. Given : Polarization, $\vec{P} = \left(5z^2 + 7\right)\hat{k}$

The volume charge density, $\rho_b = -\vec{\nabla}\cdot\vec{P} = -10z$

Therefore, volume bound change inside the dielectric

$$Q_b = \pi r^2 dz\left(-10z\right) = -5\left[z\right]_0^2 \pi r^2 = -5\pi r^2 L$$

Correct option is (c)

26. $$\beta_3 = \sum_{i,j}\varepsilon_{ij3}T_{ij} = \varepsilon_{113}T_{11} + \varepsilon_{223}T_{22} + \varepsilon_{333}T_{33} + \varepsilon_{123}T_{12} + \varepsilon_{213}T_{21} + \varepsilon_{233}T_{23} + \varepsilon_{323}T_{32} + \varepsilon_{133}T_{13} + \varepsilon_{313}T_{31}$$

$$= \varepsilon_{123}T_{12} + \varepsilon_{213}T_{21} = T_{12} - T_{21}$$

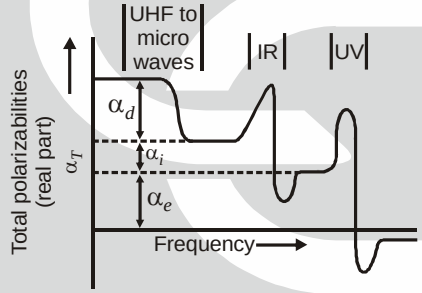
$$\Rightarrow \ \beta_3 = \sum_k \varepsilon_{12k}a_k - \sum_k \varepsilon_{21k}a_k = \varepsilon_{123}a_3 - \varepsilon_{213}a_3 = 2a_3$$

Correct option is (a)

27. From the given figure, it is clear that curve *P*, *Q* and *R* represents χ versus *T* graphs for ferromagnetic, paramagnetic and antiferromagnetic materials respectively.

Hence, correct option is (c).

28. Optical frequency range 390 THz to 770 THz
In this range dielectric constant of a material due to electronic polarizability.



Correct option is (b)

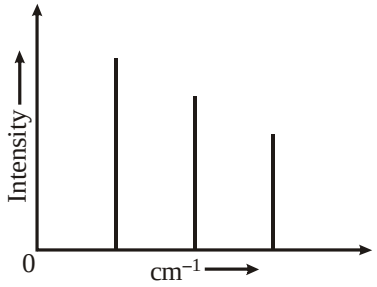
29. For hole, $\overrightarrow{k_h} = -\overrightarrow{k_e}$, $\overrightarrow{v_h} = \overrightarrow{v_e}$ and $m_h = -m_e$

For velocity part $\nabla\varepsilon_h\left(k_h\right) = \nabla\varepsilon_e\left(k_e\right) \Rightarrow \mathbf{V}_h = \mathbf{V}_e$

These relations are obtained by using mass and momentum conservation laws.

Hence, correct option is (d).

30. For diatomic molecule, the internuclear separation of the ground and first excited state are the same. The absorption spectrum will appear for the transition from lowest vibrational state of the ground state will appear as shown in figure :



Hence, correct option is (a).

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31. In all the transitions given above, the transition from $2 \to 0$ is the most suitable for a continuous wave (cw) laser because the state 2 is a metastable state with highest life time of 10^{-6} sec.
Hence, correct option is (b).

32. Time in lab frame $= \dfrac{\text{time in rest frame}}{\sqrt{1-v^2/c^2}}$

$$= \frac{2\times10^{-6}}{\sqrt{1-(0.95)^2}} = 6.4\times10^{-6}\ \text{sec}.$$

Hence correct answer is (a)

33. The nuclear spin of Cesium (I) = 7/2

For D-lines of the Cesium atom, the transitions will be from $^2P_{3/2} \to {}^2S_{1/2}$ and $^2P_{1/2} \to {}^2S_{1/2}$

For $^2P_{3/2}$ term, we have $J = \dfrac{3}{2},\ I = \dfrac{7}{2}$

Since, F = $|I + J|$ to $|I - J| \Rightarrow F = 5, 4, 3, 2$ ... (i)

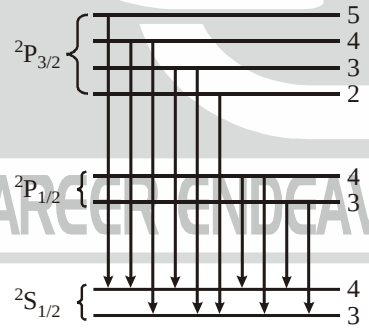
For $^2P_{1/2}$ term, we have $J = \dfrac{1}{2},\ I = \dfrac{7}{2}$.

Therefore, F = 4, 3 ... (ii)

Also for $^2S_{1/2}$ term, we have $J = \dfrac{1}{2},\ I = \dfrac{7}{2}$

Therefore, F = 4, 3 ... (iii)

Now, for hyperfine spectrum, the lines are shown with selection rule $\Delta F = 0, \pm 1$ $(0 \not\leftrightarrow 0)$



Thus, these are total 10 lines in the hyperfine spectrum.
Hence, correct option is (a).

34. The probability of occupied state for fermions is

$$P(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$$

So probability of unoccupied state is

$$P(\bar{\varepsilon}) = 1 - P(\varepsilon) = \frac{e^{(\varepsilon-\mu)/k_BT}}{e^{(\varepsilon-\mu)/k_BT}+1} = \frac{1}{1+e^{(\mu-\varepsilon)/k_BT}}$$

Correct option is (c).

35. Since, $M(A,Z)=\frac{1}{c^2}[f(A)+yZ+zZ^2]$

For stable nucleus, $\frac{dM(A,Z)}{dZ}=0$

$$\Rightarrow \frac{d}{dZ}\left[\frac{1}{c^2}\left[f(A)+yZ+zZ^2\right]\right]=0$$

$$\Rightarrow y+z(2Z)=0 \qquad \Rightarrow y+2zZ=0$$

$$\Rightarrow -4a_A+2(a_cA^{-1/3}+4a_AA^{-1})Z=0$$

$$\Rightarrow 4a_A=2(a_cA^{-1/3}+4a_AA^{-1})Z=0$$

$$\Rightarrow Z=\frac{4a_A}{2[a_cA^{-1/3}+4a_AA^{-1}]}=\frac{4a_A}{2\times 4\left[a_AA^{-1}+\frac{1}{4}a_cA^{-1/3}\right]}$$

$$=\frac{(a_A/2)}{\left(\frac{a_A}{A}+\frac{1}{4}a_cA^{-1/3}\right)}=\frac{\left(a_A/2\right)}{\frac{a_A}{A}\left(1+\frac{a_c}{4a_A}A^{2/3}\right)}=\frac{(a_{A/2})}{\frac{a_A}{A}\left(1+\frac{a_c}{4a_A}A^{2/3}\right)}=\frac{A/2}{\left(1+\frac{a_c}{4a_A}A^{2/3}\right)}$$

Correct option is (c)

36. The energy in 3-D is

$$E=\frac{3}{2}k_BT=\frac{p^2}{2m}\Rightarrow p=\sqrt{3m\,k_BT}$$

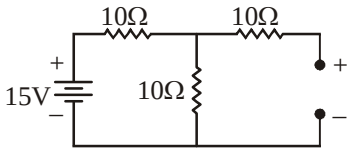
The de-Broglie wavelength is

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{3m\,k_BT}}$$

Correct option is (b).

37. For calculation of Thevenin voltage (V_{TH}) or open circuit voltage (V_{OC}) Remove R_L

Find V_{TH}



By applying voltage divider, $V_{TH}=\frac{15\times 10}{10+10}$ Volt $= V_{OC}$

$$V_{TH}=7.5 \text{ Volt}$$

For the calculation of thevenin resistance

Condition {Voltage-source is short-circuit and current-source is open-circuit }

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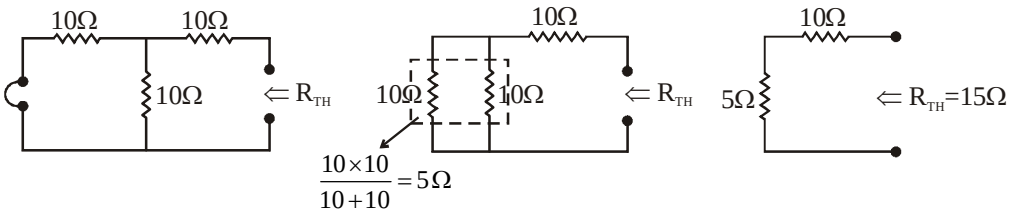
- Equivalent circuit



Correct option is (a)

38. For $|n\rangle$; $I=\frac{1}{2}, I_3=\frac{1}{2}$ for $|p\rangle, I=\frac{1}{2}, I_3=-\frac{1}{2}$

For option (a), $I=\frac{1}{\sqrt{2}}\left(|nn\rangle-|pp\rangle\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\frac{1}{2}-\frac{1}{2}\cdot\frac{1}{2}\right)=0$

$$I_3=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\frac{1}{2}-\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{4}+\frac{1}{4}\right)=\frac{1}{2\sqrt{2}}\neq 0$$

For option (b), $I=\frac{1}{\sqrt{2}}\left(|nn\rangle+|pp\rangle\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{1}{2\sqrt{2}}\neq 0$

$$I_3=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\frac{1}{2}+\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{4}-\frac{1}{4}\right)=\frac{1}{2\sqrt{2}}\neq 0$$

For option (c), $I=\frac{1}{\sqrt{2}}\left(|np\rangle-|pn\rangle\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\frac{1}{2}-\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{1}{\sqrt{4}}\left(\frac{1}{4}-\frac{1}{4}\right)=0$

$$I_3=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right)=\frac{1}{\sqrt{2}}\left(-\frac{1}{4}+\frac{1}{4}\right)=0$$

For option (d), $I=\frac{1}{\sqrt{2}}\left(|np\rangle+|pn\rangle\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)=\frac{1}{2\sqrt{2}}\neq 0$

$$I_3=\frac{1}{\sqrt{2}}\left[\left(\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\right]=\frac{1}{\sqrt{2}}\left(-\frac{1}{4}\cdot\frac{1}{4}\right)=\frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right)=-\frac{1}{2\sqrt{2}}\neq 0$$

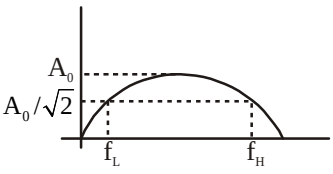
Therefore, correct option is (c)

39. **Given :** Gain of Amplifier = 1000

β "feedback" = 9.9% of output

f_L = 20 Hz "lower 3-dB frequency"

f_H = 200 KHz "upper 3-dB frequency"



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$$f_{L_{new}} = \frac{f_L}{1+A\beta} \quad \Rightarrow \quad f_{H_{new}} = f_H\left[1+A\beta\right]$$

$$f_{H_{new}} = f_H\left[1+1000\times\frac{9.9}{100}\right]$$

$$f_{H_{new}} = 100 \text{ times } f_H$$

Note: (1) Feedback decreases f_L

(2) Feedback increases f_H

(3) Feedback decreases gain.

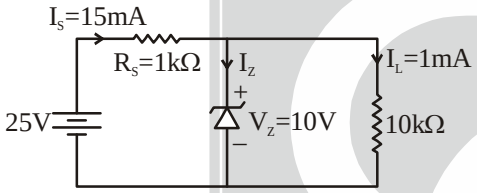
Correct option is (c)

40. Check the value of given option

(a) $I_{z_{max}} = 15\,mA,\ R_L = 10\,k\Omega$

The given circuit zener diode will conduct in reverse bias if $V_{OC} > V_{BR}$ i.e $V_{OC} = \frac{25\times10}{10+1} = 25.72\,V$

Then, $V_{R_L} = V_z = 10\,V$. Hence, $I_L = \frac{10}{10\,k-\Omega} = 1$ mA

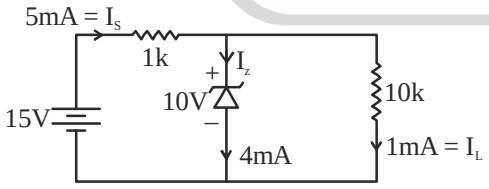


$I_S = I_Z + I_L \qquad \left(\because\ I_L = 1\,mA,\ I_S = \frac{25-10}{1} = 15\right)$

So, $I_Z = 14\,mA$

Hence false $\because \quad I_Z = 15\,mA\,(\text{given})$

(b) $R_L = 10\,k\Omega$



$I_Z = 4\,mA$

$V_{OC} = \frac{15\times10}{10+1} = 13.63$. So, $V_{OC} > V_{BR}$. Again, zener diode will conduct in reverse bias .

So, $V_R = V_Z = 10$. So, $I_L = \frac{10}{10\,k\Omega} = 1\,mA$, $I_S = \frac{15-10}{1\,k\Omega} = 5mA$

Here, $I_S = I_Z + I_L$

So, I_Z = 5–1 = 4 mA

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Hence invalid $\because \quad I_{Z_{min}} = 5\ mA$ (given)

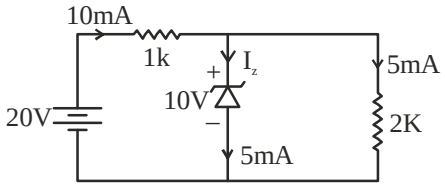
(c) $R_L = 2\,k\Omega$

Firstly check diode is conducting or not.

$$V_{OC} = \frac{20\times 2}{1+2} = \frac{40}{3} = 13.33V$$

Since, $V_{OC} > V_{BR}$

Then, zener diode will conduct i.e. $V_L = V_Z = 10V$



$I_S = I_Z + I_L$ (Here, $I_S = \frac{20-10}{1\,k\Omega} = 10\ mA$, $I_L = \frac{V_L}{2} = \frac{10}{2} = 5\ mA$)

$10 = I_Z + 5 \Rightarrow I_Z = 5$

Hence, $I_Z = I_L$

Correct option is (c)

41. Since, $Q = T_\alpha\left(1+\frac{M_{He}}{M_U}\right)$ ... (i)

where, $M_U \simeq A$ of uranium and $M_{He} \simeq A$ of He = 4

where T_α = kinetic energy of α-particle = 5.17 MeV and Q = disintegration energy = ?

Therefore, equation (i)

$$Q = 5.17\times\left(1+\frac{4}{236}\right) = 5.257\ MeV$$

Correct option is (a)

42. $x \to x' = x + \delta x$

$y \to y' = y + \delta y$

are translation, invariance of lagrangian under translation leads to conservation of linear momentum in that direction therefore, p_x and p_y are conserved.

Invariance of Lagrangian under rotation leads to conservation of angular momentum along that axis.

Therefore, L_z is also conserved.

Since potential doesn't explicitly depend on time and is also independent of velocity therefore energy is conserved.

Hence, correct answer is (b)

43. $S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Eigenvalue equation of S_y corresponding to $-\frac{\hbar}{2}$

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$$\frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=-\frac{\hbar}{2}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$-ix_2=-x_1 \Rightarrow x_1=ix_2$

Assuming $x_2=1,\ x_1=i$

Therefore, the normalized eigen function of S_y corresponding to eigenvalue $-\frac{\hbar}{2}$ is $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\ 1\end{pmatrix}$

Correct option is (d)

44. $(AB)^{\dagger}=B^{\dagger}A^{\dagger}=B\cdot A\left(\text{If } A \text{ and } B \text{ are hermitian}\right)$

$=AB\left(\text{If A and B commutes}\right)$

Correct option is (c)

45. Consider the set of vectors in 3-dimensional real vector space $\mathbb{R}^3$

$S=\{(1,1,1),(1,-1,1),(1,1,-1)\}$

Volume spanned by these vectors

$$v=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}=1[0]+1[2]+1[1+1]=2+2=4\neq 0$$

Therefore, S is a set of linearly independent vector and they will form a basis in $\mathbb{R}^3$

Correct option is (b)

46. The density of state in 3-D is

$g(p)\,dp=\frac{4\pi V}{h^3}p^2dp$,

where $p=\sqrt{2mE} \Rightarrow dp=\sqrt{\frac{m}{2E}}dE$

$\therefore \quad g(E)\,dE=g(p)\,dp=\frac{4\pi V}{h^3}2mE\sqrt{\frac{m}{2E}}\,dE$

The number of particles is

$N=\int_0^{\infty} f(E)\,g(E)\,dE$, where $f(E)=\begin{cases}1, & E\le E_F\\ 0, & E>E_F\end{cases}$ *at* $0K$

$\therefore \quad N=\int_0^{E_F}\frac{8\pi V}{h^3}2mE\sqrt{\frac{m}{2E}}\,dE$

$$=\frac{4\sqrt{2}\pi V}{h^3}\int_0^{E_F}m^{3/2}E^{1/2}dE=\frac{4\sqrt{2}\pi V}{h^3}m^{3/2}\frac{2E_F^{3/2}}{3}$$

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$\Rightarrow \quad E_F \propto N^{2/3}$

Correct option is (a).

47. $L = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{1}{2}kx_1 \cdot x_2$

$\therefore \quad T = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right), \quad V = \frac{1}{2}k\left(\frac{x_1x_2}{2} + \frac{x_2x_1}{2}\right)$

Corresponding matrices are

$$\hat{T} = m\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad \hat{V} = k\begin{pmatrix}0 & 1/2\\ 1/2 & 0\end{pmatrix}$$

for frequency of normal modes

$$\det\left|-\omega^2\hat{T} + \hat{V}\right| = 0 \qquad \Rightarrow \quad \begin{vmatrix}-m\omega^2 & k/2\\ k/2 & -m\omega^2\end{vmatrix} = 0$$

$$\left(m\omega^2\right)^2 - \left(\frac{k}{2}\right)^2 = 0$$

$$m\omega^2 = \frac{k}{2} \qquad \therefore \quad \omega = \sqrt{\frac{k}{2m}}$$

Hence correct answer is (d)

48. Let the state is $|\psi\rangle = A|\psi_0\rangle + B|\psi_1\rangle$

Therefore, the average energy of the particle

$$\langle E\rangle = \sum P_i E_i = |A|^2 E_0 + |B|^2 E_1 = 20$$

$$\Rightarrow |A|^2 10 + 30|B|^2 = 20$$

$$\Rightarrow |A|^2 + 3|B|^2 = 2$$

According to normalized condition $|A|^2 + |B|^2 = 1 \Rightarrow |A|^2 = 1 - |B|^2$

Putting this value in above equation

$$1 - |B|^2 + 3|B|^2 = 2 \Rightarrow 1 + 2|B|^2 = 2$$

$$|B| = \pm\frac{1}{\sqrt{2}} \qquad \therefore |A| = \pm\frac{1}{\sqrt{2}}$$

Therefore, $|\psi\rangle = \frac{1}{\sqrt{2}}|\psi_0\rangle - \frac{1}{\sqrt{2}}|\psi_1\rangle$

$$= \frac{1}{\sqrt{2}}|E_0 = 10\text{ eV}\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\text{ eV}\rangle.$$

Correct option is (d)

49. $L = \exp(\alpha t)\left[\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right]$

equation of motion

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$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$$

$$\frac{d}{dt}\left(e^{\alpha t}m\dot{x}\right)+e^{\alpha t}kx=0$$

$$\alpha e^{\alpha t}m\dot{x}+me^{\alpha t}\ddot{x}+e^{\alpha t}kx=0$$

$$\ddot{x}+\alpha\dot{x}+\frac{k}{m}x=0$$

Hence correct answer is (d)

50. The phase velocity by $v_p=\dfrac{\omega}{(2\pi/\lambda)}=\dfrac{\omega\lambda}{2\lambda}$

The group velocity is given by $v_g=\dfrac{\Delta\omega}{\Delta k}=\dfrac{\Delta\omega}{\Delta\left(\dfrac{2\pi}{\lambda}\right)}=\dfrac{\Delta\omega}{2\pi}\dfrac{\lambda^2}{\Delta\lambda}$

Hence, correct option is (a).

51. The partition function is

$$Z=e^{-0/k_BT}+e^{-\varepsilon/k_BT}=1+e^{-\varepsilon/k_BT}$$

The Helmholtz free energy is

$$F=-k_BT\,\ell n\left(Z\right)=-k_BT\;\ell n\,(1+e^{-\varepsilon/k_BT})$$

Correct option is (a).

52. The internal energy is

$$U=-\frac{\partial}{\partial\beta}\ell n\,Z=k_BT^2\frac{\partial}{\partial T}\ell n\,Z \qquad \left[\because\ \beta=\frac{1}{k_BT}\right]$$

$$=k_BT^2\frac{\partial}{\partial T}\ell n\left(1+e^{-\varepsilon/k_BT}\right)$$

$$=k_BT^2\frac{e^{-\varepsilon/k_BT}}{1+e^{-\varepsilon/k_BT}}\left(\frac{\varepsilon}{k_BT^2}\right)$$

$$=\frac{\varepsilon e^{-\varepsilon/k_BT}}{1+e^{-\varepsilon/k_BT}}$$

The specific heat is

$$C_v=\left(\frac{\partial U}{\partial T}\right)_V=\frac{\left(1+e^{-\varepsilon/k_BT}\right)\left(\varepsilon e^{-\varepsilon/k_BT}\cdot\frac{\varepsilon}{k_BT^2}\right)-\varepsilon e^{-\varepsilon/k_BT}\left(\frac{\varepsilon}{k_BT^2}e^{-\varepsilon/k_BT}\right)}{\left(1+e^{-\varepsilon/k_BT}\right)^2}$$

$$=\frac{\varepsilon^2}{k_BT^2}\frac{e^{-\varepsilon/k_BT}}{\left(1+e^{-\varepsilon/k_BT}\right)^2}$$

Correct option is (d).

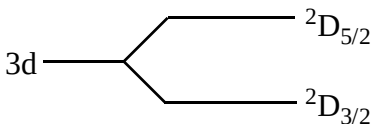
CAREER ENDEAVOUR

53. Dimension of wave function in 1-D is $(\text{length})^{-1/2}$. But in the given question, dimension of the wave function is $(\text{length})^{-1/4}$. Hence question is not correct.

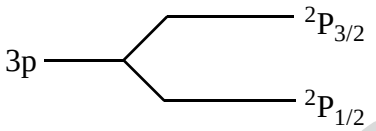
54. Dimension of wave function in 1-D is $(\text{length})^{-1/2}$. But in the given question, dimension of the wave function is $(\text{length})^{-1/4}$. Hence question is not correct.

55. For **3d** : L = 2, $S = \frac{1}{2} \Rightarrow J = \left|2-\frac{1}{2}\right|, \left|2+\frac{1}{2}\right| = \frac{3}{2}, \frac{5}{2} \Rightarrow$ Terms are ${}^2D_{5/2}$ and ${}^2D_{3/2}$

For **3p :** $L = 1, S = \frac{1}{2} \Rightarrow J = \left|1-\frac{1}{2}\right|, \left|1+\frac{1}{2}\right| = \frac{1}{2}, \frac{3}{2} \Rightarrow$ Terms are ${}^2P_{3/2}$ and ${}^2P_{1/2}$

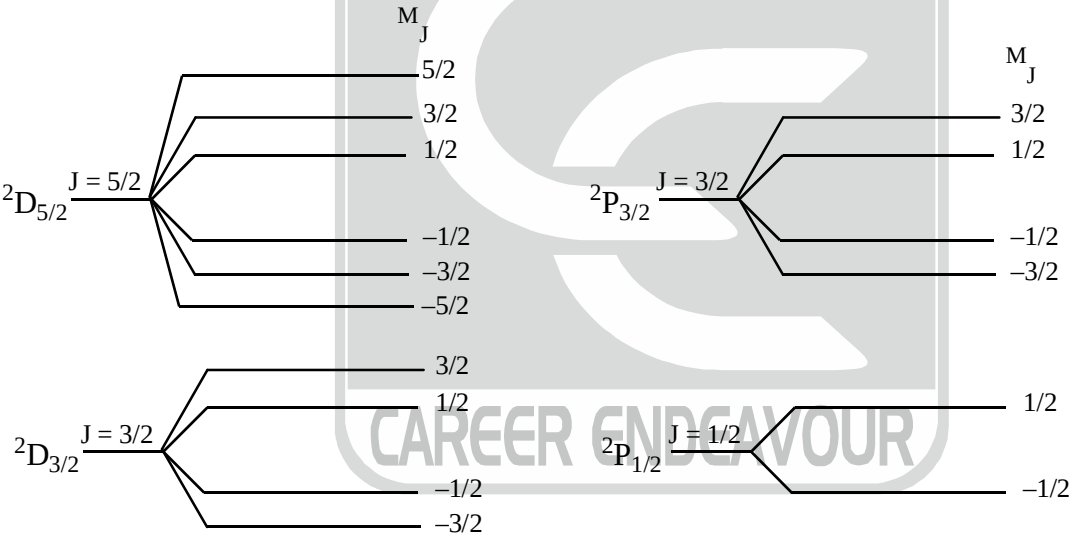


Since, transitions is between doubles, thus it is anomalous Zeeman effect. Here, each J will split into M_J values. Selection rules are



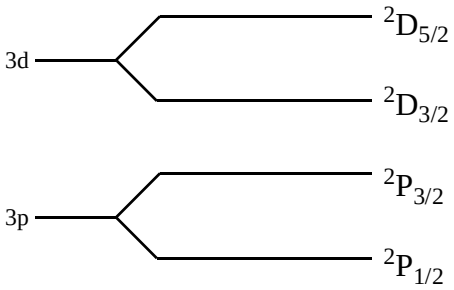
$\Delta M_J = 0, \pm 1,\ M_J = 0 \not\leftrightarrow 0$, if $\Delta J=0$

Also $\Delta M_J = 0$ for π transitions and $\Delta M_J = \pm 1$ for σ transitions. Here, both type of transitions will be seen



Hence, correct option is (d).

56.



Longest wavelength means smallest energy gap which is for $^2D_{3/2} \rightarrow {}^2P_{3/2}$

Total transitions = $4J_1 + 2J_2 + 1 = 10$ where $J_1 < J_2$

Here, $J_1 = J_2 = \dfrac{3}{2}$

Correct option is (b)

57. Volume of primitive unit cell of fcc lattice,

$$V = \vec{a}_1 \bullet (\vec{a}_2 \times \vec{a}_3) = \frac{a}{2}(\hat{j} + \hat{k}) \bullet \left[\frac{a}{2}(\hat{i} + \hat{k}) \times \frac{a}{2}(\hat{i} + \hat{j})\right]$$

$$= \begin{vmatrix} 0 & a/2 & a/2 \\ a/2 & 0 & a/2 \\ a/2 & a/2 & 0 \end{vmatrix} = 0\left(0 - \frac{a^2}{4}\right) - \frac{a}{2}\left(0 - \frac{a^2}{4}\right) + \frac{a}{2}\left(\frac{a^2}{4} - 0\right)$$

$$= \frac{a^3}{8} + \frac{a^3}{8} = \frac{a^3}{4}$$

Therefore, primitive translation vectors of reciprocal lattice are

$$\vec{b}_1 = \frac{2\pi(\vec{a}_2 \times \vec{a}_3)}{\vec{a}_1 \bullet (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi\left[\frac{a}{2}(\hat{i} + \hat{k}) \times \frac{a}{2}(\hat{i} + \hat{j})\right]}{\left(\frac{a^3}{4}\right)} = \frac{2\pi\left(\frac{a^2}{4}\right)}{\left(\frac{a^3}{4}\right)}\left[\hat{i} \times \hat{i} + \hat{i} \times \hat{j} + \hat{k} \times \hat{i} + \hat{k} \times \hat{j}\right]$$

$$= \frac{2\pi}{a}\left[0 + (+\hat{k}) + (+\hat{j}) + (-\hat{i})\right] = \frac{2\pi}{a}(-\hat{i} + \hat{j} + \hat{k})$$

$$\vec{b}_2 = \frac{2\pi(\vec{a}_3 \times \vec{a}_1)}{\vec{a}_1 \bullet (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi\left[\frac{a}{2}(\hat{i} + \hat{j}) \times \frac{a}{2}(\hat{j} + \hat{k})\right]}{\left(\frac{a^3}{4}\right)} = \frac{2\pi\left(\frac{a^2}{4}\right)}{\left(\frac{a^3}{4}\right)}\left[\hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{j} + \hat{j} \times \hat{k}\right]$$

$$= \frac{2\pi}{a}(\hat{k} - \hat{j} + \hat{i}) = \frac{2\pi}{a}(\hat{i} - \hat{j} + \hat{k})$$

Similarly, $\vec{b}_3 = \dfrac{2\pi}{a}(\hat{i} + \hat{j} - \hat{k})$

Hence, correct option is (a).

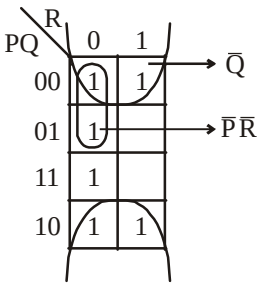
58. $$V = \vec{b}_1 \bullet (\vec{b}_2 \times \vec{b}_3) = \frac{2\pi}{a}(-\hat{i} + \hat{j} + \hat{k}) \bullet \left[\frac{2\pi}{a}(\hat{i} - \hat{j} + \hat{k}) \times \frac{2\pi}{a}(\hat{i} + \hat{j} - \hat{k})\right]$$

$$= \left(\frac{2\pi}{a}\right)^3 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \left(\frac{2\pi}{a}\right)^3 [-1(1-1) - 1(-1-1) + 1(1+1)] = \left(\frac{2\pi}{a}\right)^3 (2+2) = 4\left(\frac{2\pi}{a}\right)^3$$

Hence, correct option is (a).

CAREER ENDEAVOUR

59.

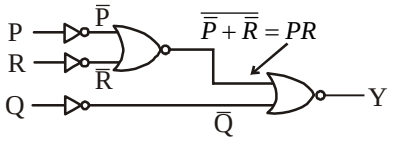


Note:- Form group of 1's in K-map

$f = \bar{P}\bar{R} + \bar{Q}$ simplified expression.

Correct option is (a)

60. (a)

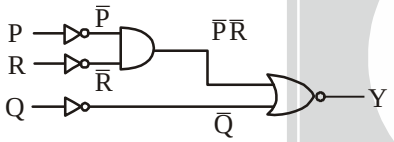


$$Y = \overline{PR + \bar{Q}}$$

$$Y = \overline{PR} \cdot Q$$

Note:- Apply Demorgan's law.

(b)



$$Y = \bar{P}\bar{R} + \bar{Q}$$

Hence K-map expression is present in option (b)

Correct option is (b)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

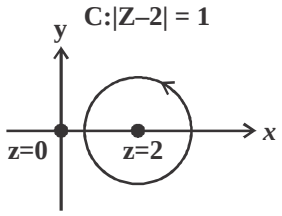
1. The number of independent component of a antisymmetric tensor P_{ij} of rank '2', is given by $\frac{1}{2}\left(N^2 - N\right)$

Here, the indices *i* and *j* are running from 1 to 5. So, $N = 5$
Therefore, number of independent components

$$\Rightarrow \quad \frac{1}{2}\left(N^2 - N\right) = \frac{1}{2}\left(25 - 5\right) = 10$$

Correct option is (b)

2. $f(z) = \dfrac{e^z \sin z}{z^2}$ has pole of order '2' at z = 0



Since, $z = 0$ lies outside the circle

So, $\oint_C \dfrac{e^z \cdot \sin z}{z^2} dz = 0$

Correct option is (d)

3. Sum of the eigenvalues = Trace of the matrix i.e. $\lambda_1 + \lambda_2 + \lambda_3 = 5$

Product of the eigenvalues = Determinant of the matrix i.e. $\lambda_1\lambda_2\lambda_3 = -5$
Only option (c) satisfy both relations.
Correct option is (c)

4. $$L\left[f(x)\right] = \int_3^\infty e^{-sx}(x-3)\,dx = (x-3)\frac{e^{-sx}}{-s}\bigg|_3^\infty - \int_3^\infty 1\cdot\frac{e^{-sx}}{-s}dx = -\frac{e^{-sx}}{s^2}\bigg|_3^\infty = \frac{e^{-3s}}{s^2}$$

Correct option is (d)

5. Electrical conductivity, thermal conductivity and metallic lustre properties are valence electron dependent while shear modulus is independent of valence electrons.
Hence, correct option is (c).

6. For fcc lattice, the amplitude of diffracted X-ray beam is given by

$$F = f\left[1 + e^{\pi i(h+k)} + e^{\pi i(k+l)} + e^{\pi i(l+h)}\right] \quad \text{... (i)}$$

For (212), $F = f\left[1 + e^{\pi i(2+1)} + e^{\pi i(1+2)} + e^{\pi i(2+2)}\right] = f(1-1-1+1) = 0$ (i.e., no diffraction)

For (111), $F = f\left[1 + e^{\pi i(1+1)} + e^{\pi i(1+1)} + e^{\pi i(1+1)}\right] = f(1+1+1+1) = 4f$

For (200), $F = f\left[1 + e^{\pi i(2+0)} + e^{\pi i(0+0)} + e^{\pi i(0+2)}\right] = f(1+1+1+1) = 4f$

For (311), $F = f\left[1 + e^{\pi i(3+1)} + e^{\pi i(1+1)} + e^{\pi i(1+3)}\right] = f(1+1+1+1) = 4f$

Hence, correct option is (a).

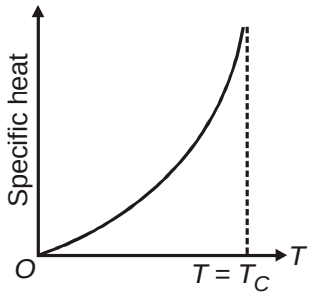
7. Hall coefficient, $R_H = \dfrac{1}{ne}$

Correct option is (c)

8. Option (d) is the correct statement of Bloch theorem.
Hence, correct option is (d).

CAREER ENDEAVOUR

9. For ferromagnetic substance, the specific heat is maximum at T_c. Thus correct graph of specific heat versus T is expressed as : $C_V = -2\lambda\, M_s\left(\frac{\partial M_s}{\partial T}\right)$, where ; $M_s = N\mu \tan h\left(\frac{\mu_0 h\lambda M_s}{k_B T}\right)$



Thus, option (d) is not correctly shown.
Hence, correct option is (d).

10. The thermal conductivity reduces in the transition from the normal state to the superconducting state because Cooper pairs have very less energy ($\approx 10^{-3}$ eV) . They can not transfer energy to the lattice.
Hence, correct option is (a).

11. In β -decay process one neutron decay into one proton and one electron

$n \rightarrow p + e^- + \overline{\nu}_e$

Quark structure of u is udd and p is uud

Thus, reaction is written as $udd \rightarrow uud + e^- + \overline{\nu}_e$ or $d \rightarrow u + e^- + \overline{\nu}_e$

Correct option is (a)

12. Since, ${}^{15}_{7}\mathrm{N}$ has Z = 7, N = 15 – 7 = 8

Therefore, 7Z, $(^1s_{1/2})^2\,(^1p_{3/2})^4\,(^1p_{1/2})^1$

$\therefore \quad J = \frac{1}{2},\ l = 1$ (for p) and parity = $(-1)^1 = -1$(odd)

Thus, spin-parity of ${}^{15}_{7}\mathrm{N} = \frac{1}{2}^-$

CAREER ENDEAVOUR

Correct option is (a)

13. (1) Lepton are produced by weak interaction. I_3 is not conserved.
(2) γ -ray produced through electromagnetic interaction.
(3) Baryon are produced by strong interaction. Isospin I and I_3 are conserved.
Correct option is (a)

14. For a gaseous species in a mixture of gases, the preferred probing tools is ESR spectroscopy.
Correct option is (c)

15. Since, to establish equilibirium
Correct option is (c)

16. Electric dipole transition between two states will be allowed if both the states have opposite parities. Since, selection rules are $\Delta\ell = \pm 1$ and $\Delta m_\ell = 0, \pm 1$
Hence, correct option is (c).

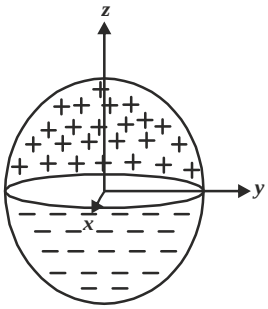
17. Using Wein's displacement law,

$$\lambda_{max} = \frac{2.897\times10^{-3}}{T}(m\text{-K}) = \frac{2.897\times10^{3}}{1000} = 2.897\times10^{-6}m = 2897\text{Å}$$

This wavelength lies in infrared region

Correct option is (b)

18. From the figure we can say that total charge is zero.



But dipole moment is not zero. So, electric field will be due to dipole moment.
Correct option is (d)

19. 12-bit DAC

Range of voltage –10 to 10 = 20V.

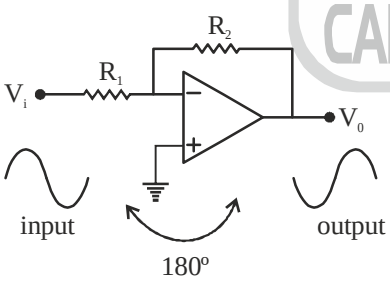
Resolution of DAC (R) = $\frac{V_{PP}}{2^n - 1}$

Voltage Resolution = $\frac{V_H - H_L}{2^n - 1} = \frac{10-(-10)}{2^{12}-1} = 5\,\text{mV}$

Correct option is (b)

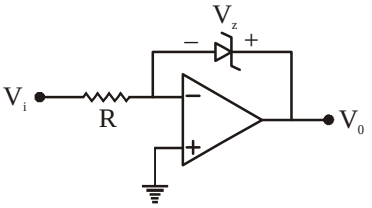
CAREER ENDEAVOUR

20. (a)



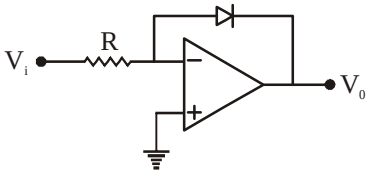
$$\frac{V_0}{V_i} = -\frac{R_2}{R_1}$$

(b)



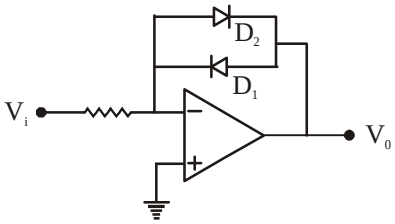
input (–)ve will make zener - ON
Since, Zener diode operates in reverse break down voltage

(c) At the time of negative input of the signal diode become off and output $V_0 = 0$



Diode in reverse bias for negative input, So $V_0 = 0$

(d)



D_2 conduct for positive input. D_1 conduct for negative input.

Correct option is (c)

21. We need two independent coordinates for each particles to know its position on the surface of the sphere.
Number of degree of freedom for N particles = $2N$

and energy per degree of freedom $= \frac{1}{2} k_B T$.

So internal energy of the system $= \frac{1}{2} k_B T \times 2N = N k_B T$.

Correct option is (c).

22. To exhibit Bose-Einstein condensation, a particle must be Boson, i.e., it must have an integral spin. Since all protons, electrons and neutrons have half integral spin, only those atoms whose $n_p + n_e + n_n$ is even will be a boson.

Particle	n_p	n_e	n_n	$n_p + n_e + n_n$
$^{1}H_{1}$	1	1	0	2 (even)
$^{4}He_{2}$	2	2	2	6 (even)
$^{23}Na_{11}$	11	11	12	34 (even)
$^{40}K_{19}$	19	19	21	59 (odd)

Since $n_p + n_e + n_n$ for $^{40}K_{19}$ is odd, it cannot exhibit Bose-Einstein condensation.

Correct option is (d).

23. Lorentz transformation satisfies group properties.
Therefore both statements P and Q are correct.
Hence correct answer is (B)

24. Any allowed wave function of the particle in bound state should satisfy the following conditions:

(1) $\psi(x)$ should be singled valued, finite, continuous everywhere in space,
(2) $\psi(x)$ should become zero at $x = \pm\infty$, $r = \infty$.

$\psi = N\dfrac{e^{-\alpha r}}{r^3}$ is not defined a $r = 0$

$\psi = N\left(1 - e^{-\alpha r}\right)$ is not zero at $r = \infty$

$\psi = Ne^{-\alpha x} e^{-\beta\left(x^2+y^2+z^2\right)}$ is zero at $x = \pm\infty$

The function given in option (d) is discontinuous at $r = R$. Therefore, the function given is option (c) will be an allowed wave function for a particle in a bound state.

Correct option is (c)

25. Momentum of the particle is,

$$p = \Delta p_{\min} = \frac{\hbar}{\Delta x_{\max}} = \frac{1.06\times10^{-34}\times3\times10^{8}}{10^{-15}\times1.6\times10^{-19}}\frac{eV}{c} = 200\frac{MeV}{c}$$

Correct option is (c)

Q.26 – Q.55 : Carry TWO marks each.

26. $f(z) = \dfrac{e^{\sqrt{z}} - e^{-\sqrt{z}}}{\sin\left(\sqrt{z}\right)} = 2\dfrac{\sinh\left(\sqrt{z}\right)}{\sin\left(\sqrt{z}\right)}$ is not analytic at $z = 0$

$$\lim_{z\to0} 2\cdot\frac{\sinh\left(\sqrt{z}\right)}{\sin\left(\sqrt{z}\right)} = 2\lim_{z\to0}\frac{\cosh\left(\sqrt{z}\right)\cdot\dfrac{1}{2\sqrt{z}}}{\cos\left(\sqrt{z}\right)\cdot\dfrac{1}{2\sqrt{z}}} = 2$$

So, $z = 0$ is a removable singular point.

Correct option is (c)

27. $\dfrac{d^2y}{dt^2} - y = 2\cdot\cosh t$

$C.F. = Ae^{t} + Be^{-t}$

$$P.I. = \frac{1}{D^2-1}\left(2\cdot\cosh t\right) = \frac{1}{\left(D^2-1\right)}\left(e^{t}+e^{-t}\right)$$

$$= t\frac{1}{2D}\left(e^{t}+e^{-t}\right) = \frac{t}{2}\left(e^{t}-e^{-t}\right) = t\sinh t$$

Total solution, $y = Ae^{t} + Be^{-t} + t\sinh t$

Putting the condition, $y\left(t=0\right) = 0 \Rightarrow A + B = 0$

Putting the condition, $\left.\dfrac{dy}{dt}\right|_{t=0} = 0 \Rightarrow A - B = 0$

Solving above two equations, $A = 0, B = 0$

Therefore, $y = t\sinh t$

Correct option is (d)

28. $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\,dx = \int_{-1}^{1}\left[xP_n(x)\right]\left[xP_{n+2}(x)\right]dx$

$$= \int_{-1}^{1}\left[\frac{(n+1)}{(2n+1)}P_{n+1}(x) + \frac{n}{(2n+1)}P_{n-1}(x)\right]\left[\frac{(n+3)}{(2n+5)}P_{n+3}(x) + \frac{(n+2)}{(2n+5)}P_{n+1}(x)\,dx\right]$$

$$= \int_{-1}^{1}\frac{(n+1)(n+2)}{(2n+1)(2n+5)}P_{n+1}(x)P_{n+1}(x)\,dx$$

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[Using the orthonormal property, $\int_{-1}^{1} P_m(x) P_n(x) dx = \frac{2}{2n+1}\delta_m$]

$$= \frac{(n+1)(n+2)}{(2n+1)(2n+5)} \frac{2}{(2n+3)} \neq 0$$

Correct option is (d)

29. The density of state is 2-D is

$$g(p)\,dp = \frac{2\pi A}{h^2} p\,dp,$$

where $p = \sqrt{2mE}$ or $dp = \sqrt{\frac{m}{2E}} dE$

$$\therefore \quad g(E)\,dE = \frac{2\pi A}{h^2}\sqrt{2mE}\sqrt{\frac{m}{2E}}\,dE = \frac{2\pi A}{h^2} m\,dE$$

Since, we can have an electron with spin up or spin down in each state, we need to multiply density of state by the factor of two.

$$\therefore \quad g(E) = \frac{4\pi A}{h^2} m$$

The number of particles is

$$N = \int_0^\infty f(E)\, g(E)\, dE \text{, where } f(E) = \begin{cases} 1, & E \le E_F \\ 0, & E > E_F \end{cases}$$

$$\therefore \quad N = \frac{4\pi A}{h^2} m E_F \Rightarrow \frac{N}{A} = \frac{4\pi E_F m}{h^2} = n$$

where $n = \frac{N}{A}$ = electronic density

$$\Rightarrow \quad n = \frac{4\pi m E_F}{h^2} = \frac{4\pi m E_F}{(\hbar \cdot 2\pi)^2} \qquad \left[\because \hbar = \frac{h}{2\pi}\right]$$

$$\Rightarrow \quad n = \frac{m E_F}{\pi h^2}$$

Correct option is (b)

30. **Correct option is (d).**

31. **Correct option is (d)**

32. $E = \frac{\hbar^2}{2\Im} I(I+1)$ where I is spin and $\Im$ is moment of inertia

Now, for $I = 2$, $E = 57.5$ KeV

$$\Rightarrow \quad 57.5 = \frac{\hbar^2}{2\Im} \times 2 \times 3 \Rightarrow \frac{\hbar^2}{2\Im} = \frac{57.5}{6}$$

Next level is 6 +.

Correct option is (a)

CAREER ENDEAVOUR

33. Given nuclear reaction, $\pi^- + p \to K^- + \Sigma^+$

$$\Delta I_3 = \left(-1+\frac{1}{2}\right)-\left(-\frac{1}{2}+1\right)=\left(-\frac{1}{2}\right)-\left(\frac{1}{2}\right)=-1\neq 0$$

$$\Delta S = (0+0)-(-1-1)=2\neq 0$$

$$\Delta B = (0+1)-(0+1)=0$$

Correct option is (c)

34. CH_3OH is example of symmetric top molecules, where $I_x = I_y = I$ and $I_z \neq I$

Energy level of symmetric top molecule is

$$E_J = \left[BJ\left(J+1\right)+\left(A-B\right)m_J^2\right]hc$$

where, $m_J = 0, \pm 1, \pm 2, ...\pm J$ and $J = 0,1,2,3,.....$

Also, $B=\dfrac{h}{8\pi^2 I_B c}$ and $A=\dfrac{h}{8\pi^2 I_A c}$

where, I_B and I_A is the moment of inertia about orthogonal and parallel to the molecules axis i.e. $I_B = I_x = I_y = I$ and $I_A = I_z$.

Here, J is same as ℓ, therefore, energy levels can be written as

$$E_\ell = \frac{\hbar^2}{2I}\ell\left(\ell+1\right)+\frac{\hbar^2}{2}m_\ell^2\left[\frac{I}{I_z}-\frac{I}{I_B}\right]$$

Thus, $$E_\ell = \frac{\hbar^2}{2I}\ell\left(\ell+1\right)+\frac{\hbar^2}{2}m_\ell^2\left[\frac{1}{I_z}-\frac{1}{I}\right] \quad \left[\because\ I_B = I\right]$$

Correct option is (a)

35. For the given Half harmonic oscillator potential, energy eigenvalues of the particle are

$$E_n = \left(2n+\frac{3}{2}\right)\hbar\omega$$

Expectation value of the energy will be

$$\left\langle E\right\rangle = P_0E_0 + P_1E_1 = \frac{1}{5}\times\left(\frac{3}{2}\hbar\omega\right)+\frac{4}{5}\times\left(\frac{7}{2}\hbar\omega\right)=\frac{31}{10}\hbar\omega$$

Correct option is (a)

36. Here the molecular has been compared with their corresponding wave number or we can say that energy of these spectra has been normalized in the units of cm^{-1}. So, in terms of energy rotational spectra corresponds to microwave radiation which has the lowest energy. Then comes vibrational spectra (IR region) slightly greater than both rotational and vibration.

Molecular dissociation corresponds to very high energy.

So, $E_{rotational} < E_{vibrational} < E_{electronic} < E_{molecular\ dissociation}$

Rotational $\to 10^o - 10^2\ cm^{-1}$

Electronics $\to 10^5 - 10^6\ cm^{-1}$

Molecular $\to 10^6\ cm^{-1}$ and above

Correct option is (b)

CAREER ENDEAVOUR

37.

Quantity	T	P
E	E	−E
B	−B	B

Correct option is (a)

38. The interaction energy between two magnetic dipole moment is given by

$$w_{12} = \frac{\mu_0}{4\pi}\left(3\frac{(\vec{m}_1\cdot\vec{r}_{12})\vec{r}_{12}}{r_{12}^5} - \frac{\vec{m}_1}{r_{12}^3}\right)\cdot\vec{m}_2$$

$$= \frac{\mu_0}{4\pi}\left\{3\frac{(\vec{m}_1\cdot\vec{r}_{12})(\vec{m}_2\cdot\vec{r}_{12})}{r_{12}^5} - \frac{\vec{m}_1\cdot\vec{m}_2}{r_{12}^3}\right\}$$

$$= \frac{\mu_0}{4\pi}\left[\frac{3m^2\cos^2 45^\circ}{d^3} - 0\right] \qquad (\because\ \vec{m}_1 \perp \vec{m}_2)$$

$$= \frac{\mu_0}{4\pi}\frac{3m^2}{d^3}\frac{1}{2} = \frac{\mu_0}{8\pi}\frac{3m^2}{d^3}$$

Correct option is (d)

39. When the loop will be pulled out of the region of the magnetic field, then amount of flux passing through the loop will gets changed. This change in flux will induced an emf in the circuit

$$V_{emf} = -\frac{d\phi}{dt}$$

At $t = 0$, loop was completly inside the magnetic field

So, $\phi(t=0) = \phi_0 = \text{constant}\,(say)$

Given circuit is op-amp integrator circuit

$$V_{out} = \frac{1}{RC}\int V_{emf}\cdot dt \qquad \left[\because\ V_{emf}\ \text{will acts as an input voltage for op-amp}\right]$$

$$= -\frac{1}{RC}\int \frac{d\phi}{dt}\cdot dt$$

$\Rightarrow \quad -\frac{1}{RC}\phi$ [This ϕ will be depending on u also]

$\Rightarrow$ V_{out} is independent of ϕ_0

Correct option is (a)

40. Since current flow in the circuit is constant I_0.
So, initially capacitor will start to charging

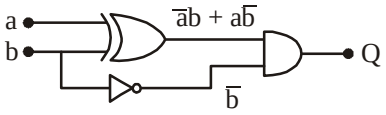
$$\therefore \quad I = \frac{dq}{dt} \Rightarrow q = It$$

$$\therefore \quad v_0 = v_c = qC = CIt \qquad \text{(linear with time)}$$

After certain time capacitor will be fully charged.
So, Voltage will not change with time.

Correct option is (d)

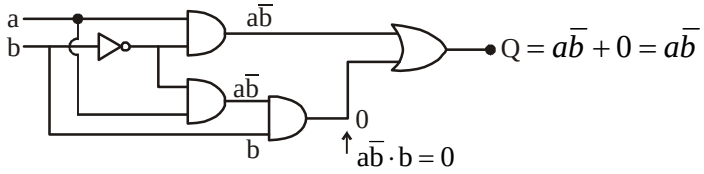
CAREER ENDEAVOUR

41. (a) 

$$Q = \left(\bar{a}b + a\bar{b}\right)\bar{b} \quad \Rightarrow \quad Q = a\bar{b}$$

Note: $x \cdot \bar{x} = 0 \quad \Rightarrow \quad 0 + x = x$

(b)

(c) 

$$Q = a\bar{b} + 0 \quad \Rightarrow \quad Q = a\bar{b}$$

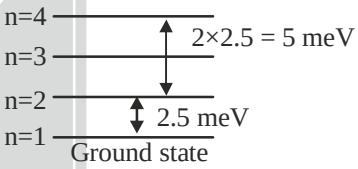
(d)

Option (d) is different except other.

42. Number of molecules in a given energy states is given by Boltzmann law.

$$N_2 = N_1 e^{-\frac{(E_2 - E_1)}{k_B T}}$$

where, N_2 = number of molecules in energy state E_2
N_1 = number of molecules in energy state E_1
T = temperature of the system
k_B = Boltzmann constant.



Thus, $N_4 = N_2 e^{-\frac{(E_4 - E_2)}{k_B T}}$

Ratio of molecules,

$$\frac{N_4}{N_2} = e^{-\frac{(E_4 - E_2)}{k_B T}}$$

As energy is equally spaced, therefore,

$$\Delta E = E_4 - E_2 = 2 \times 2.5\, meV = 5\, meV$$

$$\Rightarrow \quad \frac{N_4}{N_2} = e^{\left(\frac{-5\times10^{3}\, eV}{8.62\times10^{-5}\, eVk^{-1}\times 300K}\right)} = e^{-0.193} = 0.826 \simeq 0.8$$

Correct option is (c)

43. The enthalpy is defined as

$$H = U + PV$$

$$\Rightarrow \quad dH = dU + PdV + VdP \text{ where } dU = TdS - PdV$$

CAREER ENDEAVOUR

$\therefore \quad dH = TdS + VdP$

So we have $T = \left(\frac{\partial H}{\partial S}\right)_P$ and $V = \left(\frac{\partial H}{\partial P}\right)_S$

Correct option is (b).

44. $\left\langle S_x^2 \right\rangle = \left\langle s,m \mid S_x^2 \mid s,m \right\rangle = \frac{1}{4}\left\langle s,m \mid \left(S_+^2 + S_-^2 + S_+S_- + S_-S_+\right) \mid s,m \right\rangle$

$= \frac{1}{4}\left\langle s,m \mid S_+S_- \mid s,m \right\rangle + \frac{1}{4}\left\langle s,m \mid S_-S_+ \mid s,m \right\rangle$

$= \frac{1}{4}\left[(s+m)(s-m+1)\right]\hbar^2 + \frac{1}{4}\left[(s-m)(s+m+1)\right]\hbar^2 = \frac{1}{2}\left[s(s+1) - m^2\right]\hbar^2$

Correct option is (a)

45. $V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3, \quad k, \lambda > 0$

for equilibrium $\frac{dV}{dx} = 0 \qquad kx - \lambda x^2 = 0$

$x = 0, \frac{k}{\lambda}$

$\frac{d^2V}{dx^2} = k - 2\lambda x$

at $x = 0, \ \frac{d^2V}{dx^2} = k > 0$

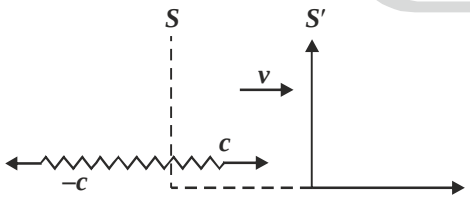
Therefore $x = 0$ stable equilibrium point.

at $x = \frac{k}{\lambda} \ \frac{d^2V}{dx^2} = -k < 0$

Therefore, $x = \frac{k}{\lambda}$ is unstable equilibrium point.

Hence correct answer is (b)

46.



The position of two photon in the s-frame when they are detected at

$x_1 = 10c\ m, \ x_2 = -10c\ m$

So, the time internal between two detection in s' frame is

$$\Delta t' = t'_2 - t'_1 = \frac{v(-x_2 + x_1)/c^2}{\sqrt{1 - v^2/c^2}} = \frac{0.6c \times 20c \times \frac{1}{c^2}}{\sqrt{1 - (0.6)^2}} = 15\ s$$

Correct option is (a)

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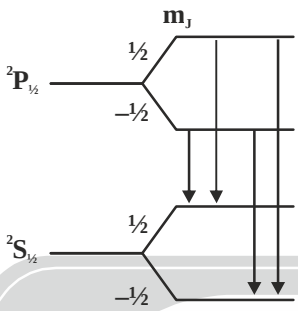
47. First order correction to ground state energy is,

$$E^{(1)} = \langle \psi_1 | V_p | \psi_1 \rangle = \frac{2V_0}{L}\int_0^L \sin^2\frac{\pi x}{L}\sin\frac{2\pi x}{L}dx = \frac{V_0}{L}\int_0^L\left(1-\cos\frac{2\pi x}{L}\right)\sin\frac{2\pi x}{L}dx = 0$$

Second order correction to the ground state energy is always less than equal to zero i.e. $E^{(2)} \le 0$

Correct option is (a)

48. Consider the transition, $^2P_{½} \to {}^2S_{½}$. These terms are correspond to $J_1 = \frac{1}{2}$ and $J_2 = \frac{1}{2}$. In a weak magnetic field it breaks into $2J_1+1 = 2J_2+1 = 2$ components



The selection rule of transition

$\Delta m_J = 0, \pm 1$

Thus, in total four spectral lines are observed in transition from $^2P_{1/2} \to {}^1S_{1/2}$

Also, by formula, total number of transitions = $4J_1 + 2J_2 + 1 = 4\times\frac{1}{2} + 2\times\frac{1}{2} + 1 = 4$

Correct option is (c)

49. Given transition, $m_J = +\frac{1}{2} \to m_J = -\frac{1}{2}$ is corresponds to $\Delta m_J = -1$

Transition $\Delta m_J = \pm 1$ corresponds to σ components. Electric field of the emitted radiation oscillates perpendicular to the applied magnetic field. Classically, this situation corresponds to two parallel dipoles oscillating with phase difference 90º. Superposition of two dipoles produces circulating current. Thus, in direction of the magnetic field, circularly polarized light is emitted.

For $\Delta m_J = +1$, it is clockwise and

$\Delta m_J = -1$, it is anti-clockwise circular

Correct option is (a)

50. The partition function of the photon gas is

$$\ell n\, Z = \frac{\pi^2}{45}\frac{V(k_BT)^3}{\hbar^3c^3}$$

The integral energy is

$$U = k_BT^2\frac{\partial}{\partial T}\ell n\, Z = k_BT^2\frac{\partial}{\partial T}\left[\frac{\pi^2}{45}\frac{V(k_BT)^3}{\hbar^3c^3}\right]$$

$$= k_B T^2 \frac{\pi^2}{45} \frac{V}{\hbar^3 c^3} k_B^3 \cdot 3T^2 = \frac{3\pi^2 V}{45\hbar^3 c^3} k_B^4 \cdot T^4$$

Now, $C_v = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3\pi^2 V}{45\hbar^3 c^3} k_B^4 \cdot 4T^3 \Rightarrow C_v \propto T^3$

Correct option is (b).

51. The free energy is

$$F = -k_B T \ \ell n\, Z = -k_B T \left[\frac{\pi^2}{45} \frac{V(k_B T)^3}{\hbar^3 c^3}\right]$$

The pressure is given by $P = -\left(\frac{\partial F}{\partial V}\right)_{N,T} = k_B T \left[\frac{\pi^2}{45} \frac{(k_B T)^3}{\hbar^3 c^3}\right] = \frac{\pi^2}{45} \frac{(k_B T)^4}{\hbar^3 c^3}$

Correct option is (c).

52. Let $\vec{E} = \vec{E}_0 \, e^{i\left(\vec{k}\cdot\vec{r} - \omega t\right)}$

We know that Maxwell's equations,

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

Replacing $\vec{\nabla}$ by ik and $\frac{\partial}{\partial t}$ by $-i\omega$, we get

$$i\vec{k} \cdot \vec{E} = 0 \Rightarrow \vec{k} \cdot \vec{E} = 0$$

$$i\vec{k} \cdot \vec{B} = 0 \Rightarrow \vec{k} \cdot \vec{B} = 0$$

$$i\vec{k} \times \vec{E} = i\omega\mu\vec{H} \Rightarrow \vec{k} \times \vec{E} = \mu\omega\vec{H}$$

$$i\vec{k} \times \vec{H} = i\omega\varepsilon\vec{E} \Rightarrow \vec{k} \times \vec{H} = -\varepsilon\omega\vec{E}$$

Correct option is (d)

53. $\vec{S} = \vec{E} \times \vec{H}^* = \vec{E} \times \left(\vec{k} \times \vec{E}^*\right) \frac{1}{\mu\omega}$

$$= \frac{1}{\mu\omega}\left[\vec{k}\left(\vec{E}.\vec{E}^*\right) - \left(k \cdot E\right)\vec{E}^*\right] = \frac{1}{\mu\omega}\vec{k}\left|E\right|^2 = \frac{k}{\mu\omega}\left|E\right|^2 \hat{k}$$

Since μ is negative.

$$\vec{S} = -\frac{k}{\mu\omega}\left|E\right|^2 \hat{k}$$

Therefore, k and S are anti-parallel.

Correct option is (b)

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54. $L = \frac{1}{2} m\ell^2 \dot{\theta}^2 - mg\ell(1-\cos\theta)$

Corresponding Hamittonian of system is

$$H = \frac{p_\theta^2}{2m\ell^2} + mg\ell(1-\cos\theta)$$

Hamilton's equations are

$$\dot{\theta} = \frac{\partial H}{\partial p_\theta} \text{ and } \dot{p}_\theta = -\frac{\partial H}{\partial \theta}$$

Therefore

$$\dot{\theta} = \frac{p_\theta}{m\ell^2}, \quad \dot{p}_\theta = -mg\ell\sin\theta$$

Hence correct answer is (a)

55. Poisson Bracket $\{\theta, \dot{\theta}\}$

$$= \left\{\theta, \frac{p_\theta}{m\ell^2}\right\} = \frac{1}{m\ell^2}\{\theta, p_\theta\} = \frac{1}{m\ell^2} \times 1 = \frac{1}{m\ell^2}$$

Hence correct answer is (b)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

1. Two similar matrices A and B have same eigenvalues, same determinant and same trace.
Correct option is (c)

2. **Correct option is (a)**

3. Quantum mechanical operator for the momentum of a particle moving in one dimension is

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$$

Correct option is (b)

4. The Carnot cycle is a reversible cycle and hence the entropy change of the universe is zero.

i.e., $(\Delta S)_{\text{universe}} = 0$

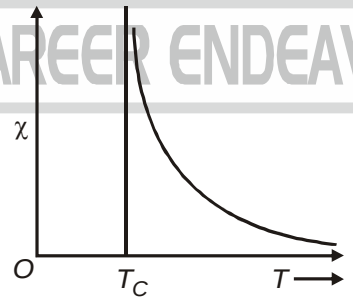
Correct option is (d).

5. The key points are :
(i) The specific heat diverges for both I and II order phase transition.
(ii) The entropy has a finite discontinuity for I order phase transition whereas it is continuous for II order phase transition.
Correct option is (b).

6. Surface energy $\left(E_s\right) = a_s A^{2/3} \Rightarrow E_s \propto A^{2/3}$
Correct option is (c)

7. In case of two level, the rate of upward transitions is equal to the rate of downward transitions which gives $N_2 = N_1$.
Thus, there will be no population inversion.
Correct option is (a).

8. For ferromagnetic materials, $\chi = \dfrac{C}{(T - T_C)}$, for $T > T_C$.



Correct option is (b).

9. The order of magnitude of the energy gap of a typical superconductor is $\simeq 1$ meV.
Correct option is (d). (Reference → C. Kittel)

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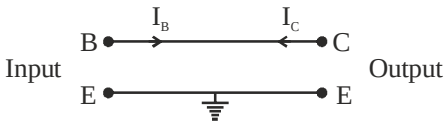
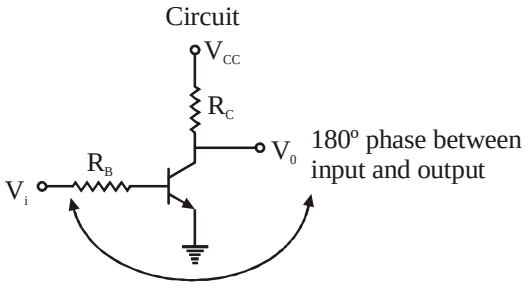
10. Common emitter configuration / amplifier





- Output is taken from collector
- It has 180° phase shift between input and output.
- Amplifier is an application in active region. So, one junction is forward bias and other junction is reversed bias.

Correct option is (b)

11. If $\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues of matrix, then

Trace = $\lambda_1 + \lambda_2 + \lambda_3 = 11$

Determinant = $\lambda_1\ \lambda_2\ \lambda_3 = 36$

So, the possible values of λ_1, λ_2 and λ_3 are (2, 3, 6)

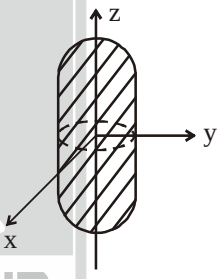
Correct option is (d)

12. Linear, $I_1 = I_2, I_3 = 0$

Symmetric top, $I_1 = I_2 \neq I_3$

Spherical top, $I_1 = I_2 = I_3$

Asymmetric top, $I_1 \neq I_2 \neq I_3$



Correct answer is (c)

13. For the region $x > 0$, the 1-D time independent Schrodinger equation will be

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0 \Rightarrow \frac{d^2\psi}{dx^2} - \alpha^2\psi = 0 \text{ where } \alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

Solution will of the form: $\psi = Ae^{\alpha x} + Be^{-\alpha x}$

Since, the wave function should be zero at $x = \infty$, therefore $A = 0$ and the space part of the wave function for the region $x > 0$ will be of the form $\psi = Be^{-\alpha x}$.

Correct option is (b)

14. $\left[L_x L_y, L_z\right] = \left[L_x, L_z\right]L_y + L_x\left[L_y, L_z\right] = \left(-i\hbar L_y\right)L_y + L_x\left(i\hbar L_x\right) = i\hbar\left(L_x^2 - L_y^2\right)$

Correct option is (c)

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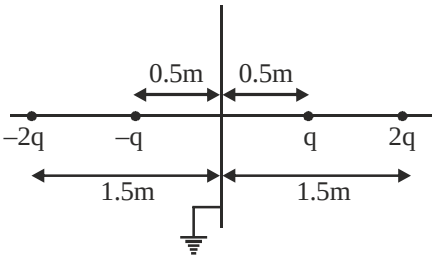
15. $\left\langle \frac{1}{r^2} \right\rangle = \iiint \psi^*(r) \frac{1}{r^2} \psi(r)\, dV = \frac{1}{\pi a^3} \iiint \frac{1}{r^2} e^{-2r/a} r^2 \sin\theta dr d\theta d\varphi$

$$= \frac{1}{\pi a^3} \int_{r=0}^{\infty} e^{-2r/a} dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\varphi=0}^{2\pi} d\varphi = \frac{2}{a^2}$$

Correct option is (d)

16. $F = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{4} + \frac{q^2}{1} + \frac{2q^2}{1}\right]$

$$= \frac{1}{4\pi\varepsilon_0} \frac{7q^2}{2}$$



Correct option is (a)

17. $B = \frac{\mu_0}{2} \vec{K} \times \hat{n}$, where $\vec{K}$ is the surface current density

$$= +\frac{\mu_0 K}{2} \hat{i} \text{ for } z > 0$$

$$= -\frac{\mu_0 K}{2} \hat{i} \text{ for } z < 0$$

Correct option is (a)

18. **Correct option is (c)**

19. We know that, $\vec{\nabla} \times \vec{A} = \vec{B}$

$\therefore$ $\vec{\nabla} \times \left(B_0 x \hat{J}\right) = B_0 \hat{k}$ is uniform field.

$\vec{\nabla} \times \left(-B_0 y \hat{J}\right) = -B_0 \hat{k}$ is uniform field.

$\vec{\nabla} \times \left(\frac{B_0 x}{2} \hat{i} + \frac{B_0 y}{2} \hat{j}\right) = 0$

$\vec{\nabla} \times \left(-\frac{B_0 y}{2} \hat{i} + \frac{B_0 x}{2} \hat{j}\right) = \frac{B_0}{2}\left(\hat{k}\right)$ is uniform field.

Correct option is (c)

20. Neutrons cannot produce ionization because they are electrically neutral. They can be detected because of the secondary particles produced in a nuclear reaction in the detector medium.

Correct option is (d)

21. In weak field, the higher total angular momentum state splits into (2J + 1) states. Possible J values for one electron $\left(1s^1\right)$, $s = \frac{1}{2}$, $\Rightarrow J = \ell - \frac{1}{2}, \ell + \frac{1}{2}$

Higher J value i.e. $\ell + \frac{1}{2}$ will split into $2J + 1 = 2\left(\ell + \frac{1}{2}\right) + 1 = 2\ell + 2$

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Correct option is (a).

22. The selection rules for orbital angular momentum :

For one electron, most of the transitions take place when ℓ value changes by one unit, i.e., $\Delta\ell = \pm 1$ because, according to Laporte rule, the parity should change in a electric dipole transition.

The selection rules for total orbital angular momentum:

For total angular momentum, the selection rule for the electric dipole transition is $\Delta L = 0, \pm 1$.

The selection rules for total spin angular momentum :

For total spin angular momentum, the selection rule for electric dipole transition is $\Delta S = 0$.

Correct option is (b).

23. If there are *p* atoms in a primitive cell, then total number of branches are 3*p*. Out of 3*p* branches, 3 will be acoustical branches and remaining (3*p*-3) will be optical branches. The number of branches does not depend on the number of primitive cells.

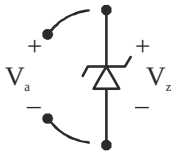
Correct option is (c)

24. The fermi level of an intrinsic semiconductor is given by

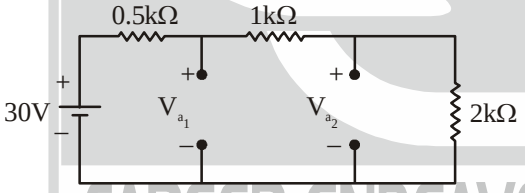
$$E_F = \frac{E_c + E_v}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_h^*}{m_e^*}\right)$$

Correct option is (c)

25. Voltage across $2k\Omega$



if $V_a \geq V_z$ means zener ON

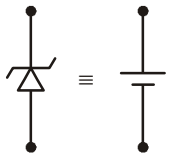


For given circuit to check zener in breakdown region.

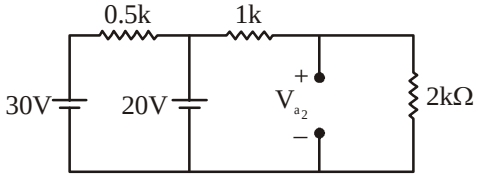
$$V_{a_1} = \frac{30 \times 3}{0.5 + 3} \quad \Rightarrow \quad V_{a_1} = 25.71$$

$$V_{z_1} = 20 \text{ Volt}$$

$V_{a_1} > V_{z_1}$ so, Zener - ON



equivalent circuit

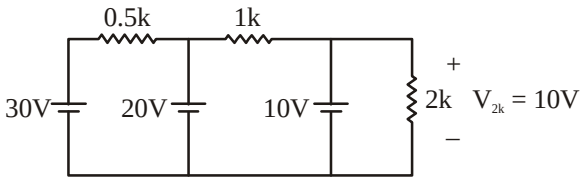


$$V_{a_2} = \frac{20 \times 2}{3} \quad \Rightarrow \quad V_{a_2} = 13.33 \text{ Volt}$$

$V_{a_2} > V_{z_2}$ So, $V_{a_2} = 10\,V$

zener - ON

Final circuit



$$I_{2k} = \frac{10}{2} = 5 \text{ mA}$$

Correct option is (d)

Q.26 – Q.55 : Carry TWO marks each.

26. Unit vector normal to the surface $\hat{n} = \left.\frac{\vec{\nabla}\phi}{\left|\vec{\nabla}\phi\right|}\right|_{(1,1,1)} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$

Correct option is (d)

27. $\int_S \vec{r} \cdot \hat{n}\, dS = \int_V (\vec{\nabla} \cdot \vec{r})\, dV$ (Using divergence theorem)

$$= 3\int_V dV = 3V = 3\pi a^2 h$$

Correct option is (b)

28. $\frac{dy}{dx} = -\frac{x}{y+1} \Rightarrow \int (y+1)\, dy = -\int x\, dx$

$$\Rightarrow \frac{y^2}{2} + y = -\frac{x^2}{2} + c$$
$$\Rightarrow x^2 + y^2 + 2y = 2c$$
$$\Rightarrow x^2 + (y+1)^2 = 2c + 1$$

This represents a family of circles having center at (0, –1) and radii of $\sqrt{2c+1}$ units.

Correct option is (a)

29. $V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$

Generalised force is given as

$$Q = -\frac{\partial V}{\partial q} + \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}}\right)$$

$$= \frac{2(1+\dot{q})}{q^3} + \frac{d}{dt}\left(\frac{1}{q^2}\right) = \frac{2(1+\dot{q})}{q^3} - \frac{2\dot{q}}{q^3} = \frac{2}{q^3}$$

Correct answer is (c)

30. Frequency of normal mode is given as

$$\omega = \sqrt{\frac{k}{\mu}},\ \mu = \frac{m_1 m_2}{m_1 + m_2} = \textit{reduced mass}$$

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$$= \frac{m \cdot 2m}{m + 2m} = \frac{2m}{3}$$

$$\therefore \quad \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{2m/3}} = \sqrt{\frac{3k}{2m}}$$

Hence correct answer is (a)

31. $Q = q^{\alpha} \cos(\beta\, p)$

$P = q^{\alpha} \sin(\beta\, p)$

For canonical transformation

Poisson Bracket $\{Q, P\}_{q,p} = 1$

or, $\frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q} = 1$

or, $\alpha q^{\alpha-1} \cos(\beta p) \cdot q^{\alpha} \beta \cos(\beta p) + q^{\alpha} \beta \sin(\beta p) \cdot \alpha q^{\alpha-1} \sin(\beta p) = 1$

or, $\alpha\beta q^{2\alpha-1} = 1 \cdot q^0$

$\therefore \quad 2\alpha - 1 = 0 \quad \Rightarrow \quad \alpha = 1/2$

$\therefore \quad \alpha\beta = 1 \quad \Rightarrow \quad \frac{1}{2}\beta = 1 \quad \Rightarrow \quad \beta = 2$

Correct answer is (d)

32. Initial : $\underset{m}{\bigcirc\!\!\rightarrow} 0.6c \qquad \underset{0.6c}{\leftarrow\!\!\bigcirc} m$

Initial momentum of two particles is zero. Therefore final momentum will also be zero. So final entity will be at rest.

final : ⊘ M rest

Apply conservation of energy.

$$E_{initial} = E_{final}$$

$$2 \times \frac{mc^2}{\sqrt{1 - (0.6)^2}} = Mc^2$$

$$\therefore \quad M = \frac{2m}{0.8} = \frac{5}{2} m$$

Correct answer is (c)

33. First order correction to ground state energy is

$$E^{(1)} = \int_0^{a/2} \psi_1^*(x) V'(x) \psi_1(x)\, dx = \frac{2V_0}{a} \int_0^{a/2} \sin^2 \frac{\pi x}{a} \cos \frac{\pi x}{a}\, dx = \frac{2V_0}{3\pi}$$

[Assume $\sin \frac{\pi x}{a} = p$ to solve the integration]

Correct option is (a)

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34. Given : $\kappa_T = 10^{-10}\,(Pa)^{-1}$

By definition, $\kappa_T = -\frac{1}{V}\left(\frac{dV}{dP}\right)$, where $V = \frac{m}{\rho}$; ρ is the density

$$\therefore \quad \frac{dV}{V} = \frac{-\frac{m}{\rho^2}d\rho}{m/\rho} = -\frac{d\rho}{\rho}$$

$$\therefore \quad \kappa_T = \left(\frac{d\rho}{\rho}\right)\frac{1}{dP} \Rightarrow dP = \frac{(d\rho/\rho)}{\kappa_T}$$

For $\frac{d\rho}{\rho} = 1\% = 0.01$ and $\kappa_T = 10^{-10}$, we have

$$dP = \frac{0.01}{10^{-10}} = 10^8\,Pa$$

Correct option is (c).

35. Number of microstates for N non-interacting particles with spin s is

$\Omega = (2s+1)^N = 3^N \quad [\because \text{ for } s = 1]$

The entropy of the system is $S = k_B \ln \Omega = N k_B \ln 3$

Correct option is (d).

36. The partition function is $Z = \sum_i g(\varepsilon_i)\, e^{-\varepsilon_i/k_BT} = 4e^{-\varepsilon/k_BT} + 2e^{-2\varepsilon/k_BT}$

The fraction of particles in the upper level $= \frac{2e^{-2\varepsilon/k_BT}}{4e^{-\varepsilon/k_BT} + 2e^{-2\varepsilon/k_BT}} = \frac{1}{2e^{\varepsilon/k_BT} + 1}$

Correct option is (b).

37. $\sigma = -\varepsilon_0 \left.\frac{\partial\phi}{\partial r}\right|_{r=a}$

$$= +\varepsilon_0 E_0 \cos\theta + \frac{\varepsilon_0 2E_0 a^3}{r^3}\cos\theta \Big|_{r=a} = \varepsilon_0\left(E_0\frac{\sqrt{3}}{2} + 2E_0\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}\varepsilon_0 E_0$$

Correct option is (a)

38. We have nucleus ${}^{17}_{8}\mathrm{O}$,

Number of protons (Z) = 8 (even) and number of neutrons (N) = 9 (odd)

9N = $(^1s_{1/2})^2\ (^1p_{3/2})^4\ (^1p_{1/2})^2\ (^1d_{5/2})^1$

$J = \frac{5}{2},\ l = 2$ (for d)

Parity = $(-1)^2$ = +1 (positive)

Spin parity of ground state $= \frac{5^+}{2}$

Correct option is (d)

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39. **Correct option is (a)**

40. For baryon Ω^-, strangeness $(S) = -3$ and isospin (I_3)

Correct option is (b)

41. According to Heisenberg uncertainty relation

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi} = \frac{h}{2\pi} \text{ (suppose)} \quad \because \quad \Delta t = 1\,ns = 1\times10^{-9}\,s$$

$$\therefore \quad \Delta E = \frac{h}{2\pi\times\Delta t} = \frac{6.6\times10^{-34}}{2\times3.14\times10^{-9}} \text{ Joule}$$

$$\Rightarrow \Delta E = \frac{h}{2\pi\times\Delta t} = \frac{6.6\times10^{-34}}{2\times3.14\times10^{-9}\times1.6\times10^{-19}} \text{ eV} = 0.65 \times 10^{-6} \text{ eV} \approx 10^{-6} \text{ eV}$$

Correct option is (c)

42. Total degeneracy, $D = D_1D_2$, where D_1 is degeneracy of $2p^2$ and D_1 is degeneracy of $3d^1$.

Now, $$D_1 = \frac{(4\ell_1+2)!}{(4\ell_1+2-N)!N!} = \frac{(4\times1+2)}{(4\times1+2-2)!2!} = \frac{6!}{4!2!} = \frac{6\times5}{2} = 15$$

And $$D_2 = \frac{(4\times2+2)!}{(4\times2+2-1)!1!} = \frac{10!}{9!} = 10 \quad \Rightarrow D = D_1D_2 = 15\times10 = 150$$

Correct option is (d).

43. The line spacing in the infrared rotational absorption spectrum $2B = 20\ cm^{-1}$ = constant $\Rightarrow$ $B = 10\ cm^{-1}$

Since, the distance of first Stokes lines from Rayleigh line is $6B$. Therefore, the position of first Stokes line in the rotational Raman spectrum = $6 \times 10\ cm^{-1} = 60\ cm^{-1}$.

Correct option is (c).

44. Given that : $\lambda = 2.1$ Å, $\theta = 30°$, bcc structure

From Bragg's law : $2d \sin\theta = \lambda \Rightarrow d = \frac{\lambda}{2\sin\theta} = \frac{2.1\text{ Å}}{2\times\sin 30°} = \frac{2.1\text{ Å}}{2\times\frac{1}{2}} = 2.1\text{ Å}.$

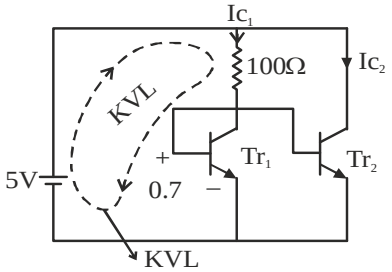
For first diffraction peak $(hkl) = (100)$

$$\because \quad d = \frac{a}{\sqrt{h^2+k^2+l^2}} \Rightarrow a = d\times\sqrt{h^2+k^2+l^2} \Rightarrow a = 2.1\text{ Å}\times\sqrt{1^2+0^2+0^2} \Rightarrow a = 2.1\text{ Å}$$

Therefore, volume of the unit cell $= a^3 = (2.1\text{ Å})^3 = 9.261\text{ Å}^3 \simeq 9.3\text{ Å}^3$

Correct option is (c).

45. Given $V_{BE} = 0.7$ V (identical transistor)



CAREER ENDEAVOUR

Here, $Ic_2 \equiv Ic_1$ because it is current mirror circuit.

By applying KVL through dotted loop

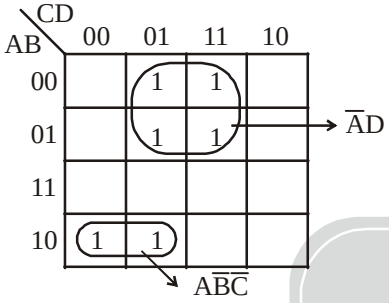
$5 = Ic_1 \times 100 + 0.7$

$Ic_1 = 43\,mA \equiv Ic_2$

Correct option is (d)

46. $Y = \underbrace{\bar{A}\,B\,\bar{C}\,D}_{0101} + \overbrace{A\,\bar{B}\,\bar{C}\,\bar{D}}^{1000} + \overbrace{\bar{A}\,\bar{B}\,\bar{C}\,D}^{0001} + \overbrace{\bar{A}\,\bar{B}\,C\,D}^{0011} + \underbrace{\bar{A}\,B\,C\,D}_{0111} + \underbrace{A\,\bar{B}\,\bar{C}\,D}_{1001}$

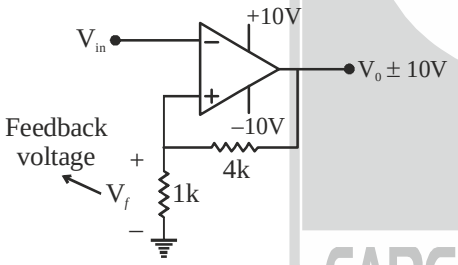
It is a 4 variable boolean function which can be minimized by using K-map



$f = A\bar{B}\bar{C} + \bar{A}D$

Correct option is (c)

47. Lets redraw the circuit.



V_0

$V_f = \dfrac{10 \times 1}{1+4}$ $\quad V_f = \dfrac{-10 \times 1}{1+4}$ $\quad V_f > V_{in}$ $\quad +V_{sat}$

$V_f < V_{in}$ $\quad -V_{sat}$

$V_f = V_{UT} = 2\,\text{Volt}$ $\quad V_f = -2\ \text{Volt}$

$V_{UT} = 2V$ $\quad V_{LT} = -2V$

Upper threshold $\quad$ Lower threshold

O/P of this $(+)ve$ feedback circuit is a square wave [Schmitt Trigger]

Note: (1) When input wave crosses V_{UT}, V_0 makes transition from $+V_{SAT}$ to $-V_{SAT}$

(2) When input wave drops below V_{LT}, V_0 makes transition from $-V_{SAT}$ to $+V_{SAT}$.

Correct option is (a)

CAREER ENDEAVOUR

48. $f(z)=\dfrac{z\sin z}{(z-\pi)^2}$ (putting $z-\pi=t$)

$$=\frac{(t+\pi)\sin(t+\pi)}{t^2}=\frac{(t+\pi)[-\sin t]}{t^2}$$

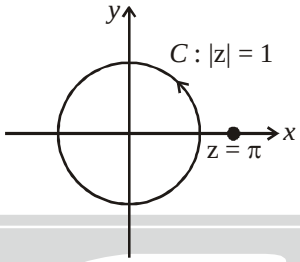
$$\Rightarrow\ f(z)=-\frac{\sin t}{t}-\pi\frac{\sin t}{t^2}=\frac{1}{t}\left[t-\frac{t^3}{3!}+\frac{t^5}{5!}-.....\right]-\pi\cdot\frac{1}{t^2}\left[t-\frac{t^3}{3!}+\frac{t^5}{5!}-.....\right]$$

Highest negative power in the Laurrent series expansion is 1/*t*

So, $f(z)$ has a simple pole at $z=\pi$

Correct option is (d)

49. Since, $z=\pi$ lies outside the circle.



Therefore, $\oint_C \dfrac{z\sin z}{(z-\pi)^2}dz=0$

Correct option is (b)

50. Density of states $=\dfrac{L}{\pi\left(\dfrac{dE}{dk}\right)}=\dfrac{L}{\pi\left[\dfrac{d}{dk}\{E_0-\beta-2\gamma\cos(ka)\}\right]}=\dfrac{L}{2\pi\,\gamma\,a\sin ka}.$

Hence, correct option is (b).

51. Effective mass $(m^*)=\dfrac{\hbar^2}{\left(\dfrac{d^2E}{dk^2}\right)}=\dfrac{\hbar^2}{\dfrac{d}{dk}\left(\dfrac{d}{dk}\{E_0-\beta-2\gamma\cos(ka)\}\right)}=\dfrac{\hbar^2}{\dfrac{d}{dk}[2\gamma a\sin(ka)]}$

$$\Rightarrow\quad m^*=\frac{\hbar^2}{2\gamma a^2\cos(ka)}$$

Hence, correct option is (b).

52. ψ_2 will be orthogonal to ψ_1 if

$$\langle\psi_1|\psi_2\rangle=0\Rightarrow\left[\langle\varphi_0|-2\langle\varphi_1|+3\langle\varphi_2|\right]\left[|\varphi_0\rangle-|\varphi_1\rangle+\alpha|\varphi_2\rangle\right]=0\ \Rightarrow 1+2+3\alpha=0\Rightarrow\alpha=-1$$

Correct option is (c)

53. Normalized form of ψ_2: $|\psi_2\rangle=\dfrac{1}{\sqrt{3}}|\varphi_0\rangle-\dfrac{1}{\sqrt{3}}|\varphi_1\rangle-\dfrac{1}{\sqrt{3}}|\varphi_2\rangle$

Therefore, $\langle E\rangle=P_0E_0+P_1E_1+P_2E_2=\dfrac{1}{3}\times\left(\dfrac{1}{2}\hbar\omega\right)+\dfrac{1}{3}\times\left(\dfrac{3}{2}\hbar\omega\right)+\dfrac{1}{3}\times\left(\dfrac{5}{2}\hbar\omega\right)=\dfrac{3}{2}\hbar\omega$

Correct option is (d)

CAREER ENDEAVOUR

54. $\vec{B} = B_0 \sin\left[\left(x+y\right)\frac{k}{\sqrt{2}} + \omega t\right]\hat{k}$, wavevector $\vec{K} = \frac{k}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

We know the solution,

$$\vec{E} = -\frac{1}{\omega\varepsilon_0\mu_0}\left[\vec{K}\times\vec{B}\right] = -\frac{1}{\omega}c^2\left[\vec{K}\times\vec{B}\right]$$

$$= -cB_0 \sin\left[\left(x+y\right)\frac{k}{\sqrt{2}} + \omega t\right]\left[\left(\hat{i}+\hat{j}\right)\frac{1}{\sqrt{2}}\times\hat{k}\right]$$

$$= cB_0 \sin\left[\left(x+y\right)\frac{k}{\sqrt{2}} + \omega t\right]\left(\hat{j}-\hat{i}\right)\frac{1}{\sqrt{2}}$$

$$= cB_0\frac{\left(\hat{j}-\hat{i}\right)}{\sqrt{2}}\sin\left[\left(x+y\right)\frac{k}{\sqrt{2}} + \omega t\right]$$

Correct option is (a)

55. $\langle S\rangle = \left\langle\left(E\times H^*\right)\right\rangle = \frac{cB_0^2}{T\mu_0}\int_0^T \sin^2\left[\left(x+y\right)\frac{k}{\sqrt{2}} + \omega t\right]\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}dt$

$$= \frac{cB_0^2}{2\sqrt{2}\mu_0}\left(\hat{i}+\hat{j}\right) \quad \left(\because \frac{1}{T}\int_0^T \sin^2\left(\left(x+y\right)\frac{k}{\sqrt{2}} + \omega t\right)dt = \frac{1}{2}\right)$$

Correct option is (c)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

1. Given vectors are $\vec{a} = 3\hat{i} + 2\hat{j}, \quad \vec{b} = \hat{i} + 2\hat{j}$

If $\vec{a}$ and $\vec{b}$ are linearly dependent then we must have

$$\ell\vec{a} + m\vec{b} = 0, \quad \ell \neq 0, m \neq 0$$

$$\therefore \quad \ell\left(3\hat{i} + 2\hat{j}\right) + m\left(\hat{i} + 2\hat{j}\right) = 0$$

$$\therefore \quad 3\ell + m = 0 \text{ and } 2\ell + 2m = 0$$

These equations do not have non-zero solution i.e., $\ell = 0, m = 0$

Therefore, $\vec{a}$ and $\vec{b}$ are linearly independent.

Further, $\vec{a}.\vec{b} \neq 0$, not orthogonal

$|a| \neq 1, \ |\vec{b}| \neq 1$ not normalized.

Correct option is (a)

2. Moment of inertia of a thin rod about an axis passing through its one end and perpendicular to its length is $\frac{mL^2}{3}$, where m = mass, L = length of rod

Therefore, for given system

$$\text{Moment of inertia} = \frac{M_1L^2}{3} + \frac{M_2L^2}{3} = \left(M_1 + M_2\right)\frac{L^2}{3}$$

Correct option is (c)

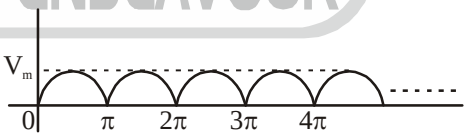
3. $$E = E_0\cos\left(\omega t - kz\right) \Rightarrow u_E = \frac{1}{2}\in_0 E^2 \Rightarrow u_E = \frac{1}{2}\in_0 E_0^2\cos^2\left(\omega t - kz\right)$$

$$\left\langle u_E \right\rangle = \frac{1}{2}\in_0 E_0^2\left\langle\cos^2\left(\omega t - kz\right)\right\rangle = \frac{1}{4}\in_0 E_0^2 \qquad \left[\because \left\langle\cos^2\left(\omega t - kz\right)\right\rangle = \frac{1}{2}\right]$$

Correct option is (a)

4. V_{DC} for full wave rectifier

$$V_{DC} = \frac{1}{\pi}\int_0^{\pi} V_m \cdot \sin t \cdot dt = \frac{2V_m}{\pi}$$



$V_{DC} = \frac{2V_m}{\pi}$, putting the value of V_m, we get

$$V_{DC} = \frac{2\times 10}{\pi} = 6.36 \text{ Volt}$$

DC value is also referred as average value.

Correct optino is (c)

CAREER ENDEAVOUR

5. Energy eigenvalues of the particle confined in a two dimensional square well potential of width $2a$ is

$$E_{n_x n_y} = \frac{\pi^2\hbar^2}{2m(2a)^2}\left(n_x^2 + n_y^2\right) = \frac{\pi^2\hbar^2}{8ma^2}\left(n_x^2 + n_y^2\right)$$

Energy of the first excited state of the particle is $E_{21}\left(\text{or } E_{12}\right) = \frac{\pi^2\hbar^2}{8ma^2}\left(1^2 + 2^2\right) = \frac{5\pi^2\hbar^2}{8ma^2}$

Note : The question in the original paper is wrong.

6. The isothermal compressibility is defined as $\kappa_T = -\frac{1}{V_0}\left(\frac{\partial V}{\partial P}\right)_{T_0}$.

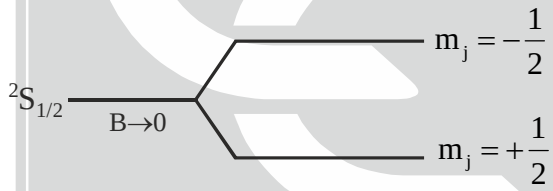
It actually explains how does the volume of a substance change when pressure on it changes at constant temperature and hence explains how easily a material can be compressed.
Correct option is (a).

7. For $^2S_{1/2}$ state : $\ell = 0$, $\left(2s+1 = 2 \Rightarrow s = \frac{1}{2}\right)$, $J = \frac{1}{2}$

For weak field, this state will split into two $(2J+1=2)$ energy levels.

The energy of splitted levels is given by $E = -g\mu_B\left(\vec{J}\cdot\vec{B}\right) = -g\mu_B BJ\cos\theta = -g\mu_B Bm_j$

For $^2S_{1/2}$ state : $g = 1 + \frac{J(J+1) - \ell(\ell+1) + s(s+1)}{2J(J+1)} = 1 + \frac{\left(\frac{1}{2}\times\frac{3}{2} - 0\times1 + \frac{1}{2}\times\frac{3}{2}\right)}{\left(2\times\frac{1}{2}\times\frac{3}{2}\right)} = 1 + \frac{\left(\frac{3}{4}+\frac{3}{4}\right)}{\left(\frac{6}{4}\right)} = 1+1 = 2.$



Therefore, the energy of upper level, $\left(m_j = -\frac{1}{2}\right) = -(2)(\mu_B)\left(-\frac{1}{2}\cdot B\right) = +\mu_B B$

The energy of lower level, $\left(m_j = +\frac{1}{2}\right) = -(2)(\mu_B)\left(+\frac{1}{2}\cdot B\right) = -\mu_B B$

Therefore, the difference in energy levels $= \mu_B B + \mu_B B = 2\mu_B B$.

Hence, correct option is (b).

8. The Fermi energy of a Fermi gas can be given as

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} \Rightarrow E_F \propto n^{2/3} \qquad \text{...(i)}$$

Now, this Fermi energy is related to velocity v_F at the Fermi surface as $E_F = \frac{1}{2}mv_F^2$

$$\Rightarrow \quad \frac{1}{2}mv_F^2 = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} \quad \Rightarrow \quad v_F^2 \propto n^{2/3} \quad \Rightarrow \quad v_F \propto n^{1/3}.$$

Hence, correct option is (d).

9. Electron, photon and neutrino do not have substructures and hence are the fundamental particles.
Correct option is (c)

10. **Correct option is (a)**

11. Reflectivity is given as $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1 - \frac{3}{2}}{1 + \frac{3}{2}}\right)^2 = \frac{1}{25} \equiv \frac{1}{25} \times 100 = 4\%$

Correct option is (a)

12. $\sigma = ne\mu \quad \Rightarrow \rho = \frac{1}{ne\mu} = \frac{1}{10^{15} \times 1.6 \times 10^{-19} \times 1800} = 3.472\ \Omega cm$

Correct option is (c)

13. $\left(v_x + v_y\right)^2 = v_x^2 + v_y^2 + 2v_x . v_y$

$\therefore \quad \left\langle \left(v_x + v_y\right)^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + 2\left\langle v_x . v_y \right\rangle$

From equipartition theorem,

$$\frac{1}{2} m \left\langle v_x^2 \right\rangle = \frac{1}{2} k_B T \ \Rightarrow \left\langle v_x^2 \right\rangle = \frac{k_B T}{m}$$

Similarly, $\left\langle v_y^2 \right\rangle = \frac{k_B T}{m}$

$$\left\langle v_x v_y \right\rangle = \left\langle v_x \right\rangle \left\langle v_y \right\rangle = 0 \text{ as } \left\langle v_x \right\rangle = 0 \text{ and } \left\langle v_y \right\rangle = 0$$

$\therefore \quad \left\langle \left(v_x + v_y\right)^2 \right\rangle = \frac{k_B T}{m} + \frac{k_B T}{m} + 0 = \frac{2k_B T}{m}$

Correct option is (d)

14. From mathematical concepts of conic section it is known that for parabola $\varepsilon = 1$
Correct option is (b)

15. Arsenic is penta-valent.
Note : In N-type semi conductor as doping increases Fermi level moves towards the conduction band. Hence, it is lies nearly donar energy level.
Correct option is (b)

16. The position of first stokes line of a rotational Raman spectrum $6B = 12.96\ cm^{-1} \Rightarrow \quad B = 2.16\ cm^{-1}$

Hence, the rotational constant $B = 2.16\ cm^{-1}$

Hence, correct option is (c).

17. $E = \frac{k\, N^{5/3}}{V^{2/3}} \quad \Rightarrow P = -\left(\frac{\partial E}{\partial V}\right)_{S,N} = \frac{2}{3} k \left(\frac{N}{V}\right)^{5/3} \Rightarrow P = \frac{2}{3} . \frac{k}{V} . \frac{N^{5/3}}{V^{2/3}} = \frac{2}{3} . \frac{E}{V}$

$\therefore \quad PV = \frac{2}{3} E$

Correct option is (b)

CAREER ENDEAVOUR

18. For a system of identical Fermions, total wave function should be anti-symmetric in nature. Since, the given spatial part of the wave function is symmetric under particle exchange, therefore the spin part of the wave function should be anti-symmetric under particle exchange.

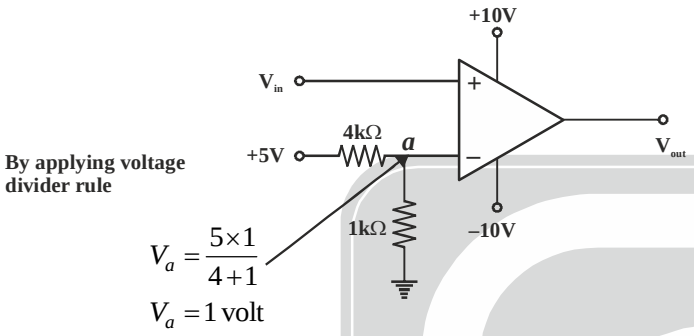
Correct option is (b)

19. $\vec{E} = -\vec{\nabla}\phi - \dfrac{\partial \vec{A}}{\partial t} \Rightarrow \vec{E} = -\hat{i}z$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ tz & 0 & 0 \end{vmatrix} = -\hat{j}(0-t) = \hat{j}t$$

Correct option is (d)

20.



By applying voltage divider rule

$$V_a = \frac{5 \times 1}{4+1}$$

$$V_a = 1 \text{ volt}$$

Input condition	V_0
$V_+ > V_-$	$+V_{SAT}$
$V_- > V_+$	$-V_{SAT}$

- When V_{in} is greater than V_a; output $V_0 = +V_{SAT}$

Correct option is (a)

21. The nuclear forces are same for neutron-neutron, neutron-proton and proton-proton inside nucleus. Therefore, it suggests that these forces are not isospin dependent. The spin and quadrupole moment is non-zero, therefore, they have spin dependence along with tensor components.

Correct option is (d)

22. Given : $V(x) = x(x-2)^2$, $m = 1$

Time period $= 2\pi\sqrt{\dfrac{m}{k}} \qquad k \neq 0$

$$k = \left.\frac{d^2V}{dx^2}\right|_{\text{at stable point}} = \left.\frac{d^2V}{dx^2}\right|_{x=2} = \frac{d}{dx}\left[(x-2)^2 + 2x(x-2)\right]_{x=2}$$

$$= \left[2(x-2) + 2(x-2) + 2x\right]_{x=2} = 4$$

$\therefore$ time period $= 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{1}{4}} = \pi$

Correct option is (b)

23. Expression for potential energy of thermal expansion is given by $U(x) = (cx^2 - gx^3 - fx^4)$. This is a non-harmonic approximation in the lattice vibration in solids. Here x is the displacement of their equilibirium position.

Hence, correct option is (d).

24. Since the potential under which the particle is moving i.e. truncated harmonic oscillator potential, is not symmetric about $x = 0$, therefore the wave function for the different states of the particle should not have a definite parity.

Energy eigenvalues of a particle moving under a truncated harmonic oscillator potential are

$$E_n = \left(2n + \frac{3}{2}\right)\hbar\omega$$

Therefore, ground state energy and first excited state energy will be $\frac{3}{2}\hbar\omega$ and $\frac{7}{2}\hbar\omega$ respectively.

Correct option is (d)

25. For symmetric tensor, $A_{ij} = A_{ji}$

Therefore, if 3 of the off-diagonal elements are known then the other 3 off-diagonal elements are also known.
Hence, number of independent components = 3 diagonal + 3 off-diagonal = 6

Correct option is (c)

Q.26 – Q.55 : Carry TWO marks each.

26. Energy eigenvalues corresponding to unperturbed Hamiltonian $H_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is $E_0 = 1, 1$.

Since the unperturbed Hamiltonian $H_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and perturbing Hamiltonian $H' = \begin{bmatrix} \delta & \delta \\ \delta & \delta \end{bmatrix}$ commutes with each other, therefore the eigenvalues of the perturbing Hamiltonian $H' = \begin{bmatrix} \delta & \delta \\ \delta & \delta \end{bmatrix}$ will give the first order correction to energy.

CAREER ENDEAVOUR

Eigenvalue equation of H':

$$|H' - \lambda I| = 0 \Rightarrow \begin{vmatrix} \delta - \lambda & \delta \\ \delta & \delta - \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 2\delta, 0$$

Energy eigenvalues of the perturbed system using the first order perturbation approximation, will be $1 + 2\delta$ and 1 respectively.

Correct option is (a)

27. The magnetic susceptibilty of a ferromagnetic material is given by

$$\chi = \frac{C}{(T - \lambda C)} \qquad ...(i)$$

At $T = 0$ K; $\chi_{T=0K} = \frac{C}{(0 - \lambda C)} = \frac{C}{(-\lambda C)} = -\frac{1}{\lambda}$

$$\Rightarrow \lambda = -\frac{1}{\chi_{T=0K}} = -\left(-2\times10^4\right) \Rightarrow \quad \lambda = 2\times10^4$$

And also $T_C = \lambda C \Rightarrow C = \frac{T_C}{\lambda} = \frac{600}{2\times10^4} = 300\times10^{-4} = 3\times10^{-2}$

$\Rightarrow \quad C = 3\times10^{-2}$

Hence, correct option is (c).

28. $\vec{E} = 10\,\hat{j}\exp\left[i\left(6x+8z\right)\right] \Rightarrow E\left(x,z,t\right) = 10\,\hat{j}\exp\left[i\left(6x+8z-10ct\right)\right] \quad \left[\because\ \omega = ck = 10c\right]$

$\vec{k} = 6\hat{x}+8\hat{z} \ \Rightarrow\ \left|\vec{k}\right| = 10$

$\therefore \qquad \vec{B} - = \frac{1}{c}\left[\hat{k}\times\vec{E}\left(x,z,t\right)\right] = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i\left(6x+8z-10ct\right)\right]$

Correct option is (a)

29. Trace of the matrix = $\lambda_1 + \lambda_2 + \lambda_3 = 0$

Only option (b) satisfies this.

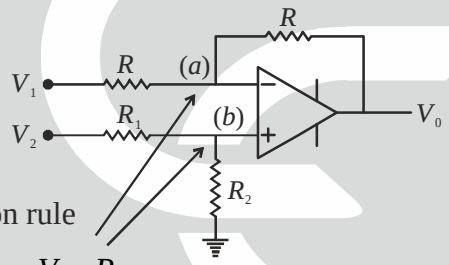
Correct option is (b)

30.

P. Infrared region	3. Vibrational transitions of molecules
Q. Ultraviolet visible region	1. Electronic transitions involving valence electrons
R. X-ray region	4. Transitions involving inner shell electrons
S. γ - rays region	2. Nuclear transitions

Hence, correct option is (c).

31.



By applying voltage division rule

$V_b = V_a = \frac{V_2\times R_2}{R_1+R_2}$ (Since, $V_a = V_b$ due to virtual short)

(Since, $V_a = V_b$ due to virtual short)

Given $V_0 = -V_1 + \frac{V_2}{2}$; $V_a = V_b = \frac{V_2\times R_2}{R_1+R_2}$

Applying KCL at node - (a)

$\frac{V_a - V_1}{R} + \frac{V_a - V_0}{R} = 0$

$2V_a - V_1 = V_0$; Put V_a value

$\frac{2V_2\times R_2}{R_1+R_2} - V_1 = V_0$

Compare this equation with given V_0 value

$\Rightarrow \ \frac{1}{2} = \frac{2R_2}{R_1+R_2} \qquad \Rightarrow \ \frac{1}{4} = \frac{R_2}{R_1+R_2} \qquad \Rightarrow \ 4 = \frac{R_1+R_2}{R_2}$

$\Rightarrow \ 4 = 1+\frac{R_1}{R_2} \quad \Rightarrow \ \frac{R_1}{R_2} = 3$

Correct option is (d)

CAREER ENDEAVOUR

32. The given electronic configuration $= 2s^1\ 3d^1$

For $2s^1$ electron : $\ell = 0, \quad s = \frac{1}{2}$

For $3d^1$ electron : $\ell = 2, \quad s = \frac{1}{2}$

Thus, the total angular momentum (J_1) for $2s^1$ electron

$$J_1 = |\ell + s| \text{ to } |\ell - s| = \left(0 + \frac{1}{2}\right) \text{to} \left(0 - \frac{1}{2}\right) = \frac{1}{2}.$$

The total angular momentum (J_2) for $3d^1$ electron

$$J_2 = |\ell + s| \text{ to } |\ell - s| = \left(2 + \frac{1}{2}\right) \text{to} \left(2 - \frac{1}{2}\right) = \frac{5}{2}, \frac{3}{2}.$$

Thus, total angular momentum J in j–j , coupling is given by

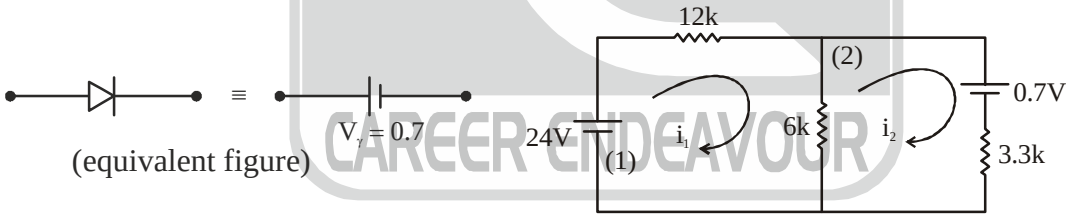
For $J_1 = \frac{1}{2}$ and $J_2 = \frac{3}{2}$: $\quad J = |J_1 + J_2| \text{ to } |J_1 - J_2| = \left(\frac{1}{2} + \frac{3}{2}\right) \text{to} \left|\frac{1}{2} - \frac{3}{2}\right| = 2, 1.$

For $J_1 = \frac{1}{2}$ and $J_2 = \frac{5}{2}$: $\quad J = \left|\frac{1}{2} + \frac{5}{2}\right| \text{ to } \left|\frac{1}{2} - \frac{5}{2}\right| = 2, 3.$

Thus, we can write it as $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

Hence, correct option is (a).

33. Given $V_{\gamma_{(si)}} = 0.7$ volt



Solve this circuit $i_2 = 1$ mA

Applying KVL rule,

Loop-(1) : $24 = 12\, i_1 + 6(i_1 - i_2)$ ... (i)

Loop-(2) : $-0.7 = 3.3\, i_2 + 6(i_2 - i_1)$ ... (ii)

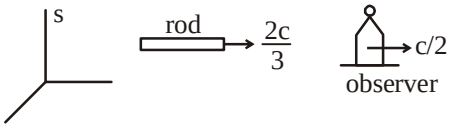
Solving (i) and (ii), we get $i_2 = 1$ mA

Correct option is (b)

34. Charged particles interact through electromagnetic interactions. In weak all particles interact and neutrino interact through weak.

Correct option is (a)

35.



velocity of rod with respect to observer

$$v_{ro} = \frac{v_r - v_0}{1 - \frac{v_r \cdot v_0}{c^2}} = \frac{\frac{2c}{3} - \frac{c}{2}}{1 - \frac{\frac{2c}{3} \cdot \frac{c}{2}}{c^2}} = \frac{c}{4}$$

length of rod in observer's frame is

$$= \ell_0 \sqrt{1 - \left(\frac{v_{r0}}{c}\right)^2} = \ell_0 \sqrt{1 - \left(\frac{c/4}{c}\right)^2} = \frac{\ell_0 \sqrt{15}}{4} = 0.97\, \ell_0$$

Correct option is (d)

36. Relation between lattice parameters : a, b and c ; and interplanar spacings of (hkl) plane is given by

$$d_{hkl} = \frac{1}{\sqrt{\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)}} \quad \text{... (i)}$$

For simple cubic crystal, $a = b = c = a_c$, $d_{(101)} = ?$

$$\therefore \quad d_{hkl} = (d_{101})_c = \frac{1}{\sqrt{\frac{1^2}{a_c^2} + \frac{0^2}{a_c^2} + \frac{1^2}{a_c^2}}} = \frac{1}{\sqrt{\frac{2}{a_c^2}}} = \frac{a_c}{\sqrt{2}} \quad \text{... (ii)}$$

For tetragonal crystal, $a_t = b_t = \sqrt{2}\, a_c$, $c_t = 2 a_c$

$$\therefore \quad d_{hkl} = (d_{101})_t = \frac{1}{\sqrt{\frac{1^2}{\left(\sqrt{2}\, a_c\right)^2} + \frac{0^2}{\left(\sqrt{2}\, a_c\right)^2} + \frac{1^2}{\left(2 a_c\right)^2}}}$$

$$= \frac{1}{\sqrt{\frac{1}{2 a_c^2} + \frac{1}{4 a_c^2}}} = \frac{1}{\sqrt{\frac{3}{4 a_c^2}}} = \frac{2 a_c}{\sqrt{3}} \quad \text{... (iii)}$$

$$\therefore \quad \frac{(d_{101})_c}{(d_{101})_t} = \frac{\frac{a_c}{\sqrt{2}}}{\frac{2 a_c}{\sqrt{3}}} = \frac{\sqrt{3}\, a_c}{2\sqrt{2}\, a_c} = \sqrt{\frac{3}{8}}.$$

Hence, correct option is (c).

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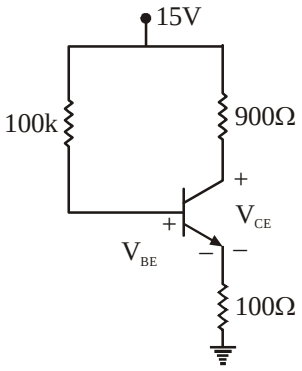
37. $\beta_{dc} = 100$



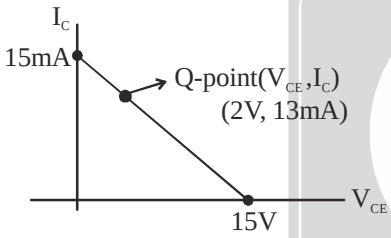
- For drawing DC load line $\beta \rightarrow$ High

Applying KVL at emitter collector loop.

$15 = 900\, I_C + V_{CE} + 100\, I_C$; Load line equation

Put $V_{CE} = 0 \Rightarrow I_C = 15$ mA

Put $I_C = 0 \Rightarrow V_{CE} = 15V$



Now KVL at Base-Emitter loop

$15 = 100 \times I_B + 0.7 + 0.1 \times I_E$

$I_E = \gamma I_B$; $I_E = (\beta + 1) I_B$; $I_E = 101 \times I_B$

$I_B = 0.13$ mA

$I_C = \beta \cdot I_B = 100 \times 0.13$

$I_C = 13$ mA

Now KVL at o/p (output)

$15 = 0.9 \times I_C + V_{CE} + 0.1 \times I_E$

Put value of I_C and I_E

$V_{CE} = 2V$

Correct option is (a)

38. The partition function is

$$Z = \sum_i g(\varepsilon_i) e^{-\beta \cdot \varepsilon_i} = 1e^{-\beta \cdot 0} + 2e^{-\beta \cdot \varepsilon} + 1e^{-\beta \cdot 2\varepsilon}$$

$$= 1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon} = (1 + e^{-\beta\varepsilon})^2$$

Correct option is (c).

CAREER ENDEAVOUR

39. Since surface is charge free

$\therefore \quad \vec{D}_2 \cdot \hat{n} = \vec{D}_1 \cdot \hat{n}$

$\therefore \quad 5\varepsilon_0 E_{2n} = 2\varepsilon_0 5$

$E_{2n} = 2\hat{k}$

And $E_{2\tau} = E_{1\tau}$; $E_{2\tau} = \left(2\hat{i} - 3\hat{j}\right)$

$\therefore \quad \vec{E}_2 = \left(2\hat{i} - 3\hat{j} + 2\hat{k}\right)$

$\therefore \quad \vec{D}_2 = \varepsilon_0 5\left(\vec{E}_2\right) = \varepsilon_0\left(10\hat{i} - 15\hat{j} + 10\hat{k}\right)$

Correct option is (b)

40. Radial probability density for the ground state of Hydrogen atom is

$$P(r) = |\psi_{100}|^2 r^2 = \frac{1}{4\pi a_0^3} r^2 e^{-2r/a_0}$$

For small 'r', the term 'r^2' will dominate and for large 'r', the term 'e^{-2r/a_0}' will dominate.

Correct option is (d)

41. For $\left({}^1p_{1/2}\right)$, $O^{15} + {}_0n^1 \rightarrow O^{16}$

Therefore, binding energy of n in $\left({}^1p_{1/2}\right)$

$= B.E.\left(O^{16}\right) - B.E.\left(O^{15}\right)$ (as n has zero B.E.)

= 127.62 – 111.96 = 15.66 MeV

For $\left({}^1d_{5/2}\right)$, $O^{16} + {}_0n^1 \rightarrow O^{17}$

Therefore, binding energy of n in $\left({}^1p_{1/2}\right)$

$= B.E.\left(O^{17}\right) - B.E.\left(O^{16}\right) = 131.76 - 127.62 = 4.14\ MeV$

Therefore, energy difference = 15.66–4.14 = 11.52 MeV

Correct option is (b)

42. $L(\theta, \phi) = \frac{1}{2} ma^2 \left(\dot{\theta}^2 + \sin^2\theta\, \dot{\phi}^2\right) - mga\cos\theta$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = ma^2\,\dot{\theta} \quad \Rightarrow \quad \dot{\theta} = \frac{p_\theta}{ma^2}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = ma^2 \sin^2\theta\,\dot{\phi} \quad \Rightarrow \quad \dot{\phi} = \frac{p_\phi}{ma^2\sin^2\theta}$$

Hamiltonian is defined as

$H = \sum p_i \dot{q}_i - L$

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$$H = p_\theta \dot{\theta} + p_\phi \dot{\phi} - \frac{1}{2} ma^2 \left(\dot{\theta}^2 + \sin^2 \theta \, \dot{\phi}^2 \right) + mga \cos \theta$$

put $\dot{\theta}$ and $\dot{\phi}$ to get $H = \frac{1}{2ma^2} \left(p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta} \right) + mga \cos \theta$

Correct option is (b)

43. $\vec{F} = \vec{r} \times \vec{B}$

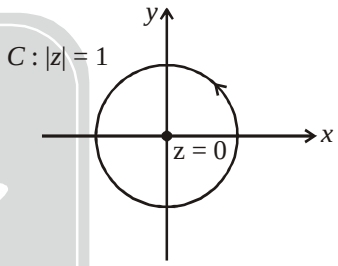
$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S \left(\vec{\nabla} \times \vec{F} \right) \cdot d\vec{S} = \iint \left[\vec{\nabla} \times \left(\vec{r} \times \vec{B} \right) \right] \cdot d\vec{S}$$

$$= \iint_S \left[\left(\vec{B} \cdot \vec{\nabla} \right) \vec{r} - \left(\vec{r} \cdot \vec{\nabla} \right) \vec{B} + \vec{r} \left(\vec{\nabla} \cdot \vec{B} \right) - B \left(\vec{\nabla} \cdot \vec{r} \right) \right] \cdot d\vec{S}$$

$$= \iint \left(\vec{B} - 0 + 0 - 3\vec{B} \right) \cdot d\vec{S} = \iint -2\vec{B} \cdot d\vec{S} - = -2B_0 \pi = -2\pi B_0$$

Correct option is (c)

44. $f(z) = e^{1/z} = \left[1 + \frac{1}{z} + \frac{1}{2!} \frac{1}{z^2} + ... \right]$



$f(z)$ has an essential singular point at $z = 0$.

$\oint_C e^{-1/z} dz = 2\pi i \left[\text{Residue of } f(z) \text{ at } z = 0 \right]$

$= 2\pi i \left[\text{co-efficients of } 1/z \right] = 2\pi i$

Correct option is (d)

45. The number of microstates when N particles are distributed into N_1 and N_2 particles are :

$\Omega = \frac{N!}{N_1! N_2!}$, where $N_1 + N_2 = N$

Given : $N_1 = N_2 = \frac{N}{2}$, $\therefore \quad \Omega = \frac{N!}{\frac{N!}{2} \frac{N!}{2}}$

The entropy is $S = k_B \, \ell n \, \Omega = k_B \left[\ell n (N)! - 2 \ell n \left(\frac{N}{2} \right)! \right]$

Using Stirling approximation, $\ell n \, N! = N \, \ell n \, N - N$ (for large N)

$\therefore \quad S = k_B \left[(N \, \ell n \, N - N) - 2 \left(\frac{N}{2} \ell n \frac{N}{2} - \frac{N}{2} \right) \right]$

$= k_B \left[N \ell n \, N - N \, \ell n \frac{N}{2} \right] = N \, k_B \, \ell n \, 2$

Correct option is (d).

46. The vibrational terms (energies in wave number unit m^{-1} or cm^{-1}) of diatomic molecules are given by

$$G(v) = \omega_e \left(v + \frac{1}{2} \right) - \omega_e x_e \left(v + \frac{1}{2} \right)^2$$

where, v is called vibrational quantum number

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ω_e is the wave-number spacing of energy levels that would occur if the potential were a parabola.

$\omega_e x_e$ is called an harmonicity constant that arise if potential curve is not a perfect parabola.

Thus, $G(0) = \frac{1}{2}\omega_e - \frac{\omega_e x_e}{4}$ ... (1)

$G(1) = \frac{3}{2}\omega_e - \frac{9}{4}\omega_e x_e$ ... (2)

$G(2) = \frac{5}{2}\omega_e - \frac{25}{4}\omega_e x_e$ ... (3)

Fundamental band corresponds to the transition, $v(=0) \to v(1)$, therefore, wave number

$$v_0 = G(1) - G(0) = \omega_e - 2\omega_e x_e$$

First spectral lines (or first overtone band) corresonds to

$v(0) \to v(2)$, thus $v_1 = G(2) - G(0) = 2\omega_e - 6\omega_e x_e$

Ratio of frequencies,

$$\frac{v_1}{v_0} = \frac{2(\omega_e - 3\omega_e x_e)}{\omega_e - 2\omega_e x_e} = 1.96 \quad [\text{Given}]$$

$\Rightarrow \quad x_e = 0.01923 \simeq x_e = 0.02$

Correct option is (b)

47. The average speed, $v_{\text{av.}} = \sqrt{\frac{8}{\pi}\frac{k_B T}{m}}$ ... (i)

The most probable speed, $v_p = \sqrt{\frac{2k_B T}{m}}$ ... (ii)

and the r.m.s. speed, $v_{\text{r.m.s.}} = \sqrt{\frac{3k_B T}{m}}$ ... (iii)

From equations (i) and (ii), $\frac{v_p}{v_{\text{av.}}} = \frac{\sqrt{2}}{\sqrt{8/\pi}} = \frac{\sqrt{\pi}}{2}$

$\Rightarrow \quad v_p = \frac{\sqrt{\pi}}{2} v_{\text{av.}} = \frac{\sqrt{\pi}}{2} \times 400 = 354.49 \text{ m/s} \simeq 355\ m/s$

Now from equations (i) and (iii), $\frac{v_{\text{r.m.s.}}}{v_{\text{av.}}} = \frac{\sqrt{3}}{\sqrt{8/\pi}} \Rightarrow v_{\text{r.m.s.}} = \sqrt{\frac{3\pi}{8}}\, v_{\text{av.}} = \sqrt{\frac{3\pi}{8}} \times 400 = 434.16 \text{ m/s} \simeq 434\ m/s$

Correct option is (a).

48. $\hat{H}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[e^{ikx} + 2e^{-ikx}\right] = -\frac{\hbar^2}{2m}\left[(ik)(ik)e^{ikx} + (-ik)(-ik)2e^{-ikx}\right] = \frac{\hbar^2 k^2}{2m}\left[e^{ikx} + 2e^{-ikx}\right] = \frac{\hbar^2 k^2}{2m}\psi$.

Energy of the particle is $E = \frac{\hbar^2 k^2}{2m}$.

Correct option is (c)

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49. Real part of the wave function is $\mathrm{Re}(\psi) = \cos kx + 2\cos kx$.

Probability current density for the real part of the wave function is

$$J = -\frac{i\hbar}{2m}\left[\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right] = 0 \quad (\text{Since } \psi = \psi^* \text{ for real wavefunction})$$

Correct option is (d)

50. $$\omega = A\left|\sin\frac{Ka}{2}\right| \qquad \text{... (i)}$$

At the boundary of first Brillouin zone, i.e. at $K = \pm\frac{\pi}{a}$, equation (i) gives

$$\omega = A\left|\sin\left(\pm\frac{\pi}{a}\cdot\frac{a}{2}\right)\right| = A\left|\sin\left(\pm\frac{\pi}{2}\right)\right| = A \qquad \text{... (ii)}$$

It can also be solved as : Group velocity $v_g = \frac{d\omega}{dK} = \frac{d}{dK}\left(A\left|\sin\frac{Ka}{2}\right|\right) = \frac{Aa}{2}\left|\cos\frac{Ka}{2}\right|$ ... (iii)

At the boundary of first Brillouin zone i.e. for $K = \pm\frac{\pi}{a}$, equation (ii) gives

$$\Rightarrow \quad v_g = \frac{Aa}{2}\cos\left(\frac{\pi}{2}\right) \quad \Rightarrow \quad v_g = 0.$$

Standing waves produced at Brillouin zone boundary.

Hence, correct option is (a).

51. The dispersion relation (ω versus K) for one-dimensional monatomic lattice is given by

$$\omega = \sqrt{\frac{4C}{M}}\left|\sin\frac{Ka}{2}\right| \qquad \text{... (iv)}$$

Comparing equation (iv) and equation (i), we get (where, C is force constant)

$$A = \sqrt{\frac{4C}{M}} \quad \Rightarrow \quad A^2 = \frac{4C}{M} \Rightarrow \quad C = \frac{MA^2}{4}.$$

Correct option is (a).

52. Since, $E = \frac{1}{4\pi\varepsilon_0}\bullet\frac{p}{a^3}$

$$\Rightarrow \quad a^3 = \frac{p}{4\pi\varepsilon_0 E} = \frac{qd}{4\pi\varepsilon_0 E}$$

$$\Rightarrow \quad d = \frac{4\pi\varepsilon_0 a^3 E}{q} = \frac{4\times 3.14\times 8.854\times(0.5\times10^{-10})^3\times 30\times10^5\times10^{-12}}{1.6\times10^{-19}} = 260.63\times10^{-18}\ \text{m}$$

$$\Rightarrow \quad d = 2.60\times10^{-16}\ \text{m}.$$

Correct option is (c).

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53. Since, dipole moment $(p) = q \times d = \alpha E$

$$\Rightarrow \quad \alpha = \frac{qd}{E} = \frac{1.6\times10^{-19}\times2.6\times10^{-16}}{30\times10^{5}} = 0.139\times10^{-40}$$

$$\Rightarrow \quad \alpha = 1.4\times10^{-41}.$$

Correct option is (b).

54. Given $y = ax^2 \quad \Rightarrow \quad \dot{y} = 2ax\dot{x}$

$$L = T - V$$

$$= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - mgy$$

$$= \frac{1}{2}m\left(\dot{x}^2 + 4a^2x^2\dot{x}^2\right) - mgax^2$$

$$L = \frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2 - mgax^2$$

Correct answer is (b)

55. Equation of motion is

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left[m\left(1+4a^2x^2\right)\dot{x}\right] - 4ma^2\dot{x}^2x + 2mgax = 0$$

$$\Rightarrow \quad m\left(1+4a^2x^2\right)\ddot{x} + 8ma^2x\dot{x}^2 - 4ma^2\dot{x}^2x + 2mgax = 0$$

$$\Rightarrow \quad m\left(1+4a^2x^2\right)\ddot{x} + 4ma^2\,x\dot{x}^2 + 2mgax = 0$$

or $\left(1+4a^2x^2\right)\ddot{x} + 4a^2x\,\dot{x}^2 + 2gax = 0$

Correct answer is (b)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

1. For a symmetric periodic function i.e., $f(-x) = f(x)$, fourier co-efficient $b_n = 0$
The Fourier series contains constant term and cosine terms.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n k x)$$

Correct option is (b)

2. Stress = Force/Area, there are two directions involved with it, i.e., one is direction of force and the other one is direction of area.
Strain = change in length/original length, there are two directions involved with it, i.e., one is the direction of length and the direction along which length is changing.

Moment of inertia $= \dfrac{\text{Angular momentum}}{\text{Angular velocity}}$ i.e., two directions involved one of angular momentum and other of angular velocity

Pressure = Force/Area ; But here direction of force and direction of area are same. Therefore, it is a first order tensor.

Correct option is (d)

3. $\bigcirc\!\!\rightarrow$ 0.85c (electron) $\quad \bigcirc\!\!\rightarrow$ c (photon)

velocity of electron w.r.t. photon is given as

$$v_{ep} = \frac{v_e - v_p}{1 - \dfrac{v_e \cdot v_p}{c^2}} = \frac{0.85c - c}{1 - 0.85} = \frac{c(0.85 - 1)}{1 - 0.85} = -c$$

$\therefore \quad v_{ep} = -c$

Correct option is (b)

4. Energy in case of blackbody radiation is given by

$$\langle E \rangle = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$

If $\hbar \to 0$, we have $\dfrac{0}{0}$ form.

So using L. Hospital's rule, we have

$$\langle E \rangle = \lim_{\hbar \to 0} \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1} = \lim_{\hbar \to 0} \frac{\omega}{e^{\hbar\omega/k_B T} \cdot \dfrac{\omega}{k_B T}} = \frac{k_B T}{1} = k_B T$$

Correct option is (d)

5. The free energy is continuous but its derivative is discontinuous in first order phase transition.
Correct option is (c)

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6. Once the gases are allowed to freely exchange of energy, both sides will attain thermal equilibrium by maintaining the same temperature. Besides this, due to unbalanced pressure, both sides will attain mechanical equilibrium by balancing the pressure on both sides of the movable wall.

Correct option is (a)

7. **Given:** $S(E) = aE(E_0 - E)$.

The entropy is defined as

$$dS = \frac{dE}{T}$$

$$\Rightarrow \quad \frac{1}{T} = \frac{dS}{dE} = aE_0 - 2aE$$

$$\Rightarrow \quad T = \frac{1}{a(E_0 - 2E)}$$

For $E_0 < 2E$, T is negative.

Correct option is (a)

8. The number of microstates for N independent spin s particles are

$$\Omega = (2s+1)^N = 2^N \qquad [\because \quad s = 1/2]$$

The entropy is

$$S = k \ln \Omega = Nk \ln 2$$

Correct option is (d)

9.

$n \longrightarrow$	p^+	$+$ e^{-1}	$+$ $\bar{\nu}_e$	
$B:1$	1	0	0	$\Delta B = 0$
$L:0$	0	+1	−1	$\Delta L = 0$
$I_3: -½$	½	0	0	$\Delta I = -1 \neq 0$
$S:0$	0	0	0	$\Delta S = 0$

Correct option is (c)

10. Baryon number of up quark is 1/3.
Isospin number of up quark is 1/2.

Correct option is (d)

11. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range r_0.

Given that $\cot(kr_0 + \delta) \simeq -\frac{\gamma}{k}$ here, δ = phase shift

$$\tan(kr_0 + \delta) = -\frac{k}{\gamma} \Rightarrow \quad kr_0 + \delta = -\frac{k}{\gamma}$$

$$\boxed{\delta \simeq -\frac{k}{\gamma} - kr_0}$$

Correct option is (a)

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12. According to Fermi-Selection rules $\Delta I = 0$

Gamow Teller rules $\Delta I = \pm 1, 0 \ (\text{except } 0 \leftrightarrow 0)$

Therefore, the transition $2^+ \to 3^+$, $\Delta I = +1$, parity does not change. The given transition is not allowed by Fermi but allowed by Gamow Teller selection rule.

Correct option is (c)

13. At a surface current (using boundary condition), normal component of magnetic field is continuous and tangential component of magnetic field is discontinuous.
For vector potential : Both the normal component and tangential component are continuous

Correct option is (d)

14. Equation for plane waves are given as

$$\psi_1(r,t) = A_1 e^{i(k_1 r - \omega t)}; \qquad \psi_2(r,t) = A_2 e^{i(k_2 r - \omega t)}$$

Phase difference = $\delta = \left(\vec{k}_1 - \vec{k}_2\right) \times$ path difference

Condition for interference maxima $= \left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} = 2m\pi$

Correct option is (b)

15. Consider a scalar function ϕ which is satisfying the Laplace equation, $\vec{\nabla}^2 \phi = 0$

$$\vec{\nabla} \cdot \left(\vec{\nabla}\phi\right) = 0$$

Also, $\vec{\nabla} \times \left(\vec{\nabla}\phi\right) = 0$

Hence, $\vec{\nabla}\phi$ has both zero divergence and zero curl.

Correct option is (c)

16. Circularly polarized light can be decomposed into two perpendicular components. One in the plane of incidence and the other perpendicular to the plane. At Brewster angle the reflected light contains only perpendicular component and refracted light has larger component in the plane of incidence than its component perpendicular to the plane. As a result the reflected light will be plane polarized light where as refracted light will be elliptically polarized light.

Correct option is (c)

17. We know that, $\left[L^2, L_z\right] = 0, \left[L_x, L_y\right] = i\hbar L_z$

$$\left[L_z, L_+\right] = \left[L_z, L_x + iL_y\right] = \left[L_z, L_x\right] + i\left[L_z, L_y\right] = i\hbar L_y + i\left(-i\hbar L_x\right)$$

$$= i\hbar\left[L_y - iL_x\right] = \hbar L_+$$

$$\left[L_z, L_-\right] = \left[L_z, L_x - iL_y\right] = -i\hbar L_-$$

Correct option is (d)

18. $L = \alpha\dot{q}^2 + \beta q^2$

canonical momentum is given as $p_q = \dfrac{\partial L}{\partial \dot{q}} = 2\alpha\dot{q}$

Since q is not cyclic, p_q is not conserved

Correct answer is (d)

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19. Given 6-bit A/D converter analog to digital convertor.

Maximum convertion time = 32 μ sec

Note: Not mention which type of A to D convertor is given.

Flash ADC > Successive approximation ADC > Counter type ADC > Dual slope or integrating type

$C_T = 1$ $C_T = nT_C$ $C_T = 2^{n-1}T_C$ $C_T = 2^n T_C$

Assuming counter type ADC: $C_T = 2^{n-1}T_C$ $T_C = ?$

$\Rightarrow 32\times10^{-6} = 2^{6-1}\times T_C$

$T_C = 1\times10^{-6} \quad \Rightarrow \quad f_C = 1\,MHz$

Assuming Dual slope or integrating type ADC : $C_T = 2^n T_C$

$\Rightarrow 32\times10^{-6} = 2^6\times T_C \Rightarrow T_C = 0.5\times10^6 \Rightarrow f_C = 2$ MHz

Note: Option (a) or (b) is correct depending on the type of ADC.

20. With the increase in temperature, the concentration of minority carriers starts increasing. Eventually, a temperature is reached called the critical temperature (85 ºC in case of germanium and 200 ºC in case of silicon) when the number of covalent bonds that are broken is very large and the number of holes is approximately equal to the number of electrons. The extrinsic semiconductor now behaves essentially like an intrinsic semiconductor. Therefore, at 200 ºC, silicon will have the Fermi-level in the middle of energy gap.

Hence, correct option is (c).

21. Considering the BCS theory of superconductors the statement "(c) Quantization of magnetic flux in superconducting ring in the unit of (*h*/*e*)" is not correct. It correctly predicts the Meissner effect, isotope effect and band gap in the superconductor. It is quantised in units of $\left(\frac{h}{2e}\right)$

Hence, correct option is (c).

22.

Group-I	**Group-II**
P. Phonon	iv. elastic wave
Q. Plasmon	iii. collective electron oscillations
R. Polaron	ii. electron + elastic deformation
S. Polariton	i. photon + lattice vibration

Hence, correct option is (b).

23. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is

$${}^{N_i+g_i-1}C_{N_i}, \quad N_i = 4,\ g_i = 5(\text{degeneracy})$$

$$\Rightarrow \quad {}^{4+5-1}C_4 = {}^8C_4 = \frac{8!}{4!\times4!} = \frac{8\times7\times6\times5\times4!}{24\times4!} = 70$$

Correct answer is (70)

24. Ripple Rejection of voltage regulator = –50 dB

- input ripple is 1 mV
- output ripple voltage in μV

$$\text{Ripple Rejection} = \frac{\text{output ripple voltage}}{\text{input ripple voltage}}$$

$-50\ \text{dB} = 20\log_{10}\left(\text{Ripple Rejection}\right)$

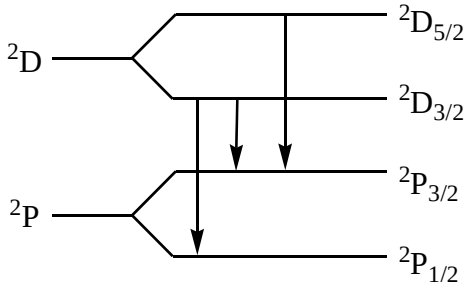
$\text{Ripple rejection} = 3.162\times10^{-3}$

$3.162\times10^{-3} = \dfrac{\text{O/P ripple voltage}}{1\times10^{-3}}$

$\text{O/P ripple voltage} = 3.162\times10^{-6}\ \text{Volt} \simeq 3.16\ \mu\text{V}$

Correct answer is (3.16)

25.



Selection rules for sodium (alkali spectra) are $\Delta L = \pm1, \Delta S = 0, \Delta J = 0, \pm1$ (but $J = 0 \not\leftrightarrow J = 0$). So, only three spectral lines are allowed.

Correct answer is (3)

Q.26 – Q.55 : Carry TWO marks each.

26. $L[\delta(t)] = \int_0^\infty \delta(t)\, e^{-st}dt = 1$

$L[1] = \int_0^\infty 1\cdot e^{-st}dt = \left[\dfrac{e^{-st}}{-s}\right]_0^\infty = \dfrac{1}{s}$

$L[\sin kt] = \int_0^\infty \sin kt\, e^{-st}dt = \dfrac{k}{s^2+k^2}$

$L[t] = \int_0^\infty t\cdot e^{-st}dt = \dfrac{1}{s^2}$

$L[te^{kt}] = L[t]_{s\to s-k} = \left.\dfrac{1}{s^2}\right|_{s\to s-k} = \dfrac{1}{(s-k)^2}$

Correct option is (c)

27. $\vec{\nabla}\left[\vec{A}\cdot\left(\vec{B}\times\vec{r}\right)\right] = \vec{\nabla}\left[\vec{r}\cdot\left(\vec{A}\times\vec{B}\right)\right] = \vec{\nabla}\left[\left(\vec{A}\times\vec{B}\right)\cdot\vec{r}\right]$

If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\left(\vec{A}\times\vec{B}\right)$ is also constant vector.

For constant vector $\vec{a}$, $\vec{\nabla}\left(\vec{a}\cdot\vec{r}\right) = \vec{a}$

Therefore, $\vec{\nabla}\left[\left(\vec{A}\times\vec{B}\right)\cdot\vec{r}\right] = \left(\vec{A}\times\vec{B}\right)$

Correct option is (b)

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28. For $n=0, \Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$

For $n=1, \Gamma\left(\frac{3}{2}\right)=\frac{1}{2}\Gamma\left(\frac{1}{2}\right)=\frac{1}{2}\sqrt{\pi}$

For $n=2, \Gamma\left(\frac{5}{2}\right)=\frac{3}{2}\Gamma\left(\frac{3}{2}\right)=\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}$

All these satisfies the form given in option (c)

Correct option is (c)

29. For one dimensional motion, Newton's equation of motion is written as

$$F=\gamma^3 m_0\frac{dv}{dt}$$

$$F=\frac{m_0}{\left(1-v^2/c^2\right)^{3/2}}\frac{dv}{dt}$$

$$F=\frac{m}{1-v^2/c^2}\cdot\frac{dv}{dt}, \quad m=\frac{m_0}{\sqrt{1-v^2/c^2}}$$

Correct option is (c)

30. $\left\langle e^{i\vec{a}\cdot\vec{p}}\right\rangle=\left\langle e^{i\left(a_x p_x+a_y p_y+a_z p_z\right)}\right\rangle$

where, $a=\sqrt{a_x^2+a_y^2+a_z^2}$ & $p=\sqrt{p_x^2+p_y^2+p_z^2}$

$\therefore \quad \left\langle e^{i\vec{a}\cdot\vec{p}}\right\rangle=\left\langle e^{ia_x p_x}\right\rangle\left\langle e^{ia_y p_y}\right\rangle\left\langle e^{ia_z p_z}\right\rangle$

$$=\left(\frac{m}{2\pi kT}\right)^{3/2}\int_{-\infty}^{+\infty}e^{mia_x v_x}e^{-\frac{mv_x^2}{2kT}}dv_x\times\int_{-\infty}^{+\infty}e^{mia_y v_y}\,e^{-\frac{mv_y^2}{2kT}}dv_y\times\int_{-\infty}^{+\infty}e^{ima_z v_z}\,e^{-\frac{mv_z^2}{2kT}}dv_z$$

Here, we have used Maxwell-Boltzmann distribution function $f\left(v_x, v_y, v_z\right)$ and it is equal to

$$f\left(v_x, v_y, v_z\right)=\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{m}{2kT}\left(v_x^2+v_y^2+v_z^2\right)}$$

Now, $\int_{-\infty}^{\infty}e^{mia_x v_x-\frac{mv_x^2}{2kT}}dv_x=e^{-\frac{m}{2kT}a_x^2k^2T^2}\int_{-\infty}^{+\infty}e^{-\frac{m}{2kT}\left(v_x-ia_x kT\right)^2}dv_x=\sqrt{\frac{2\pi kT}{m}}\,e^{-\frac{m}{2}a_x^2kT}$

$\therefore \quad \left\langle e^{i\vec{a}\cdot\vec{p}}\right\rangle=\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2\pi kT}{m}\right)^{3/2}e^{-\frac{m}{2}\left(a_x^2+a_y^2+a_z^2\right)kT}=e^{-\frac{m}{2}a^2kT}$

Correct option is (c)

31. The electromagnetic form factor $\mathrm{F}\left(\mathrm{q}^2\right)=\exp\left(-\frac{\mathrm{q}^2}{2\mathrm{Q}^2}\right)$

And we know that, $F\left(q\right)=\int e^{iq\cdot r}\left(r\right)dr$

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$$F(q) = 1 - \frac{q^2}{6}\langle r^2 \rangle + \ldots\ldots\ldots\ldots (q \cdot r << 1)$$

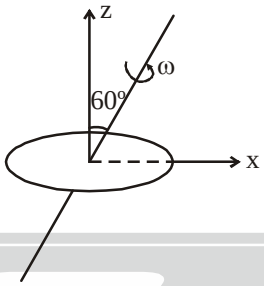
$$\langle r^2 \rangle = -6 \frac{\partial F}{\partial q^2}\bigg|_{q=0} = -6 \times \exp\left(-\frac{q^2}{2Q^2}\right) \times \left(-\frac{1}{2Q^2}\right)\bigg|_{q=0}$$

$$\Rightarrow \quad \frac{3}{Q^2} \Rightarrow \sqrt{\langle r^2 \rangle} = \frac{\sqrt{3}}{Q}$$

Correct option is (c)

32. Angular momentum of disc is

$$L = \sqrt{(I_{xx}\omega_x)^2 + (I_{yy}\omega_y)^2 + (I_{zz}\omega_z)^2}$$



Here, $I_{xx} = \frac{MR^2}{4} = I_{yy}$

$$I_{zz} = \frac{MR^2}{2}$$

$$\omega_x = \omega \sin 60^\circ = \omega \frac{\sqrt{3}}{2}$$

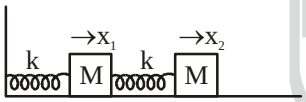
$$\omega_y = 0, \ \omega_z = \omega \cos 60^\circ = \omega/2$$

$$\therefore \quad L = \sqrt{\left(\frac{MR^2}{4}\omega\frac{\sqrt{3}}{2}\right)^2 + 0 + \left(\frac{MR^2}{2}\cdot\frac{\omega}{2}\right)^2}$$

$$= \frac{MR^2\omega}{4}\sqrt{\frac{3}{4}+1} = \frac{\sqrt{7}\,MR^2\omega}{8}$$

Correct option is (c)

33.



Let x_1 and x_2 be displacement of masses from their respective equilibrium positions. Therefore,

Kinetic energy $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}M\dot{x}_2^2$

Potential energy $V = \frac{1}{2}k(x_1 - 0)^2 + \frac{1}{2}k(x_2 - x_1)^2$

$$= \frac{1}{2}k\left(2x_1^2 + x_2^2 - 2x_1x_2\right)$$

Corresponding matrices are

$$\hat{T} = M\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \hat{V} = k\begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}$$

For frequency (ω) of normal modes

$$\det\left|-\omega^2\hat{T} + \hat{V}\right| = 0$$

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$$\begin{vmatrix} 2k - M\omega^2 & -k \\ -k & k - M\omega^2 \end{vmatrix} = 0$$

or $\left(2k - M\omega^2\right)\left(k - M\omega^2\right) - k^2 = 0$

$2k^2 - 3Mk\omega^2 + M^2\omega^4 - k^2 = 0$

$M^2\omega^4 - 3Mk\omega^2 + k^2 = 0$

$\therefore \quad \omega^2 = \dfrac{3Mk \pm \sqrt{9M^2k^2 - 4M^2k^2}}{2M^2}$

$\omega^2 = \dfrac{k}{2M}\left(3 \pm \sqrt{5}\right) \Rightarrow \omega = \sqrt{\dfrac{k}{2M}\left(3 \pm \sqrt{5}\right)}$

Correct option is (c)

34. We have one dimension charge density

$$\rho = Q\left\{\delta\left(x - x_0\right) - \delta\left(x + x_0\right)\right\}$$

Therefore, the total charge,

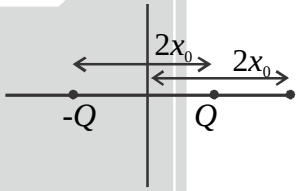
$$Q = \int \rho dx = Q\int \delta\left(x - x_0\right)dx - Q\int \delta\left(x + x_0\right)dx$$

$$= Q\left(\text{at } x = x_0\right) + \left(-Q\right)\left(\text{at } x = -x_0\right)$$

Therefore, the field at $\left(2x_0, 0, 0\right)$ point is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q}{x_0^2} - \frac{Q}{\left(3x_0\right)^2}\right\}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{9-1}{9x_0^2}\right] = \frac{2Q}{9\pi\varepsilon_0 x_0^2}\hat{x}$$



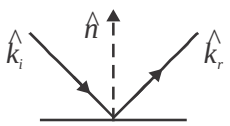
Correct option is (a)

35. We know that $\vec{k}_i, \vec{k}_r, \hat{n}$ all are in the same plane.

The vector $\left(\hat{k}_i \times \hat{n}\right)$ has its direction perpendicular

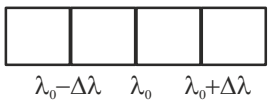
to the plane containing the vectors $\hat{k}_i, \hat{n}, \hat{k}_r$

$\therefore \quad \left(\hat{k}_i \times \hat{n}\right) \cdot \hat{k}_r = 0$ because $\left(\hat{k}_i \times \hat{n}\right) \perp \hat{k}_r$



Correct option is (c)

36. $5\,^1D_2 \longrightarrow 5\,^1P_1$



Given : wavelength, $\lambda = 643.8\ nm$

Wavelength is related to wave-number by

$$\nu = \frac{1}{\lambda} \Rightarrow \Delta\nu = -\frac{\Delta\lambda}{\lambda^2} \qquad \ldots (1)$$

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$$\Rightarrow \Delta\lambda = -\lambda^2 \Delta\nu$$

In normal Zeeman effect experiment. Wave number separation between consecutive Zeeman level are equal and is given by

$$\Delta\nu = \frac{eB}{4\pi m_e c}$$

where, B is the applied magnetic field.

Putting, $e = 1.6\times10^{-19}$ C, $m_e = 9.1\times10^{-31}$ kg, $c = 3\times10^{8}$ m/s, we get

$$\Delta\nu = 46.7\ Bm^{-1} \quad \text{where, } B \text{ is in Tesla} \qquad \ldots (2)$$

As the spectrometer has resolution to observe a minimum spread in wavelength $\Delta\lambda_{\min} = 0.01\ nm$, therefore from equation (1) and equation (2).

$$\Delta\lambda = 0.01\ nm = (643.8\ nm)^2\ \Delta\nu\ = (643.8\ nm)^2 \times 46.7\ B\ m^{-1}$$

Thus, minimum magnetic field,

$$B_{\min} = \frac{0.01\ nm}{(643.8\ nm)^2 \times 46.7} = \frac{0.01\times10^{9}}{(643.8)^2 \times 46.7} = 5.166\times10^{-10}\times10^{9} = 0.52\ T$$

Correct option is (b)

37. Energy level of harmonic oscillator

$$E_v = h\nu_{osci}\left(v + \frac{1}{2}\right) \quad \text{where, } v = \text{vibrational quantum number}$$

ν_{osci} is classical frequency given by

$$\nu_{osc} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}; \qquad k = \text{force constant, } \mu = \text{reduced mass}$$

Selection rule, $\Delta v = \pm1$, gives

$$\Delta E_v = h\nu_{osci} = \text{equispaced energy level}$$

Given : $\Delta E_v = 8.44\times10^{-2}\ eV = 8.44\times10^{-2}\times1.6\times10^{-19}\ J = 13.504\times10^{-21}\ J$

Also, $\Delta E_v = h\cdot\frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \Rightarrow k = \frac{4\pi^2\mu(\Delta E_v)^2}{h^2}$

$$k = \frac{4\times(3.14)^2\times1.14\times10^{-26}\ kg\times(13.504\times10^{-21})^2}{(6.626\times10^{-34})^2} = 186.74\ N/m$$

Correct option is (c)

38. Primitive vectors, $\vec{a} = 2(\hat{j}+\hat{k})$ Å, $\vec{b} = 2(\hat{k}+\hat{i})$ Å, $\vec{c} = 2(\hat{i}+\hat{j})$ Å ... (i)

Volume of unit cell, $V = \vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{vmatrix} = 0[0-4] - 2[0-4] + 2[4-0] = 8+8 = 16\ \text{Å}^3$

Now, translation vectors of reciprocal lattice

$$\vec{a}^* = \frac{2\pi(\vec{b}\times\vec{c})}{\vec{a}\cdot(\vec{b}\times\vec{c})} = \frac{2\pi\left[2(\hat{k}+\hat{i})\times2(\hat{i}+\hat{j})\right]\text{Å}^2}{16\ \text{Å}^3} = \frac{8\pi}{16}\left[\hat{k}\times\hat{i}+\hat{k}\times\hat{j}+\hat{i}\times\hat{i}+\hat{i}\times\hat{j}\right]\text{Å}^{-1}$$

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$$= \frac{\pi}{2}\left[\hat{j} - \hat{i} + \hat{k}\right] \text{Å}^{-1} = \frac{\pi}{2}\left[-\hat{i} + \hat{j} + \hat{k}\right] \text{Å}^{-1}$$

Similarly, $\vec{b}^* = \frac{\pi}{2}\left[\hat{i} - \hat{j} + \hat{k}\right] \text{Å}^{-1}$ ... (ii)

and $\vec{c}^* = \frac{\pi}{2}\left[\hat{i} + \hat{j} - \hat{k}\right] \text{Å}^{-1}$ ... (iii)

Therefore, volume of the reciprocal lattice,

$$V^* = \vec{a}^* \cdot \left(\vec{b}^* \times \vec{c}^*\right) = \frac{\pi}{2}\left(-\hat{i} + \hat{j} + \hat{k}\right) \cdot \left[\frac{\pi}{2}\left(\hat{i} - \hat{j} + \hat{k}\right) \times \frac{\pi}{2}\left(\hat{i} + \hat{j} - \hat{k}\right)\right]$$

$$= \left(\frac{\pi}{2}\right)^3 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \frac{\pi^3}{8} \times \left[-1(1-1) - 1(-1-1) + 1(1+1)\right] = \frac{\pi^3}{8} \times 4 = \frac{\pi^3}{2}$$

which is the volume of primitive unit cell of bcc with cube edge $\left(\pi^3/2\right)$.

Correct option is (a).

39. Total energy, $E = -\frac{\alpha e^2}{4\pi \varepsilon_0 r} + \frac{B}{r^9}$ ... (i)

For equilibrium, $\left.\frac{dE}{dr}\right|_{r=r_0} = 0$, $(r = r_0$ (equilibrium separation))

$$\Rightarrow \quad -\frac{\alpha e^2}{4\pi \varepsilon_0}\left(-\frac{1}{r_0^2}\right) - \left(\frac{9B}{r_0^{10}}\right) = 0 \qquad \Rightarrow \quad \frac{\alpha e^2}{4\pi \varepsilon_0 r_0^2} = \frac{9B}{r_0^{10}}$$

$$\Rightarrow \quad r_0^8 = \frac{(9B)(4\pi \varepsilon_0)}{\alpha e^2} \qquad \Rightarrow \quad B = \frac{\alpha e^2 r_0^8}{36\pi \varepsilon_0}.$$

Correct option is (a).

40. Sides of box = 10^{-12} m

Charge of proton = 1.6×10^{-19} coulomb

Mass of proton $(m_p) = 1.67 \times 10^{-27}$ kg

Planck's constant $h = 6.63 \times 10^{-34}$ J-s

Minimum K.E. $(E_K) = \left(\frac{\hbar^2}{2m}\right)\frac{\pi^2}{L^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{h^2 \cdot 3}{8mL^2}$ $(n_x = n_y = n_z = 1$ for minimum energy)

$$= \frac{(6.63 \times 10^{-34})^2 \times 3}{8 \times 1.67 \times 10^{-27} \times (10^{-12})^2} = 3 \times 3.29 \times 10^{-17} = 9.9 \times 10^{-17} \text{ J.}$$

Correct option is (c).

41. $f(z) = \frac{16z}{(z+3)(z-1)^2}$ has a pole of order 2 at $z = 1$

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Residue of $f(z)$ at $z=1$

$$=\frac{1}{(2-1)!}\frac{d}{dz}\left[(z-1)^2\frac{16z}{(z+3)(z-1)^2}\right]_{z=1}=\frac{(z+3)\cdot 16-16z\cdot 1}{(z+3)^2}\bigg|_{z=1}=\frac{48}{(z+3)^2}\bigg|_{z=1}=3$$

Correct answer is (3)

42. Trace of the matrix = $\lambda_1+\lambda_2+\lambda_3=12$

Determinant of the matrix = $\lambda_1\lambda_2\lambda_3=50$

The combination $\lambda_1=5,\ \lambda_2=5,\ \lambda_3=2$ satisfies the above two relations.

Correct answer is (5)

43. According to energy conservation,

$$E_{\bar{\nu}}+E_\mu=m_\pi c^2 \qquad \text{... (i)}$$

According to momentum conservation,

$$0=P_{\bar{\nu}}+P_\mu \Rightarrow P_\mu=-P_{\bar{\nu}}$$

$$E_{\bar{\nu}}=\sqrt{P_{\bar{\nu}}^2c^2+m_{\bar{\nu}}^2c^4}=P_{\bar{\nu}}c \qquad (\because\ m_\nu=0) \qquad \text{... (ii)}$$

Using (i) and (ii),

$$\Rightarrow \qquad m_\pi c^2=\sqrt{P_\mu^2c^2+m_\mu^2c^4}+P_{\bar{\nu}}c$$

$$\Rightarrow \qquad \left(m_\pi c^2-P_{\bar{\nu}}c\right)^2=P_\mu^2c^2+m_\mu^2c^4 \quad \Rightarrow\ P_{\bar{\nu}}c=\frac{\left(m_\pi^2-m_\mu^2\right)c^2}{2m_\pi} \qquad \text{... (iii)}$$

Using (ii) and (iii), $E_{\bar{\nu}}=\dfrac{\left(m_\pi^2-m_\mu^2\right)c^2}{2m_\pi}=\left[\dfrac{(139.6)^2-(105.7)^2}{2\times 139.6}\right]c^2=29.7\ MeV\simeq 30$ MeV

Correct answer is (30)

44. $$\oint_C \vec{A}\cdot d\vec{\ell}=\iint_S\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S}=\iint_S\vec{B}\cdot d\vec{S}=BA\cos\theta=0.6\times\frac{1}{2}\times\cos 60^\circ=0.6\times\frac{1}{2}\times\frac{1}{2}=0.15$$

Correct answer is (0.15)

45. $$|\psi\rangle=0.8\left|\uparrow\right\rangle+0.6\left|\downarrow\right\rangle=0.8\begin{pmatrix}1\\0\end{pmatrix}+0.6\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0.8\\0.6\end{pmatrix}$$

$$10\,\sigma_z+5\,\sigma_x=10\begin{pmatrix}1&0\\0&-1\end{pmatrix}+5\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}10&5\\5&-10\end{pmatrix}$$

$$\langle\psi|10\,\sigma_z+5\,\sigma_x|\psi\rangle=\begin{pmatrix}0.8&0.6\end{pmatrix}\begin{pmatrix}10&5\\5&-10\end{pmatrix}\begin{pmatrix}0.8\\0.6\end{pmatrix}=\begin{pmatrix}0.8&0.6\end{pmatrix}\begin{pmatrix}11\\-2\end{pmatrix}=(8.8-1.2)=7.6$$

Correct answer is (7.6)

46. $$J=\frac{i\hbar}{2m}\left(\psi\nabla\psi^*-\psi^*\nabla\psi\right),\ \psi=A\,e^{ikr}\left(\frac{r_0}{r}\right)$$

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$$= \frac{i\hbar}{2m}\left[A\, e^{ikr}\left(\frac{r_0}{r}\right)\frac{\partial}{\partial r}\left(A^*\, e^{-ikr}\left(\frac{r_0}{r}\right)\right) - \left(A\, e^{ikr}\left(\frac{r_0}{r}\right)\right)^* \frac{\partial}{\partial r}\left(A\, e^{ikr}\left(\frac{r_0}{r}\right)\right)\right]$$

$$= \frac{i\hbar}{2m}|A|^2\left[e^{ikr}\left(\frac{r_0}{r}\right)(-ik)\, e^{-ikr}\left(\frac{r_0}{r}\right) - e^{-ikr}\left(\frac{r_0}{r}\right)(ikr)\, e^{ikr}\left(\frac{r_0}{r}\right)\right]$$

$$= \frac{i\hbar}{2m}|A|^2\left[(-ik)\left(\frac{r_0}{r}\right)^2 - (ik)\left(\frac{r_0}{r}\right)^2\right] = \frac{k\hbar}{2m}|A|^2\, 2\left(\frac{r_0}{r}\right)^2 = 0.25\times\frac{\hbar k|A|^2}{m}$$

Correct answer is (0.25)

47. $I_{DSS} = 5\, mA \Rightarrow V_P = 5V \qquad \Rightarrow V_{GS} = -2.5V$

Drain current, $i_d \cong I_{DSS}\left(1+\frac{V_{GS}}{V_P}\right)^2 = 5\left(1-\frac{2.5}{5}\right)^2 = 5\times\left(\frac{1}{2}\right)^2 = \frac{5}{4} = 1.25$

Correct answer is (1.25)

48. The fermions are distributed as

E	$2E$	$3E$	$4E$
•	•		
•		•	
•			•
	•	•	
	•		•
		•	•

So the partition function is

$$Z = \sum_i e^{-\beta E_i} = e^{-\beta(E+2E)} + e^{-\beta(E+3E)} + e^{-\beta(E+4E)} + e^{-\beta(2E+3E)} + e^{-\beta(2E+4E)} + e^{-\beta(3E+4E)}$$

$$= e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$$

Correct option is (b)

49. The partition function of one distinguishable particle is

$$Z_1 = e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E}$$

Since distinguishable particles are non-interacting, the partition function of two such particles is

$$Z = (Z_1)^2 = \left(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E}\right)^2$$

Correct option is (c)

50. Unperturbed hamiltonian, $H_0 = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

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Eigenvalues of $H_0 = |H_0 - \lambda I| = 0 \Rightarrow \begin{vmatrix} 5-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow (5-\lambda)\left[(5-\lambda)(2-\lambda)\right] - 2\left(2(2-\lambda)\right) = 0$

$\Rightarrow (2-\lambda)\left[(5-\lambda)^2 - 4\right] = 0 \Rightarrow (2-\lambda)\left[25 + \lambda^2 - 10\lambda - 4\right] = 0$

$\Rightarrow (2-\lambda)\left[\lambda^2 - 10\lambda + 21\right] = 0 \Rightarrow \lambda = 2, 3, 7$

Eigenvector for $\lambda = 2$

$$\begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2 \times (3x_1 + 2x_2 + 0x_3) = 0$ ... (1)

$3 \times (2x_1 + 3x_2 + 0x_3) = 0$ ... (2)

$(2)\text{-}(1) \Rightarrow 5x_2 = 0,\ x_2 = 0,\ x_1 = 0,\ x_3 =$ arbitrary

So, eigenvector = (0, 0, 1)

Correct option is (c)

51. Perturbed Hamiltonian, $H' = \varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

Eigenvectors corresponding to eigenvalue, of the unperturbed Hamiltonain, E = 2, 3, 7 will be

$$|\phi_1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\ |\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},\ |\phi_3\rangle = \frac{1}{\sqrt{2}} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

First order correction to energy, will be

$$E'_1 = \langle\phi_1|H'|\phi_1\rangle = (0\ \ 0\ \ 1)[H']\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 + \varepsilon$$

$$E'_2 = \langle\phi_2|H'|\phi_2\rangle = \frac{1}{\sqrt{2}}(1\ \ -1\ \ 0)[H']\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = 0$$

$E'_3 = \langle\phi_3|H'|\phi_3\rangle = 2\varepsilon$

$E_1 = 2 + \varepsilon,\ E_2 = 3 + 0\varepsilon,\ E_3 = 7 + 2\varepsilon$

Pair = 3, $7 + 2\varepsilon$

Correct option is (c)

CAREER ENDEAVOUR

52. $\vec{J} = \vec{\ell} + \vec{S} \Rightarrow \vec{\ell} = \vec{J} - \vec{S}$

Taking self dot product both sides

$$\vec{\ell} \cdot \vec{\ell} = \left(\vec{J} - \vec{S}\right) \cdot \left(\vec{J} - \vec{S}\right)$$

$$= \left|\vec{J}\right|^2 + \left|\vec{S}\right|^2 - 2\vec{S} \cdot \vec{J} \qquad \left(\text{Given} : \ell = J - \frac{1}{2}\right)$$

$$\Rightarrow \left\langle \vec{S} \cdot \vec{J} \right\rangle = \frac{\left|\vec{J}\right| + \left|\vec{S}\right|^2 - \left|\vec{\ell}\right|^2}{2} = \frac{1}{2}\left[J\left(J+1\right) + \frac{1}{2} \cdot \frac{3}{2} - \left(J - \frac{1}{2}\right)\left(J + \frac{1}{2}\right)\right] = \frac{\left(J+1\right)}{2}$$

Correct option is (b)

53. $\vec{\mu} = \frac{e\hbar}{2Mc}(g_\ell \vec{l} + g_s \vec{S}) = gJ\mu_N$

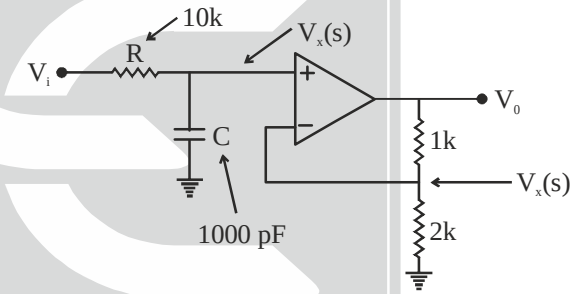
For ${}_8O^{17}$, $N = 9, \left({}^1s_{1/2}\right)^2, \left({}^1p_{3/2}\right)^4, \left({}^1p_{1/2}\right)^2, \left({}^1d_{5/2}\right)^1$

$\therefore \quad J = \frac{5}{2}$; magnetic moment = –1.91 μ_N

$$\Rightarrow \quad g\mu_N\left(\frac{5}{2}\right) = -1.91\,\mu_N \Rightarrow g = \frac{-2 \times 1.91}{5} = -\frac{3.82}{5} = -0.77$$

Correct option is (b)

54. $$V_x\left(s\right) = \frac{V_i\left(s\right) \times \frac{1}{CS}}{R + \frac{1}{CS}}$$



$$\Rightarrow V_x\left(s\right) = \frac{V_i(s)}{1 + RCS} \quad \text{... (i)}$$

$$\Rightarrow V_x\left(s\right) = V_0(s)\left(\frac{2}{2+1}\right) \quad \text{... (ii)}$$

Using (i) and (ii)

$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \frac{1.5}{1 + RCS} \equiv \frac{A_0}{1 + RCS}$$

$$S = j\omega; \quad T\left(j\omega\right) = \frac{A_0}{1 + j2\pi fRC}$$

$A_0 \equiv$ gain of this circuit

- 20 dB/decade frequency means cut off frequency.

$$T\left(s\right) = T\left(jf\right) = \frac{A_0}{1 + jf/f_H}$$

From equation (a) and (b)

$$f_H = \frac{1}{2\pi RC}, \text{ Putting the values of } R \text{ and } C$$

$$f_H = \frac{1}{2\pi \times 10 \times 10^3 \times 1000 \times 10^{-12}}$$

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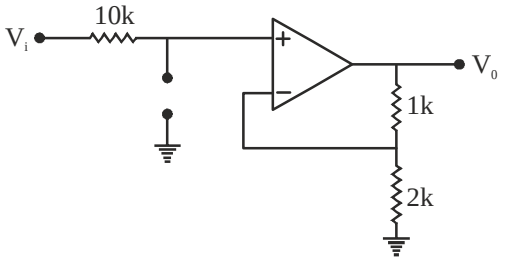
$f_H = 15.91\,KHz$

This is cut off frequency for LPF.

Circuit Analysis:

Case-(1) at low frequency $f \to 0 \Rightarrow \omega \to 0 \Rightarrow X_C \to \infty$

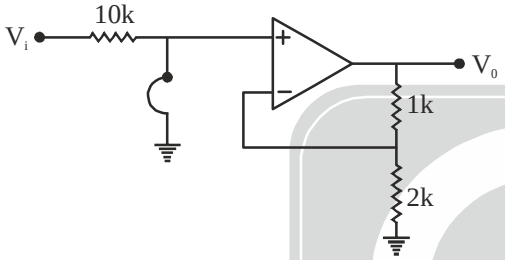
Capacitor is open circuit



At low frequency we are getting output.

Case-(2) at high frequency, $f \to \infty \Rightarrow \omega \to \infty \Rightarrow X_C \to 0$

Capacitor is short circuit.



at high frequency no output

Correct option is (a)

55. $T(jf) = \dfrac{1.5}{1 + j\omega RC}$

$T(jf) = \dfrac{1.5}{1 + j2\pi fRC}$ (given $f = 1.2 \times 10^3\,Hz$)

Closed loop gain

$$\left|T(jf)\right| = \frac{1.5}{\sqrt{1 + (2\pi fRC)^2}}$$

from circuit $R = 10 \times 10^3\,\Omega$

$C = 1000 \times 10^{-12}\,F$

$f = 1.2 \times 10^3\ Hz$

Closed loop gain $\left(T(jf)\right) \simeq 1.5$

Correct option is (b)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

1. Unit vector perpendicular to the surface

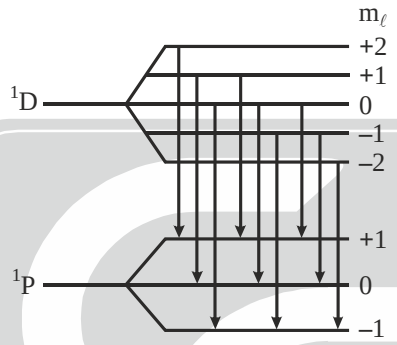
$$\hat{n} = \left.\frac{\vec{\nabla}\phi}{\left|\vec{\nabla}\phi\right|}\right|_{(1,1,1)} = \left.\frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{2\sqrt{x^2+y^2+z^2}}\right|_{(1,1,1)} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

Correct option is (d)

2. Charge is conserved for isolated system so it is relativistically invariant.

Correct answer is (b)

3. ^{1}D and ^{1}P is singlet state, because multiplicity $2s+1 = 1 \Rightarrow s = 0$
Therefore, total angular momentum J = L + S is equal to the totat angular momentum L.
Each energy level splits into 2L+ 1 levels as shown in figure below.



Selection rule : $\Delta m_\ell = 0, \pm 1$

Energy of each level is equal $\Delta E = m_\ell \mu_B B$. Because of the uniform splitting of the levels. There are only three different transition energies

$$E_0 + \frac{e\hbar B}{2m_e},\ E_0,\ E_0 - \frac{e\hbar B}{2m_e}, \text{ corresponding to } \Delta m_\ell = +1,\ 0,\ -1 \text{ respectively}$$

The energy separation between two neighbouring levels in P and D states are equal.
Thus, these will be only 3 normal Zeeman splitting components of $^1\text{P} \rightarrow {}^1\text{D}$ transition.
Hence, correct option is (a).

4. We know that $\tau_{\text{Lab}} = \dfrac{\tau_{\text{proper}}}{\sqrt{1-v^2/c^2}}$

$$\therefore\ \tau_{\text{proper}} = \tau_{\text{Lab}}\sqrt{1-v^2/c^2} = 5\times10^{-8}\sqrt{1-(0.9)^2} = 2.18\times10^{-8}$$

Hence correct answer is (2.18)

5. When light is incident at Brewster angle, then the reflected light and the refracted light make 90° angle with each other.
Correct option is (c)

6. $$\omega = \sqrt{\frac{K}{\mu}},\ \mu = \frac{m_1 \cdot m_2}{m_1 + m_2} = \frac{m \cdot 3m}{m+3m} = \frac{3m}{4}$$

$$\omega = \sqrt{\frac{K}{(3m/4)}} = \sqrt{\frac{4K}{3m}} = \sqrt{\frac{4\times 0.3}{3\times 0.1}} = 2 \text{ rad/s}$$

$$\therefore \quad v = \frac{\omega}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi} = 0.32\ Hz$$

Hence correct answer is (0.32)

7. We have, $\vec{E} = \hat{x}\,10\cos\left(6\pi\times 10^7 t - 0.4\pi z\right)\frac{V}{m}$

Compared with electric field $\vec{E} = \hat{x}E_0\cos\left(\omega t - kz\right)\frac{V}{m}$

We get, $\omega = 6\pi\times 10^7, \quad k = 0.4\pi$

Phase velocity, $V_p = \frac{\omega}{k} = \frac{6\pi}{0.4\pi}\times 10^7 = 1.5\times 10^8\ m/\sec$

Correct answer is (1.5)

8. $$AA^{\dagger} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, A is an unitary matrix

Correct option is (d)

9. $p_{A\leftarrow}\ \circ\!\rightsquigarrow \lambda_1 \qquad p_A = p_{photon} = \frac{h}{\lambda_1}$

$p_{B\leftarrow}\ \circ\!\rightsquigarrow \lambda_2 \qquad p_B = p_{photon} = \frac{h}{\lambda_2}$

$$\therefore \quad \frac{p_A}{p_B} = \frac{\lambda_2}{\lambda_1} = \frac{500}{1500} = \frac{1}{3} = 1:3$$

Hence correct answer is (c)

10. Given : $P \propto V^{-5/3} \quad \Rightarrow \quad P = CV^{-5/3}$

where C is a constant of proportionality.

Bulk modulus is defined as

$$B = -V\left(\frac{\partial P}{\partial V}\right)_T = \frac{5}{3}CV^{-8/3}V = \frac{5}{3}CV^{-5/3}$$

$$\Rightarrow \quad B \propto V^{-5/3}$$

Correct option is (d)

CAREER ENDEAVOUR

11.

	p +	$\bar{p}$	$\longrightarrow$	Λ^0 +	Λ^0
$Q:+1$		-1		0	0 $\quad \Delta Q = 0$
$B:1$		-1		1	1 $\quad \Delta B = -2 \neq 0$
$L:0$		0		0	0 $\quad \Delta L = 0$

Therefore, it is not allowed reaction because Baryon number is not conserved.

	π^- +	p	$\longrightarrow$	π^0 +	n
$Q:-1$		$+1$		0	0 $\quad \Delta Q = 0$
$B:0$		1		0	1 $\quad \Delta B = 0$
$L:0$		0		0	0 $\quad \Delta L = 0$
$I_3:-1$		$\frac{1}{2}$		0	$-\frac{1}{2}$ $\quad \Delta I_3 = 0$

Therefore, it is an allowed reaction.

	n	$\longrightarrow$	p +	e^- +	ν_e
$Q:0$			$+1$	-1	0 $\quad \Delta Q = 0$
$L_e:0$			0	$+1$	$+1$ $\quad \Delta L_e \neq 0$

Therefore, it is not allowed reaction because electronic lepton number is not conserved

	μ^+	$\longrightarrow$	e^- +	γ
$L_e:0$			-1	0 $\quad \Delta L_e \neq 0$
$L_\mu:-1$			0	0 $\quad \Delta L_\mu \neq 0$

It is not allowed reaction because electronic and muonic lepton numbers are not conserved.

Correct option is (b)

12. $(ds)^2 = 2(dx^1)^2 + (dx^2)^2 + \sqrt{3}(dx^1)\,(dx^2)$

General form of metric tensor g_{ij} in 2-D will be $(ds)^2 = g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + 2\,g_{12}(dx^1)\,(dx^2)$

(Here $g_{12} = g_{21}$)

So, metric tensor $g_{ij} = \begin{bmatrix} 2 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$

Correct option is (b)

13. Two spin up fermion cannot occupy the same quantum state and the spin wave function is symmetric. So, the space part wave function must be anti-symmetric, since the total wave function of a fermion is anti-symmetric.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)\right]$$

Correct option is (d)

CAREER ENDEAVOUR

14. The Coulomb gauge condition is

$$\bar{\nabla}\cdot\bar{A}=0$$

$$\Rightarrow \quad \frac{\partial}{\partial x}(\alpha x)+\frac{\partial}{\partial y}(2y)+\frac{\partial}{\partial z}(-3z)=0$$

$$\Rightarrow \quad \alpha+2-3=0$$

$$\Rightarrow \quad \alpha=1$$

Correct answer is (1)

15. The Hamiltonian of a classical harmonic oscillator is given by

$$H=\frac{p_x^2}{2m}+\frac{1}{2}m\omega^2x^2+\frac{p_y^2}{2m}+\frac{1}{2}m\omega^2y^2$$

Since we have four quadratic terms in the Hamiltonian and according to the law of equipartition of energy each would contribute $\frac{1}{2}k_BT$ to the average energy.

So average energy per particle

$$=4\times\frac{1}{2}k_BT=2k_BT$$

Correct answer is (2)

16. Fermions have half integral spin. Proton, electron, neutron are fermions and have 1/2-spin.

α particle $\equiv {}_2^4He$ nucleus $\equiv (2p+2n)\Rightarrow$ integer spin (BOSON)

${}_4^7$Be nucleus $\equiv (4p+3n)\Rightarrow$ half-integer spin (FERMION)

H atom $\equiv {}_1^1$ H atom $\equiv (1p+1e^-)\Rightarrow$ integer spin (BOSON)

Deutron $\equiv {}_1^2$H nucleus $\equiv (1p+1n)\Rightarrow$ integer spin (BOSON)

Correct option is (b)

17. Baryons have charges +2, +1, –1 but donot have –2 charge state.

Therefore, X^{--} does not represent a Baryon.

Correct option is (d)

18. According to Poisson's equation

$$\frac{dA}{dt}=\{A,H\}+\frac{\partial A}{\partial t}$$

If $A=q,\ \frac{\partial A}{\partial t}=0$

$$\therefore \quad \frac{dq}{dt}=\{q,H\} \ \Rightarrow\ \dot{q}=\{q,H\}$$

If $A=p,\ \frac{\partial A}{\partial t}=0$

$$\frac{dp}{dt}=\{p,H\} \quad \Rightarrow \quad \dot{p}=\{p,H\}$$

Hence correct answer is (a)

CAREER ENDEAVOUR

19. Let the plane is $\frac{x}{OA}+\frac{y}{OB}+\frac{z}{OC}=1$, where OA, OB and OC are intercepts on x, y and z-axis respectively.

Now, for (hkl) plane, $OA=\frac{a}{h},\ OB=\frac{a}{k},\ OC=\frac{a}{l}$

Thus above equation implies that $\frac{xh}{a}+\frac{yk}{a}+\frac{zl}{a}=1$

$\Rightarrow \quad xh+yk+zl=a$

For (1, 0, 0), $ah=a \Rightarrow h=1$

For (0, 0, 1), $al=a \Rightarrow l=1$

For $\left(\frac{1}{2},\frac{1}{2},\frac{1}{4}\right)$, $\frac{a}{2}\cdot h+\frac{a}{2}\cdot k+\frac{a}{4}\cdot l=a \Rightarrow \frac{a}{2}\cdot 1+\frac{a}{2}\cdot k+\frac{a}{4}\cdot 1=a \Rightarrow k=\frac{1}{2}$

Thus, $(hkl)=\left(1\frac{1}{2}1\right)=(2\,1\,2)$

Hence, correct option is (a).

20. The specific heat $C=C_{\text{el.}}+C_{\text{lattice}}$

Since, $(C_{\text{el.}})_{\text{superconductor}}=a\exp\left(-\frac{\Delta}{k_BT}\right)$, where a is a constant and Δ is energy gap.

i.e., the specific heat of a superconductor is discontinuous at transition temperature.
Therefore, the electronic specific heat of a metal in the superconducting state varies with temperature in an exponential manner.
Hence, correct option is (a).

21. $\bar{L}\cdot\bar{S}=L_xS_x+L_yS_y+L_zS_z$

So, S_z, L^2, S^2 will commute with $\bar{L}\cdot\bar{S}$ but $\left(\bar{L}+\bar{S}\right)^2=\bar{J}^2$ will not commute with $\bar{L}\cdot\bar{S}$

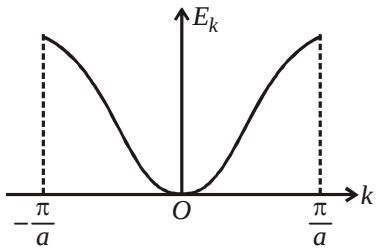
Correct option is (d)

22. The velocity v can be given by

$$v=\frac{\hbar k}{m}=\frac{p}{m} \qquad \text{... (i)}$$

This shows that, for a free electron, v is proportional to k. However, in the band theory, E_k is generally not proportional to k. The variation of E with k based on the band theory is shown in figure (below).

Here, $v=\frac{1}{k}\cdot\frac{dE}{dk}$ ... (ii)



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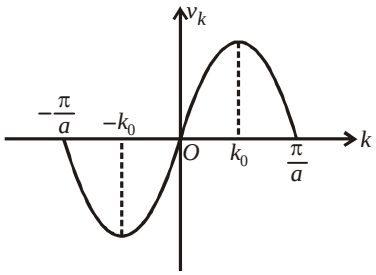
This graph shows that the slope dE/dk of the $E(k)$ curve is not constant but changes with k. Using this curve and employing equation (ii), one can obtain v versus k as shown in figure (below).



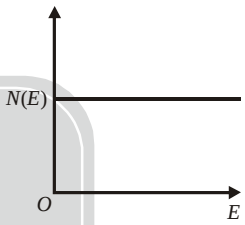
This curve indicates that the velocity of the electron is zero for $k = 0$ and $\pm \frac{\pi}{a}$, where the slope dE/dk is zero.

i.e. at the top and bottom of the energy band (first Brillouin zone). For $k = k_0$, where k_0 corresponds to the inflection point of $E(k)$ curve, the absolute value of the velocity attains a maximum value equal to free electron velocity. Beyond this point, the velocity decreases with increase in energy.

Hence, correct option is (b).

23. Since the density of states, $N(E)$ is independent of energy (E) for a free electron gas in two dimensions.

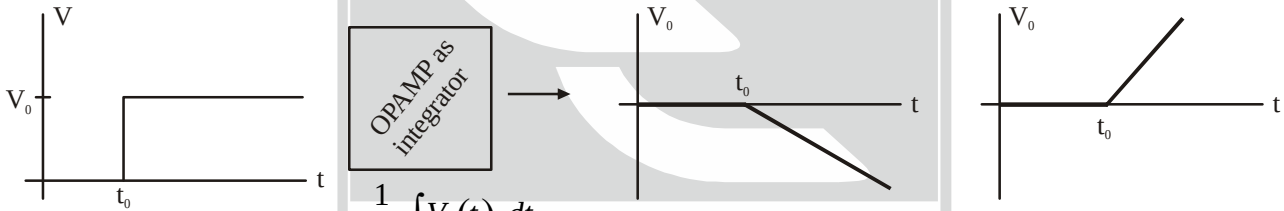
Therefore, $N(E)$ versus E graph can be represented as



Where $N(E) = \frac{mA}{\pi\hbar^2} = \text{constant}$

Hence, correct option is (c).

24.



$-\frac{1}{RC}\int V_i(t)\cdot dt$ Considering- Non integrator

Opamp as integrator

Most appropriate option is (a)

25. Given Modulus = 75

$M \le 2^n$ n is number of flip-flops

$75 \le 2^n$ for $n = 6$ Hence (n = 6 not possible)

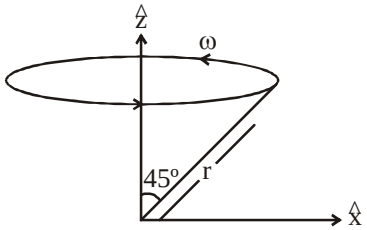
n = 7

Correct answer is (7)

Q.26 – Q.55 : Carry TWO marks each.

26. In spherical polar coordinate the Lagrangian of system is

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right) - mgr\cos\theta$$



Here $\theta = 45^\circ$, $\dot{\phi} = \omega$

momentum conjugate to r is

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

Hence correct answer is (a)

27. $$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}}e^{-(r/a_0)}$$

$$\therefore \quad \left\langle \hat{Q} \right\rangle = \left\langle \psi \left| z^2 - r^2 \right| \psi \right\rangle = \int_0^\infty \int_0^\pi \left(r^2\cos^2\theta - r^2\right)\frac{1}{\pi a_0^3}e^{-\frac{2r}{a_0}}2\pi r^2 dr d\theta \sin\theta$$

$$= -\frac{2\pi}{\pi a_0^3}\int_0^\infty r^4 e^{-\frac{2r}{a_0}}dr \int_0^\pi \sin^3\theta d\theta$$

$$= -\frac{2}{a_0^3}\times\frac{4!}{(2/a_0)^5}\times\frac{1}{4}\int_0^\pi (3\sin\theta - \sin 3\theta)d\theta$$

$$= -\frac{2}{a_0^3}\times\frac{4\times3\times2}{32}\times a_0^5\times\frac{1}{4}\times\frac{16}{3} = -2a_0^2$$

Correct option is (d)

28. For Nickel, number density, $n = 8\times10^{23}$ atoms/cm^3

And its electronic configuration $= 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^8\,4s^2$

$$\mu_B = 9.21\times10^{-21}\ Am^{-2}$$

$$n = 8\times10^{29}/m^3$$

Magnetic moment of Ni = 0.6 μ_B (can be calculated using concept of atomic physics)

Saturation magnetization = N μ = $8\times10^{29}\times0.6\times9.2\times10^{-21}$

$= 44.16\times10^8 \simeq 4.4\times10^9$ Am^{-1}

Correct answer is (4.4)

29. $$V(r) = -\frac{a}{r} + \frac{a\,r_0^2}{3r^3}$$

Time period of oscillation is given as

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ where } k = \frac{d^2V}{dr^2} \text{ at stable position}$$

At equilibrium position $(r = r')$

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$$\left.\frac{dV}{dr}\right|_{r=r'} = 0 \quad \Rightarrow \quad \frac{a}{r'^2} - \frac{ar_0^2}{r'^4} = 0$$

putting $r' = r_0$

$$\left.\frac{d^2V}{dr^2}\right|_{r=r'} = -\frac{2a}{r'^3} + \frac{4ar_0^2}{r'^5} = -\frac{2a}{r_0^3} + \frac{4ar_0^2}{r_0^5} \Rightarrow \frac{2a}{r_0^3} = k$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2a/r_0^3}} = 2\pi\sqrt{\frac{mr_0^3}{2a}}$$

Hence correct answer is (a)

30. At $T = 300$ K, $k_BT = 25$ meV

Let initial donor concentration $= N_D^+$ and final donor concentration $= 100\ N_D^+$

$\therefore$ Initial Fermi energy $(E_F)_{\text{initial}} = E_C - k_BT\ \ell n\left(\frac{N_C}{N_D^+}\right)$

and final Fermi energy $(E_F)_{\text{final}} = E_C - k_BT\ \ell n\left(\frac{N_C}{100\ N_D^+}\right)$

$$\therefore \quad (E_F)_{\text{final}} - (E_F)_{\text{initial}} = E_C - k_BT\ \ell n\left(\frac{N_C}{100\ N_D^+}\right) - E_C + k_BT\ \ell n\left(\frac{N_C}{N_D^+}\right)$$

$$= k_BT\ \ell n\left(\frac{N_C}{N_D^+} \times \frac{100\ N_D^+}{N_C}\right) = k_BT\ \ell n\,100 = 2\times(k_BT)\times \ell n\,10$$

$$= 2 \times 2.303 \times 25 \text{ meV} \simeq 115 \text{ meV}$$

Hence, correct answer is 115 meV.

31. The energy of the two dimensional harmonic oscillator,

$$E = \hbar\omega\left(n_x + n_y + 1\right)$$

Given : $E = 4\hbar\omega$

$\therefore \quad n_x + n_y + 1 = 4$

$$n_x + n_y = 3$$

n_x	n_y	
1	2	$=3$
2	1	$=3$
0	3	$=3$
3	0	$=3$

} 4-fold degeneracy, therefore $g = 4$

Correct answer is (4)

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32. $\psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$

$L^2\left|\ell, m\right\rangle = \hbar^2 \ell\left(\ell+1\right)\left|\ell, m\right\rangle$

$\left\langle L^2\right\rangle = \frac{8}{21}\times\left(0.\hbar^2\right) + \frac{9}{21}\times\left(2\hbar^2\right) + \frac{4}{21}\times\left(6\hbar^2\right) = 2\hbar^2$

Correct answer is (2)

33. $V(r) = -\frac{k}{r}$

Effective potential $V_{eff} = V\left(r\right) + \frac{\ell^2}{2mr^2} \Rightarrow V_{eff} = -\frac{k}{r} + \frac{\ell^2}{2mr^2}$

for circular orbit $\left.\frac{dV_{eff}}{dr}\right|_{r=r_0} = 0 \Rightarrow \frac{k}{r_0^2} - \frac{\ell^2}{mr_0^3} = 0, \quad \therefore \quad \ell = \sqrt{mkr_0}$

Hence correct answer is (d)

34. Rotational energy of the rigid diatomic molecules is given by

$E = \frac{h^2}{8\pi^2 I} J\left(J+1\right)$

In terms of wave number unit,

$v = \frac{E}{hc} = \frac{h}{8\pi^2 Ic} J\left(J+1\right) = B\left(J\left(J+1\right)\right) cm^{-1}$

where, J is rotational quantum number.

I_A and I_B is moment of inertia of rigid rotator A and B respectively.

Also, $E_A = \frac{h^2}{8\pi^2 I_A} J_A\left(J_A+1\right)$ and $E_B = \frac{h^2}{8\pi^2 I_B}\left[J_B\left(J_B+1\right)\right]$

$\because \quad E_A = E_B$ and $I_A = 6I_B$

$\Rightarrow \quad \frac{J_A\left(J_A+1\right)}{6I_B} = \frac{J_B\left(J_B+1\right)}{I_B} \Rightarrow J_A\left(J_A+1\right) = 6J_B\left(J_B+1\right)$

This equation is satisfied for

$J_A = 3$ and $J_B = 1$

Correct option is (b)

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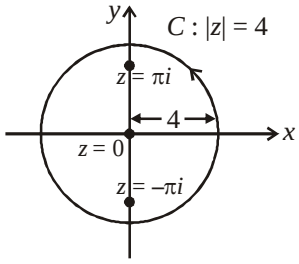
35. $f(z) = \frac{z^2}{e^z + 1}$



Condition of singularity $e^z + 1 = 0$

$\Rightarrow \; e^z = -1 = e^{\pm(2n+1)\pi i}$

$\Rightarrow \; z = \pm i\left(2n+1\right)\pi = \pm i\pi, \pm i3\pi, \pm i5\pi.....$

Only $z = \pm\pi i$ lies within the circle and it is a simple pole.

$I = 2\pi i$ [Residue of $f(z)$ at $z = \pi i$ + Residue of $f(z)$ at $z = -\pi i$]

$= 2\pi i\left[\frac{z^2}{e^z}\right]_{z=\pi i} + 2\pi i\left[\frac{z^2}{e^z}\right]_{z=-\pi i} = 4\pi^3 i$

Correct option is (c)

36. Given : Electric field in region 1 is

$$\vec{E}_1 = 7\,\hat{e}_r - 3\,\hat{e}_\theta$$

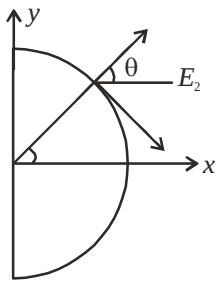
According to boundary condition,

$$E_{1\tau} = E_{2\tau}$$

$$\Rightarrow \quad -3 = E_{2\tau} = -E_2 \sin\frac{\pi}{4}$$

$$\Rightarrow \quad -3 = -\frac{E_2}{\sqrt{2}}$$

$$\Rightarrow \quad E_2 = 3\sqrt{2}$$



And $D_{1n} = D_{2n}$ (As boundary is charge free)

$$\Rightarrow \quad \varepsilon_1 7 = \varepsilon_2 E_{2n} = \varepsilon_2 E_2 \cos\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{\varepsilon_2}{\varepsilon_1} = \frac{7\sqrt{2}}{E_2} = \frac{7}{3} = 2.33$$

37. $\dfrac{d^2y}{dt^2} - y = 0$

Assume, the trial solution to be $y = c \cdot e^{mt}$

$\Rightarrow m^2 - 1 = 0 \Rightarrow m = \pm 1$

So, $y = Ae^t + Be^{-t}$

Putting $y(t=0) = 1$, we get $A + B = 1$

Putting $y(t=\infty) = 0$, we get $A = 0, B = 1$

Therefore, $y(t) = e^{-t} = \cosh t - \sinh t$

Correct option is (d)

38. For canonical transformation

Poisson Bracket $\{Q, P\}_{q,p} = 1$

$$\Rightarrow \quad \frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q} = 1$$

$$\Rightarrow \quad 1 \cdot 2 - 4\text{a} \cdot 1 = 1$$

$$\Rightarrow \quad 2 - 4\text{a} = 1$$

$$\Rightarrow \quad 4\text{a} = 1 \quad \Rightarrow \quad \text{a} = \frac{1}{4} = 0.25$$

Hence correct answer is (0.25)

39. Kinetic energy = 1 MeV

$$\frac{1}{2}mv^2 = 1\times10^6 \times 1.6\times10^{-19}\,J$$

$$\Rightarrow \quad v = \sqrt{\frac{3.2\times10^{-13}}{m}} = \sqrt{\frac{3.2\times10^{-13}}{1.67\times10^{-27}}} = \sqrt{1.91677\times10^{14}} = 1.384474\times10^7 \text{ m/sec}$$

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Now, for circular orbit, $qvB = \frac{mv^2}{r}$

$$B = \frac{mv}{qr} = \frac{1.67\times10^{-27}\times1.384474\times10^{7}}{1.6\times10^{-19}\times10^{-1}}$$

$$= \frac{1.67\times1.384474}{1.6} = 1.4450 \text{ Tesla}$$

Correct answer is (1.4450)

40. Two bosons particles can be distributed as

0	ε
··	
	··
·	·

So the partition function is

$$Z = e^{-\beta\cdot 0} + e^{-2\beta\varepsilon} + e^{-(0+\varepsilon)\beta} = 1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$$

The mean energy is

$$\langle E\rangle = -\frac{\partial}{\partial\beta}\ln Z = -\frac{\partial}{\partial\beta}\ln\left(1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}\right) = -\frac{\left(-\varepsilon e^{-\beta\varepsilon}-2\varepsilon e^{-2\beta\varepsilon}\right)}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}} = \frac{\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$$

None of the options are correct.

41. $$\frac{I_{\max}}{I_{\min}} = \left(\frac{a+b}{a-b}\right)^2 = \frac{9I_0}{I_0} = \frac{9}{1} \quad \Rightarrow \frac{a+b}{a-b} = \pm\frac{3}{1}$$

Taking positive value, we get

$$a+b = 3a-3b$$

$$\Rightarrow \qquad 2a = 4b \qquad \Rightarrow \frac{a}{b} = \frac{2}{1}$$

$$\therefore \qquad \frac{I_1}{I_2} = \left(\frac{a}{b}\right)^2 = \frac{4}{1} = \frac{4I_0}{I_0}$$

Correct option is (b)

42. $$|\psi_1\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

Let $|\psi_2\rangle = C_1\begin{pmatrix}1\\0\end{pmatrix} + C_2\begin{pmatrix}0\\1\end{pmatrix}$

According to Normalization condition,

$$|C_1|^2 + |C_2|^2 = 1$$

Given ψ_1 and ψ_2 are orthogonal i.e. $\langle\psi_2 \mid \psi_1\rangle = 0$

$$\Rightarrow \qquad \frac{C_1}{\sqrt{3}} + C_2\sqrt{\frac{2}{3}} = 0 \Rightarrow C_1 = -\sqrt{2}\,C_2$$

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$$\therefore \qquad |\psi_2\rangle = -\sqrt{\frac{2}{3}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

So, the probability of finding the spin up in ψ_2 is $= C_1^2 = \frac{2}{3} = 0.66$

43. Wavelength of neutrons can be given by $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1.675\times10^{-27}\times10^{3}}\text{ m} = 3.96\times10^{-10}\text{ m}$

$\Rightarrow \quad \lambda = 3.96$ Å

Here, $n = 1,\ \theta = 30°,\ \lambda = 3.96$ Å, $d = ?$

From Bragg's law : $2d\sin\theta = n\lambda$

$\Rightarrow \quad 2\times d\times\sin 30° = 1\times 3.96\times10^{-10}$

$\Rightarrow \quad 2\times d\times\frac{1}{2} = 3.96\times10^{-10} \Rightarrow \quad d = 3.96\times10^{-10}m \Rightarrow \quad d = 3.96$ Å.

Hence, correct answer is (3.96).

44. For positive p arrow should point towards positive q and for negative p arrow should point towards negative q therefore correct answer is (d).

Hence correct answer is (d)

45. We know that, $I = \langle u\rangle c = \frac{1}{2}\varepsilon_0 E_0^2 c$

$$E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2\times150\times10^{-3}}{3\times10^{8}\times8.85\times10^{-12}}} = 10.62\text{ V/m}$$

Correct answer is (10.62)

46. **[Note:** According to the option given in the question, the ratio of population of the initial to the final state has to be calculated].

The ratio of the molecule population of the initial states to final states is given by

$$\frac{N(J_i)}{N(J_f)} = \frac{(2J_i+1)}{(2J_f+1)}\times\frac{e^{\frac{-E_i}{k_BT}}}{e^{\frac{-E_f}{k_BT}}} = \frac{(2J_i+1)}{(2J_f+1)}\times e^{-\frac{(E_i-E_f)}{k_BT}} = \frac{(2J_i+1)}{(2J_f+1)}\times e^{-\frac{\Delta E}{k_BT}}$$

Here, difference in the energy states

$$\Delta E = E(J=2) - E(J=3) = \frac{hc}{\lambda}$$

where, λ is the wavelength for the transition from state $J = 2$ to $J = 3$.

Given : $\lambda = 3122\text{Å}$

$$\Rightarrow \qquad \frac{N(J_i)}{N(J_f)} = \left(\frac{2J_i+1}{2J_f+1}\right)e^{-\frac{hc}{\lambda k_BT}}$$

$$\frac{N(J_i=2)}{N(J_f=3)} = \frac{5}{7}\times e^{-\frac{\left[6.626\times10^{34}\times3\times10^{8}\right]}{3122\times10^{-10}\times1.38\times10^{-23}\times5000}} = \frac{5}{7}e^{-9.227} = 7.02\times10^{-5}$$

Correct option is (c)

47. The distribution of fermions is

$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$
•	•	•	
•	•		•
•		•	•
	•	•	•

So total number of microstates are 4, i.e., $\Omega = 4$.

The entropy is $S = k_B \ln \Omega = 2\, k_B \ln 2$

Correct option is (b)

48. For one dimensional potential box, the wavefunction is given by

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

Ground state $(n=1): \psi_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$

Perturbation, $W = \beta x$

First order correction to the ground state energy is

$$E_1^{(1)} = \langle \psi_1 | W | \psi_1 \rangle = \beta \left(\frac{2}{a}\right) \int_0^a x \sin^2\left(\frac{\pi x}{a}\right) dx$$

$$= \frac{\beta}{a} \int_0^a x \left(1 - \cos\frac{2\pi x}{a}\right) dx = \frac{\beta}{a} \int_0^a \left(x - x\cos\frac{2\pi x}{a}\right) dx$$

$$= \frac{\beta}{a} \left[\frac{x^2}{2} - \frac{xa}{2\pi} \sin\frac{2\pi x}{a} - \left(\frac{a}{2\pi}\right)^2 \cos\left(\frac{2\pi x}{a}\right)\right]_0^a$$

$$= \frac{\beta}{a} \left[\frac{a^2}{2} - 0 - \left(\frac{a}{2\pi}\right)^2 [1-1]\right] = \frac{a\beta}{2}$$

Correct option is (c)
Correct option is (a)

49. According to the law of conservation of energy,
(K.E. + rest mass energy) of reactants = (K.E. + rest mass energy) of products

$$\left(k_\mu + m_\mu^0 c^2\right) \times 2 = \left(0 + m_\pi^0 c^2\right) \times 2 \qquad \ldots (1)$$

From equation (1),

$$k_\mu = \left(m_\pi^0 - m_\mu^0\right) c^2 = 140 - 105 = 35\ MeV$$

Correct answer is (35)

50. $|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$

$$\langle E \rangle = \sum P_i E_i = P_2 E_2 + P_3 E_3 = \left(\frac{1}{4} \times \frac{5}{2} + \frac{3}{4} \times \frac{7}{2}\right) \hbar\omega = \frac{26}{8} \hbar\omega = 3.25\, \hbar\omega$$

Correct answer is (3.25)

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51. $X \xrightarrow{\beta\text{-decay}} Y$

So, if angular momentum (I) and Parity (P) of 'X' is denoted by $\frac{7^-}{2}$ then,

For a first forbidden β decay: $\Delta I = 0, 1, 2,\ \Delta\pi = 1$ (Parity change)

Possible values of I for $Y = \frac{7}{2} - 0 = \frac{7}{2}$ or $\frac{7}{2} - 1 = \frac{5}{2}$ or $\frac{7}{2} - 2 = \frac{3}{2}$

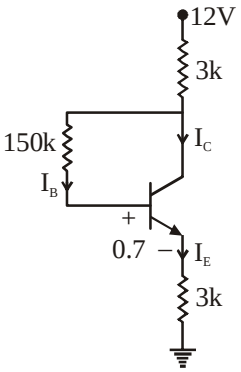
Parity changes, so possible I^P values for $Y = \frac{7^+}{2}, \frac{5^+}{2}, \frac{3^+}{2}$

Correct option is (c)

52. $\beta_{dc} = 100$

$I_C = ?$

$V_{BE} = 0.7\,V$



Apply DC analysis, it means capacitor is open circuit

KVL at input

$12 = (I_C + I_B)3 + 150 \times I_B + 0.7 + 3\,I_E$

$I_C = 100\,I_B$

$I_E = 101\,I_B$

$12 = 101 \times I_B \times 3 + 150 \times I_B + 0.7 + 3 \times 101 \times I_B$

$I_B = 0.015\,mA$

$I_C = \beta \cdot I_B = 100 \times 0.015\,mA$

$I_C = 1.5\,mA$

Correct answer is (1.5)

53. **Given :** Resolution = 1 mV

To measure 1V

Resolution $= \frac{V_{PP}}{2^n}$

Number of distinct values to measure 1V using least count of 1 mV

$2^n = \frac{1V}{1 \times 10^{-3}\,\text{Volt}} = 1000$

$\Rightarrow$ number of bits required to code 1000 different amplitudes

$2^n = 1000 \Rightarrow$ **n = 10**

Correct answer is (10)

54. $\langle 11 | L_+ L_- + L_- L_+ | 11 \rangle = 2\langle 11 | L_x^2 + L_y^2 | 11 \rangle = 2\langle 11 | L^2 - L_z^2 | 11 \rangle$

$= 2\langle L^2 \rangle - 2\langle L_z^2 \rangle = 2(2\hbar^2) - 2(\hbar^2) = 2\hbar^2$

Correct answer is (2)

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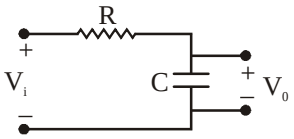
55. LPF



$$\frac{V_0(s)}{V_i(s)} = \frac{1}{1+RCS} = T(s)$$

$$T(j\omega) = \frac{1}{1+ j\omega RC}$$

$$\left|T\left(j\frac{1}{RC}\right)\right| = \frac{1}{\sqrt{2}} \simeq 0.71, \text{ magnitude at } \omega = \frac{1}{RC}$$

$$\angle T\left(j\frac{1}{RC}\right) = -\tan^{-1}\omega RC \text{ put } \omega = \frac{1}{RC}$$

$$\angle T\left(j\frac{1}{RC}\right) = -\tan^{-1}1 = -45^{\circ}$$

Phase at $\omega = \frac{1}{RC}$

Correct option is (b)

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OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

1. According to Virial theorem,

$$\langle T\rangle = \frac{n}{2}\langle V\rangle \text{ when, } V(r) \propto r^n$$

The potential energy of the satellite $V(r) \propto r^{-1}$

$$\therefore \quad \langle T\rangle = -\frac{1}{2}\langle V\rangle \Rightarrow T = -\frac{1}{2}V$$

$$V = -2T$$

Total average energy, $\langle E\rangle = \langle T\rangle + \langle V\rangle$

$$\Rightarrow \quad E = T - 2T = -T$$

Correct option is (a)

2. Lattice parameters *a*, *b*, *c* have relation $a = 2b = 3c \Rightarrow \quad a = a,\ b = \frac{a}{2},\ c = \frac{a}{3}$

and the miller indices of the plane (*hkl*) = (110) i.e., $h = 1,\ k = 1,\ l = 0$
Thereore, interplanar separation

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}} = \frac{1}{\sqrt{\frac{1^2}{a^2} + \frac{1^2}{\left(\frac{a}{2}\right)^2} + \frac{0^2}{\left(\frac{a}{3}\right)^2}}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{4}{a^2} + \frac{0}{a^2}}} = \frac{1}{\sqrt{\frac{5}{a^2}}} = \frac{a}{\sqrt{5}} = 0.447a$$

Hence, correct answer is 0.477 *a*.

3. $f(z) = u(x, y) + iv(x, y)$

The real part $u(x, y)$ and imaginary part $v(x, y)$ of a complex analytic function $f(z)$ are harmonic functions i.e. they will satisfy Laplace's equation.

Since, $f(z)$ is analytic in the domain *D*, then $f(z)$ can be Taylor expanded and the value of $\int_{z_1}^{z_2} f(z)\,dz$ will be independent of the choice of the contour.

Correct option is (c)

4. We know that, $\left[L_i, p_j\right] = i\hbar\varepsilon_{ijk}p_k$

$$\therefore \quad \left[L_x, p_y\right] = i\hbar p_z$$

Correct option is (d)

5. The $\omega - k$ relation for monatomic lattice is given by

$$\omega(k) = \sqrt{\frac{2C}{M}\left[1 - \cos(ka)\right]} \qquad \text{... (i)}$$

$$\Rightarrow \quad \omega(k) = \sqrt{\frac{2C}{M}\left[1 - \left\{1 - 2\sin^2\left(\frac{ka}{2}\right)\right\}\right]} \qquad \text{... (ii)}$$

$$\Rightarrow \quad \omega(k) = \sqrt{\frac{2C}{M}\left[1 - 1 + 2\sin^2\left(\frac{ka}{2}\right)\right]}$$

$$\Rightarrow \quad \omega(k) = \sqrt{\frac{4C}{M}} \cdot \left|\sin\frac{ka}{2}\right| \qquad \text{... (iii)}$$

Here, we have taken modulus of $\sin\frac{ka}{2}$ because $\omega(k)$ cannot be negative.

For long wavelength modes ($\lambda \gg a$), i.e., $k = \left(\frac{2\pi}{\lambda}\right) \to 0 \Rightarrow \sin\left(\frac{ka}{2}\right) \to \frac{ka}{2}$

$\therefore$ equation (iii) $\Rightarrow \quad \omega(k) = \sqrt{\frac{4C}{M}}\left[\frac{ka}{2}\right]$ ... (iv)

$\therefore$ Phase velocity, $(v_p) = \frac{\omega}{k} = \sqrt{\frac{4C}{M}} \cdot \left(\frac{a}{2}\right)$ ... (v)

and group velocity, $(v_g) = \frac{d\omega}{dk} = \sqrt{\frac{4C}{M}} \cdot \left(\frac{a}{2}\right)$ ... (vi)

$\therefore$ Ratio of phase velocity to the group velocity $\Rightarrow \quad \frac{v_p}{v_g} = \frac{\sqrt{\frac{4C}{M}} \cdot \left(\frac{a}{2}\right)}{\sqrt{\frac{4C}{M}} \cdot \left(\frac{a}{2}\right)} = 1$

Hence, correct answer is (1).

6. Since photon has zero chemical potential, the number of the photons cannot be conserved. Thus, photons can be created and annihilated.

Correct option is (a)

7. $$\vec{\nabla} \times \vec{F}_1 = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ k.\exp\left(-r^2/R^2\right) & 0 & 0 \end{vmatrix} = 0$$

$$\vec{\nabla} \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & kx^3 & -kz^3 \end{vmatrix} = 3kx^2\hat{k}$$

$$\vec{\nabla} \times \vec{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ kx^3 & ky^3 & 0 \end{vmatrix} = 0 \; ; \quad \vec{\nabla} \times \vec{F}_4 = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & \frac{k}{r} \end{vmatrix} = \frac{1}{r^2\sin\theta} r\hat{\theta}\left[-\frac{k}{r^2}\right] = -\frac{k}{r^3\sin\theta}\hat{\theta}$$

Correct option is (d)

CAREER ENDEAVOUR

8. $$\int_0^3 t^2\delta(3t-6)\,dt = \frac{1}{3}\int_0^3 t^2\delta(t-2)\,dt \left[\text{Using } \delta\left[c(t-t_0)\right] = \frac{1}{|c|}\delta(t-t_0)\right]$$

$$= \frac{1}{3}\times 4 = 1.67$$

9. Total kinetic energy (K.E.) of *N*-neutrons and *Z*-protons is given by

$$K.E. = \frac{4\pi}{5m}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{V^{2/3}}(Z^{5/3}+N^{5/3}) = \frac{4\pi}{5m}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{(CA)^{2/3}}(Z^{5/3}+N^{5/3})$$

[Volume of nucleus $V \propto$ Atomic weight of nucleus *A*. Hence, $V = CA$, where, *C* is a constant of proportionality]

$\therefore$ K.E. $\propto$ $A^{-2/3}$

$$\therefore \quad n = -\frac{2}{3} = -0.67$$

Correct answer is (–0.67)

10. Since the interparticle distance between the bosons is comparable to their de-Broglie wavelengths, their wavefuntions overlap.

Correct option is (c)

11. If α is the linear absorption coefficient of the metal sheet then the transmitted intensity is given by

$$I = I_0\exp(-\alpha L)$$

$$\therefore \quad 0.025I_0 = I_0\exp(-2\alpha)$$

$$\Rightarrow \quad \exp(-2\alpha) = 0.025$$

$$\Rightarrow \quad 2\alpha = 3.688 \Rightarrow \alpha = 1.8444\ mm^{-1} = 1844.4\ m^{-1}$$

Correct answer is (1844.4)

12. The hall coefficient of intrinsic semiconductor is given by

$$R_H = \frac{1}{|q|}\frac{n_h\mu_h^2 - n_e\mu_e^2}{(n_h\mu_h + n_e\mu_e)^2}$$

The hall voltage is given by $V_H = -\frac{1}{R_H}\frac{IB}{t}$

Given hall voltage is negative,

$$\therefore \quad R_H > 0 \Rightarrow n_h\mu_h^2 - n_e\mu_e^2 > 0$$

$$\Rightarrow \quad \mu_h > \mu_e \ (\text{in ideal intrinsic semiconductor, } n_e = n_h) \Rightarrow \quad \frac{1}{m_h^*} > \frac{1}{m_e^*} \Rightarrow m_e^* > m_h^*$$

Correct option is (d)

13. The required number of dimensions

$= (2s+1)^n$, where *s* is the spin of the particle and *n* is the number of particle.

$$= \left(\frac{1}{2}\times 2 + 1\right)^3 = 2^3 = 8.$$

Correct answer is (8)

CAREER ENDEAVOUR

14. $\mu^+ \to e^+ + \gamma$

B: $0 \to 0+0$ $\Delta B = 0$, conserved

L_μ: $-1 \to 0+0$ $\Delta L_\mu \neq 0$, not conserved

L_e: $0 \to -1+0$ $\Delta L_e \neq 0$, not conserved

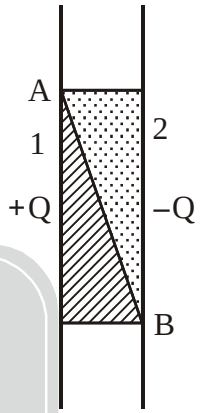
Electronic and muonic lepton number should be conserved separately. Here both are not conserved.

Correct option is (d)

15. Suppose the line AB is the interface between two dielectrics.

We can use the boundary condition for $\vec{E}$ and $\vec{D}$

Across interface, parallel component of $\vec{E}$ is continuous while perpendicular component suffers a discontinuity of $\frac{\sigma}{\varepsilon}$ where ε is the permitivity of medium.



$\Rightarrow \quad E_1^{\parallel} = E_2^{\parallel}$

Or $\quad \varepsilon_1 \vec{E}_1 = \varepsilon_2 \vec{E}_2 \Rightarrow \boxed{D_1^{\parallel} = D_2^{\parallel}}$

$\Rightarrow \quad \varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} = \sigma$

$\Rightarrow \quad D_1^{\perp} - D_2^{\perp} = \sigma \qquad (\text{if there is no charge } +Q-Q=0)$

$\Rightarrow \quad \boxed{D_1^{\perp} - D_2^{\perp} = 0} \qquad \Rightarrow \sigma = 0$

$\vec{D}$ is continuous in both cases parallel and perpendicular. While $\vec{E}$ is continuous only for parallel, not for perpendicular.

Correct option is (c)

16. Magnetic flux through the loop i.e. $\iint\limits_S \vec{B}\cdot d\vec{S} = \phi \Rightarrow \iint\limits_S \left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S} = \phi$, $\int\limits_{PQRSP} \vec{A}\cdot\overrightarrow{d\ell} = \varphi$

$\Rightarrow \quad \int\limits_{PQR} \vec{A}\cdot\overrightarrow{d\ell} + \int\limits_{RSP} \vec{A}\cdot\overrightarrow{d\ell} = \varphi$

$\Rightarrow \quad \varphi_1 + \int\limits_{RSP} \vec{A}\cdot d\vec{\ell} = \varphi \Rightarrow -\int\limits_{PSR} \vec{A}\cdot\overrightarrow{d\ell} = (\varphi - \varphi_1) \Rightarrow \int\limits_{PSR} \vec{A}\cdot\overrightarrow{d\ell} = (\varphi_1 - \varphi)$

Correct option is (b)

17. The number of image charges

$n = \left(\frac{360}{\theta} - 1\right)$ where, θ is the angle between two conductor plates

$= \left(\frac{360}{30} - 1\right) = 11$

Correct answer is (11)

CAREER ENDEAVOUR

18. $f(z)=\dfrac{1}{z\left(z+\dfrac{1}{2}\right)\cos(z\pi)}$, putting $z+\dfrac{1}{2}=t$

$$=\frac{1}{\left(t-\frac{1}{2}\right)t\cos\left[\pi\left(t-\frac{1}{2}\right)\right]}=\frac{1}{\left(t-\frac{1}{2}\right)t\sin\pi t}$$

$$=\frac{1}{-\frac{1}{2}(1-2t)}\cdot\frac{1}{t}\cdot\frac{1}{\left[\pi t-\frac{\pi^3t^3}{3!}+.....\right]}=-2\cdot\frac{1}{t}\cdot(1-2t)^{-1}\cdot\frac{1}{\pi t\left[1-\frac{\pi^2t^3}{3!}+.....\right]}$$

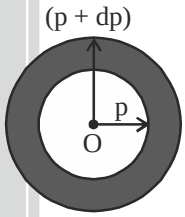
$$=-2\cdot\frac{1}{t}\cdot\frac{1}{\pi t}\cdot\left[1+2t+4t^2+.....\right]\left[1+\left(\frac{\pi^2t^2}{3!}-.....\right)+\left(\frac{\pi^2t^2}{3!}-.....\right)^2+.....\right]$$

Highest negative power term in the above Laurrent series expansion is $\dfrac{1}{t^2}$

Therefore, $f(z)$ has a pole of order 2 at $z=-\dfrac{1}{2}$

Correct option is (b)

19. The density of states in 2-D is given by



$$g(p)\,dp=\frac{2\pi Ap}{h^2}dp$$

where, $p^2=2mE \Rightarrow pdp=mdE$

$$\therefore\quad g(E)\,dE=\frac{2\pi A}{h^2}mdE$$

$$\Rightarrow\quad g(E)=\frac{2\pi A}{h^2}m$$

Since electron spin degeneracy is 2 so we should multiply density of states by a factor of 2.

Therefore, $g(E)=\dfrac{4\pi A}{h^2}m$

On comparing with $g(E)=CE^n$, we have $n=0$

Correct answer is (0)

20. According to Lorentz transform

$$t'_A=\frac{t_A-vx_A/c^2}{\sqrt{1-v^2/c^2}}=0-0=0$$

$$t'_B=\frac{t_B-vy_B/c^2}{\sqrt{1-v^2/c^2}}=\frac{0-\frac{0.6\times2}{c}}{\sqrt{0.64}}=-\frac{1.2}{0.8c}=-\frac{3}{2c}\Rightarrow ct'_B=-\frac{3}{2}$$

Correct option is (a)

CAREER ENDEAVOUR

21. Given; Source voltage

$$V_s = 1V$$

To find Drain voltage V_D

Applying KVL at input

$$V_{GS} + 1 = 0$$

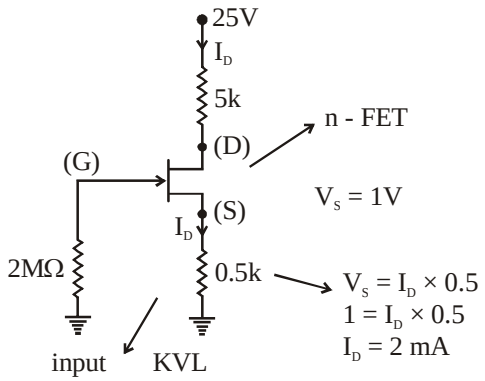
$$V_{GS} = -1 \text{ Volt}$$

KVL at output

$$25 = 5 \times I_D + V_D$$

$$V_D = 25 - 5 \times I_D \quad \text{put} \quad I_D = 2\ mA$$

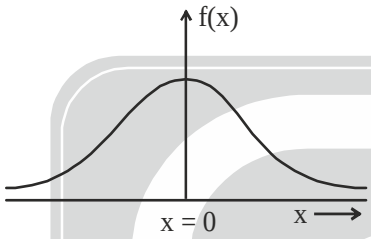
$$V_D = \mathbf{10} \text{ Volt}$$



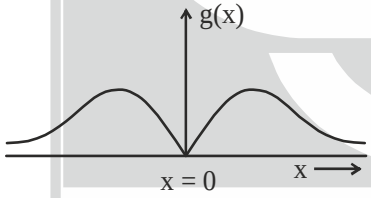
Correct answer is (10)

22. This question can be solved easily by plotting $f(x)$ and $g(x)$

$$f(x) = e^{-x^2}$$



$$g(x) = \begin{cases} xe^{-x^2}, & x \geq 0 \\ -xe^{-x^2}, & x < 0 \end{cases}$$



From graph, it is clear, $g(x)$ is not differentiable at $x = 0$ (due to presence of sharp dip at $x = 0$).

Correct option is (b)

23. Let $N_+ =$ Number of particles with spin $+\frac{1}{2}$ (or up)

and $N_- =$ Number of particles with spin $-\frac{1}{2}$ (or down).

So, we have

$$N = N_+ + N_- \qquad \text{... (i)}$$

Also, the total energy is

$$E = -\mu B(N_+ - N_-) \Rightarrow N_+ - N_- = -\frac{E}{\mu B} \qquad \text{... (ii)}$$

Solving (i) and (ii), we have

$$N_+ = \frac{1}{2}\left(N - \frac{E}{\mu B}\right) \text{ and } N_- = \frac{1}{2}\left(N + \frac{E}{\mu B}\right)$$

Total number of microstates (Ω) when N particles are distributed in two states N_+ and N_- is

$$\Omega = \frac{N!}{N_+!\,N_-!} = \frac{N!}{\frac{1}{2}\left(N-\frac{E}{\mu B}\right)!\,\frac{1}{2}\left(N+\frac{E}{\mu B}\right)!}$$

Correct option is (a)

24. (a) $(A+B)\overline{AB}$

$\Rightarrow (A+B)(\bar{A}+\bar{B})$ (By apply De-Morgan's law)

$= \bar{A}B + A\bar{B}$ XOR-logic-gate

(b) $A\bar{B} + B\bar{A}$ XOR-logic-gate

(c) $(A+B)(\bar{A}+\bar{B})$

$\underbrace{A\bar{A}}_{\substack{\uparrow\\0}} + \bar{A}B + A\bar{B} + \underset{\substack{\uparrow\\0}}{A\cdot\bar{B}}$

$\bar{A}B + A\bar{B}$ XOR logic-gate

Note: $x\bar{x} = 0$

(d) $(A+B)AB$

$= AAB + ABB = AB + AB = AB$

AND logic - gate

Note: $A + A = A$

Correct option is (d)

25. $\hat{A} = \lambda\vec{\sigma}\cdot\vec{B} = \lambda\,\sigma_x\frac{B}{\sqrt{2}} + \frac{\lambda\sigma_y}{\sqrt{2}}B = \frac{\lambda B}{\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\frac{\lambda B}{\sqrt{2}} = \frac{\lambda B}{\sqrt{2}}\begin{pmatrix}0 & 1-i\\ 1+i & 0\end{pmatrix}$

Eigenvalue equation:

$\therefore \quad |A - I\omega| = 0$

$$\Rightarrow \begin{vmatrix} -\omega & \frac{\lambda B}{\sqrt{2}}(1-i)\\ \frac{\lambda B}{\sqrt{2}}(1+i) & -\omega\end{vmatrix} = 0 \Rightarrow \omega^2 - \frac{\lambda^2B^2}{2}(2) = 0 \Rightarrow \omega^2 = \lambda^2B^2 \Rightarrow \omega = \pm\lambda B$$

Correct option is (b)

Q.26 – Q.55 : Carry TWO marks each.

26. **P-(iii):** ESR (electron spin resonance) lies in GHz [8 – 10 GHz] frequency range. Thus, it lies in microwave range.

Q-(i): NMR (nuclear magnetic resonance) lies in [100 – 1000] MHz frequency range. Thus, it is in radiofrequency range.

R-(iv): Transition between vibrational states of a molecule lies in frequency range $(10^{12}$ to 10^{14} Hz). Thus, it lies in far-infrared range.

S-(ii): Electronic transition lies in frequency range 10^{14} to 10^{15} Hz. Thus, it lies in visible range.

Hence, correct option is (d).

CAREER ENDEAVOUR

27. Given: $S = Nk_B \ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right)$

From first law of thermodynamics,

$TdS = dE + PdV - \mu dN$

At constant E and V,

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{E,V} \Rightarrow \mu = -T\left[k_B \ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) + Nk_B\left(\frac{N^{5/2}}{aVE^{3/2}}\right)\left(-\frac{5}{2}\frac{aVE^{3/2}}{N^{7/2}}\right)\right]$$

$$= -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{5}{2}\right]$$

Correct option is (a)

28. $Q = \left(M_{Eu} + M_n\right) - \left(M_{sm} + M_p\right)$

$= (151.921749 + 1.008665) - (151.919756 + 1.007825) = 2.833 \times 10^{-3}$ *a.m.u.*

Correct answer is (2.833)

29. According to energy conservation

$$Mc^2 = \frac{3Mc^2}{10\sqrt{1 - v_1^2/c^2}} + \frac{3Mc^2}{10\sqrt{1 - v_2^2/c^2}}$$

Since, initial velocity was zero and two parts have same mass.

So, $\vec{v}_1 = -\vec{v}_2$ (for momentum to be conserved)

$$\therefore \quad \frac{2 \times 3}{10\sqrt{1 - v_1^2/c^2}} = 1 \Rightarrow 1 - v_1^2/c^2 = \frac{9}{25}$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{16}{25}}\, c = \frac{4}{5}c = 0.8c$$

$$\therefore \quad \vec{v}_1 = -\vec{v}_2 = 0.8c\,\hat{z}$$

Correct option is (b)

30. $E_g = 0.72\ eV$ and $m_h^* = 6\, m_e^*$

T = 300K, $E_F = \left(\frac{E_C + E_V}{2}\right) - \left(\frac{k_B T}{2}\right)\ln\left(\frac{N_C}{N_V}\right)$

$$E_F - E_V = \left(\frac{E_C - E_V}{2}\right) - \left(\frac{k_B T}{2}\right)\ln\left(\frac{N_C}{N_V}\right) = \frac{0.72\ eV}{2} - \left(\frac{1.38 \times 10^{-23} \times 300}{2}\right) \times \frac{3}{2} \times \ln\left(\frac{m_e}{m_n}\right)$$

$$= \left(0.36 + 1.38 \times 10^{-23} \times 150 \times \frac{3}{2} \times \ln 6\right) eV \simeq 0.395\ eV$$

31. Total charge in the given figures (i) and (ii) are zero.

Therefore, value of dipole moment $\vec{p}$ is independent of origin.

CAREER ENDEAVOUR

The charge distribution in (i) and (ii) are both symmetric about origin.

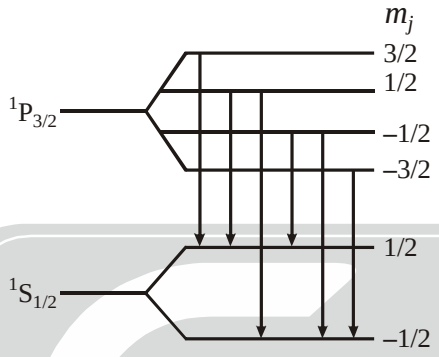
Dipole moment $\vec{p}$ due to symmetric charge distribution is zero.

Therefore, dipole moment is zero in both (i) and (ii).

Correct option is (a)

32. For $^2P_{3/2} \rightarrow {}^2S_{1/2}$:

In presence of an external weak field, the state $^2S_{1/2}$ will split into two states $\left(m_j = \frac{1}{2}, -\frac{1}{2}\right)$ and $^2P_{3/2}$ state will split into four state $\left(m_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$.



Selection rule ; $\Delta m_j = 0, \pm 1,\ 0 \not\leftrightarrow 0$

Thus, there will be total six transitions permitted in $^2P_{3/2} \rightarrow {}^2S_{1/2}$.

Hence, correct answer is (6).

33. $\psi = \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}$

First order energy correction

$$E_0^{(I)} = \int_0^{L/2} \frac{2V_0}{L} \sin^2\frac{\pi x}{L} dx = \frac{V_0}{L}\int_0^{L/2}\left(1-\cos\frac{2\pi x}{L}\right)dx = \frac{V_0}{L}\times\frac{L}{2} = \frac{V_0}{2}$$

The ground state energy is $E = \frac{\hbar^2 n^2}{2mL^2} + \frac{V_0}{2}$

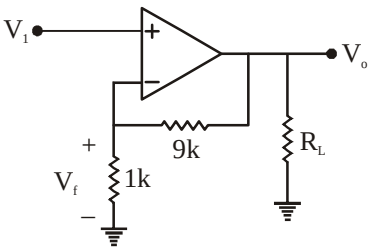
Correct option is (d)

34. $A_{OL} = 10^5$

open loop gain

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL}\beta}$$

closed loop gain



$$V_f = \frac{V_0 \times 1}{1+9} \quad \Rightarrow \text{ feedback ratio } \beta = \frac{V_f}{V_0} = \frac{1}{10}$$

CAREER ENDEAVOUR

$$A_{CL} = \frac{10^5}{1+\frac{10^5 \times 1}{10}} \cong 10 \quad \text{using } \frac{A_{OL}}{1+A_{OL}\cdot\beta}$$

Non inverting amplifier is an example of voltage series topology
Voltage series OR series-shunt
Correct option is (c)

35. $E_{initial} = (\hat{x}+i\hat{y})\,e^{i(kz-\omega t)} = \hat{x}e^{i(kz-\omega t)} + \hat{y}\,e^{i\left(kz-\omega t+\frac{\pi}{2}\right)}$

$E_{final} = (\hat{x}-i\hat{y})\,e^{i(kz-\omega t)} = \hat{x}e^{i(kz-\omega t)} + \hat{y}\,e^{i\left(kz-\omega t+\frac{3\pi}{2}\right)}$

So, phase introduce by the optical system = π
Correct option is (b)

36. Equation of motion, $m\frac{dv}{dt} = +mg - kv \Rightarrow \frac{dv}{\left(g-\frac{k}{m}v\right)} = dt$

$$\Rightarrow \quad -\frac{m}{k}\ln\left(g-\frac{k}{m}v\right) = t + c$$

At $t = 0,\ v = 10\ ms^{-1}$

$$c = -\frac{m}{k}\ln\left(g-\frac{k}{m}10\right)$$

$$\Rightarrow \quad -\frac{m}{k}\ln\left(g-\frac{k}{m}v\right) = t - \frac{m}{k}\ln\left(g-\frac{k}{m}\times 10\right)$$

$$\Rightarrow \quad \frac{m}{k}\ln\left(\frac{g-\frac{k}{m}v}{g-\frac{10k}{m}}\right) = -t \Rightarrow \frac{g-\frac{k}{m}v}{g-\frac{10k}{m}} = e^{-\frac{kt}{m}} \Rightarrow g-\frac{k}{m}v = \left(g-\frac{10k}{m}\right)e^{-\frac{kt}{m}}$$

$$\Rightarrow \quad v = \frac{m}{k}\left[g-\left(g-\frac{10k}{m}\right)e^{-\frac{kt}{m}}\right] = \frac{0.01}{0.05}\left[10-(10-50)e^{-\frac{0.05}{0.01}\times 0.2}\right] = 4.9$$

37. $L = \frac{m}{2}\overline{v}^2 + C\,\overline{r}\cdot\overline{v} - V(r)\,;\ \ \overline{p}_c = \frac{\partial L}{\partial v} = m\overline{v} + C\overline{r}$

$$\therefore \quad H = \overline{p}_C\cdot\overline{v} - L = (m\overline{v}+C\overline{r})\cdot\overline{v} - \frac{1}{2}m\overline{v}^2 - C\,\overline{r}\cdot\overline{v} + V(r)$$

$$= \frac{1}{2}m\overline{v}^2 + C\,\overline{r}\cdot\overline{v} - C\,\overline{r}\cdot\overline{v} + V(r)$$

$$= \frac{1}{2m}(p_C - C\,\overline{r})^2 + V(r)$$

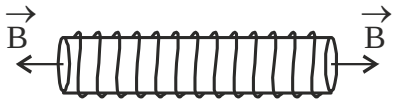
Correct option is (b)

CAREER ENDEAVOUR

38. Consider a long solenoid which is embeded into a conducting medium and is insulated from the medium. Suppose *n* is the number of turns per unit length of solenoid. So, total number of turns N = nL where L is the length of solenoid.

Magnetic field inside the solenoid $= B = \mu_0 nI = \dfrac{\mu_0 NI}{2\pi r}$

where, *r* is the radius of solenoid.



flux passing through the surface, $\phi = \int \vec{B} \cdot d\vec{S} = B\left(2\pi r^2\right)$

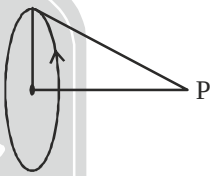
$$\phi = \left(\mu_0 nI\right) 2\pi r^2 \;\Rightarrow\; \frac{d\phi}{dt} = \mu_0 n 2\pi \frac{d\left(Ir^2\right)}{dt}$$

$$= 2\pi\mu_0 n r^2 \frac{dI}{dt} = 2\pi\mu_0 r^2 \frac{N}{2\pi r}\frac{dI}{dt}$$

Induced current will be proportional to the rate of change of flux.

So, $I_{\text{induced}} \propto \dfrac{d\phi}{dt} \propto r \;\left(\text{Inside}\right)$

Outside the solenoid,



$$\phi = \int \vec{B} \cdot d\vec{S} = B\;\; 2\pi r^2$$

Magnetic field at point due to this circular ring comes out to be

$$B \equiv \frac{1}{r^3}\left(\text{well known}\right)$$

So, $\phi_{outside} \propto \dfrac{1}{r^3} \times r^2$

$$I_{induced} \propto \frac{d\phi}{dt} \propto \frac{1}{r} \qquad \left(\text{outside}\right)$$

Correct option is (d)

CAREER ENDEAVOUR

39. We have nucleus, $^{13}C_6$

Z = 6 (even)

N = 7(odd) $\rightarrow 1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^1$

$J = \dfrac{1}{2},\; \ell = 1\left(\text{for } p\right),\; \text{parity} = \left(-1\right)^1 = -1$

Spin parity of ground state is $\dfrac{1}{2}^-$

Isospin, $I = \dfrac{M-1}{2} = \dfrac{2-1}{2} = \dfrac{1}{2}$ (M is the multiplicity ^{13}C, ^{12}C)

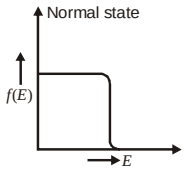
Therefore, Isospin I_3 of ^{13}C is $-\dfrac{1}{2}$ ($I_3 = +\dfrac{1}{2}$ is assigned to proton)

Correct option is (a)

40. The superconductor behaves like a normal conductor above transition temperature (T_C) and below T_C, it exhibits superconductivity.

In normal state the electrons will have $\frac{1}{2}$-integral spin and will follow Fermi-Dirac distribution function

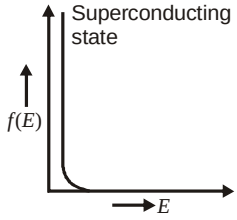
$$f(E)=\frac{1}{\left(e^{(E-E_F)/k_BT}+1\right)}$$



Therefore, $f(E)$ versus E graph $(T>0\,\text{K})$ will be

In superconducting state, electrons form cooper pairs with spin = 0.
Hence these electrons in superconducting state will follow Bose-Einstein distribution function.

$$f(E)=\frac{1}{\left(e^{(E-E_F)/k_BT}-1\right)}$$



Therefore, $f(E)$ versus E graph $(T>0\,\text{K})$ will be

Hence, correct option is (b).

41. Energy of a rigid rotator is

$$\mathrm{E}=\frac{\hbar^2}{2\mathrm{I}}\mathrm{J}\,(\mathrm{J}+1) \qquad \ldots(\mathrm{i})$$

where $\mathrm{J}=0,1,2,\ldots$

Energy of first excited state

$$(\mathrm{E}_1)=\frac{\hbar^2}{2\mathrm{I}}(1)\,(1+1)=1\,\text{meV} \;\Rightarrow\; \frac{\hbar^2}{\mathrm{I}}=1\,\text{meV} \qquad \ldots(\mathrm{ii})$$

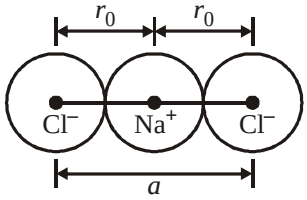
Energy of fourth excited state

$$\mathrm{E}_4=\frac{\hbar^2}{2\mathrm{I}}4(4+1)=\frac{20}{2}\cdot\frac{\hbar^2}{\mathrm{I}}=10\frac{\hbar^2}{\mathrm{I}}=10\times1\,\text{meV}=10\,\text{meV}$$

Hence, correct answer is (10)

42. The cohesive energy of NaCl can be given by :

$$U\big|_{r=r_0}=-\frac{\alpha e^2}{4\pi\,\varepsilon_0\,r_0}\left[1-\frac{1}{n}\right] \qquad \ldots(\mathrm{i}) \qquad (\text{For NaCl, } a=2r_0)$$



$$\Rightarrow\quad 7.95\times1.6\times10^{-19}=-\frac{\alpha\times(1.6\times10^{-19})^2\left[1-\frac{1}{n}\right]}{4\times3.14\times8.854\times10^{-12}\times\left(\frac{a}{2}\right)}$$

$$\Rightarrow\quad 7.95=-\frac{\alpha\times(1.6\times10^{-19})\times\left[1-\frac{1}{9}\right]}{4\times3.14\times8.854\times10^{-12}\times\left(\frac{0.563}{2}\right)\times10^{-9}} \qquad (\text{Here } n=9)$$

$$\Rightarrow\quad \alpha=-\frac{9\times7.95\times4\times3.14\times8.854\times0.563\times10^{-21}}{8\times2\times1.6\times10^{-19}} \Rightarrow\quad \alpha=-174.98\times10^{-2} \Rightarrow\quad \boxed{\alpha=-1.7498}.$$

Correct answer is (–1.750).

CAREER ENDEAVOUR

43. Since, the given Hamiltonian does not depends on time explicitly. So, total energy of the system will be conserved.

Correct option is (b)

44. The Fermi energy (E_F) is given by $E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$

$$\Rightarrow \quad E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}n\right)^{2/3} \qquad \left[\text{Let } \frac{N}{V} = n = \text{electron density}\right]$$

$$\Rightarrow \quad \left[\frac{3n}{8\pi}\right]^{2/3} = \frac{2m E_F}{h^2} \Rightarrow \quad \left(\frac{3n}{8\pi}\right) = \left(\frac{2m E_F}{h^2}\right)^{3/2}$$

$$\Rightarrow \quad n = \frac{8\pi}{3} \times \left(\frac{2m E_F}{h^2}\right)^{3/2} = \left(\frac{8 \times 3.14}{3}\right) \times \left(\frac{2 \times 9.11 \times 10^{-31} \times 5.54 \times 1.6 \times 10^{-19}}{(6.626 \times 10^{-34})^2}\right)^{3/2}$$

$$= 8.373 \times \left[\frac{3.6785 \times 10^{-50}}{10^{-68}}\right]^{3/2} = 8.373 \times 7.055 \times 10^{27}$$

$$\Rightarrow \quad n = 59.071515 \times 10^{27} \text{ per m}^3$$

$$\Rightarrow \quad \boxed{n = 5.9 \times 10^{28} \text{ per m}^3}.$$

Hence, correct answer is (5.9).

45. We have, $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{\frac{i\pi}{2}}|\psi_1\rangle\right]$

We know that,

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^\dagger\right)$$

$$\therefore \qquad \langle\psi|x|\psi\rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left[\langle\psi_0| + e^{-\frac{i\pi}{2}}\langle\psi_1|\right]\left(a + a^\dagger\right)\left[|\psi_0\rangle + e^{\frac{i\pi}{2}}|\psi_1\rangle\right]$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left[\langle\psi_0| + e^{-\frac{i\pi}{2}}\langle\psi_1|\right]\left[|\psi_1\rangle + e^{\frac{i\pi}{2}}\sqrt{2}|\psi_2\rangle + e^{\frac{i\pi}{2}}|\psi_0\rangle\right] = \sqrt{\frac{\hbar}{2m\omega}}\left[e^{\frac{i\pi}{2}} + e^{-\frac{i\pi}{2}}\right] = \sqrt{\frac{\hbar}{2m\omega}} \times 0$$

Correct answer is (0)

46. $F_c = -m\omega^2 r$ is radially outward

$$F_{CO} = -2m\omega v = -2m\omega^2 r = 2F_c$$

Correct option is (d)

CAREER ENDEAVOUR

47. $y''+\frac{1}{z}y'-\frac{m^2}{z^2}y=0$ ... (i)

$\Rightarrow \quad z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}-m^2y=0 \rightarrow$ Cauchy Euler equation.

... (ii)

Putting $z=e^x$ and replacing $z\frac{dy}{dz}$ by $\frac{dy}{dx}$ and $z^2\frac{d^2y}{dz^2}$ by $\frac{d^2y}{dx^2}-\frac{dy}{dx}$ in equation (ii)

We get, $\frac{d^2y}{dx^2}-\frac{dy}{dx}+\frac{dy}{dx}-m^2y=0$ ... (iii)

Trial solution $y=C\cdot e^{kx}$, putting inequation (iii)

$\Rightarrow \quad \left(k^2-m^2\right)=0 \Rightarrow k=\pm m$

So, $y=C_1e^{mx}+C_2e^{-mx}=C_1z^m+C_1z^{-m}$

Therefore, z^m and z^{-m} are linearly independent solutions for $m>0$.

For $m=0, k=0 \Rightarrow y=\left(C_1+C_2x\right)e^{0.x}=\left(C_1+C_2\ell nz\right)$

Therefore, 1 and *l*nz are linearly independent solutions for $m=0$

Correct option is (c)

48. The energy of a one-dimensional quantum oscillator with frequency ω is

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega, \quad n=0,1,2,\cdots$$

The partition function is

$$Z=\sum_n e^{-\beta E_n}=\sum_n e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega}=e^{-\frac{\beta\hbar\omega}{2}}\sum_n e^{-n\beta\hbar\omega}$$

Now $\sum_n e^{-n\beta\hbar\omega}=1+e^{-\beta\hbar\omega}+e^{-2\beta\hbar\omega}+\cdots$

Letting $e^{-\beta\hbar\omega}=x$. Then,

$$\sum_n e^{-n\beta\hbar\omega}=1+x+x^2+\cdots=\frac{1}{1-x}=\frac{1}{1-e^{-\beta\hbar\omega}}$$

$$\therefore \quad Z=\frac{e^{-\frac{\beta\hbar\omega}{2}}}{1-e^{-\beta\hbar\omega}}=\frac{1}{e^{\frac{\beta\hbar\omega}{2}}-e^{-\frac{\beta\hbar\omega}{2}}}=\frac{1}{2\sinh\left(\frac{\beta\hbar\omega}{2}\right)}$$

where we have used $\frac{e^x-e^{-x}}{2}=\sinh x$

The average energy U is

$$U=-\frac{\partial}{\partial\beta}\ln Z=-\frac{\partial}{\partial\beta}\ln\left[2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right]^{-1}=\frac{2\cosh\left(\frac{\beta\hbar\omega}{2}\right)}{2\sinh\left(\frac{\beta\hbar\omega}{2}\right)}\cdot\frac{\hbar\omega}{2}=\frac{1}{2}\hbar\omega\coth\left(\frac{\beta\hbar\omega}{2}\right)$$

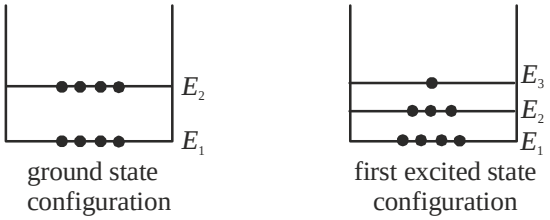
Correct option is (a)

CAREER ENDEAVOUR

49. The number of degeneration including spin is $(2s+1) = 2\times\frac{3}{2}+1 = 4$

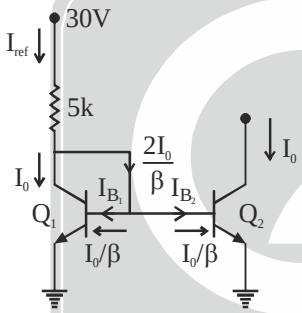
Therefore, the minimum exitation energy of the system = $\frac{\pi^2\hbar^2}{2mL^2}\left[4\times1+3\times4+1\times9-\left(4\times1+4\times4\right)\right]$

$= \frac{\pi^2\hbar^2}{2mL^2}[25-20] = \frac{5\pi^2\hbar^2}{2mL^2}$



Correct answer is (5)

50.
- Given figure is a current mirror circuit. In this circuit output current is forced to be equal to input current.
- Output current is mirror image of input current
- It is basically voltage to current converter



Given :
- $\beta = 100$
- $V_{BE} = 0.7$
- Here, Q_1 and Q_2 are identical transistor

So, $\beta_1 = \beta_2 = \beta = 100$, $V_{BE_1} = V_{BE_2} = V_{BE} = 0.7$ (Q_1 and Q_2 are in active region)

$$I_{ref} = \frac{V_{CC}-V_{BE}}{R_{ref}} = \frac{30-0.7}{5K} = \frac{29.3}{5K} = 5.86\ mA$$

$I_{ref} = I_{C_1} + I_{B_1} + I_{B_2}$ (By applying KCL) ... (1)

$\because$ $\beta_1 = \beta_2$. So, $I_{B_1} = I_{B_2}$ and $I_{C_1} = I_{C_2} = I_0$

We know that, $I_C = \beta I_B$

So, $I_B = \frac{I_C}{\beta}$

Put the value of $I_{C_1}, I_{B_1}, I_{B_2}$ in equation (1)

$$I_{ref} = I_0 + 2I_B = I_0 + \frac{2I_C}{\beta} = I_0 + \frac{2I_0}{\beta}$$

$$\therefore \qquad \boxed{I_0 = \frac{I_{ref}}{1+2/\beta}}$$

Put the value of I_{ref} and β

$$I_0 = \frac{5.86}{1+\frac{2}{100}} = \mathbf{5.74}\ mA$$

Correct answer is (5.74)

51. Shifting property:

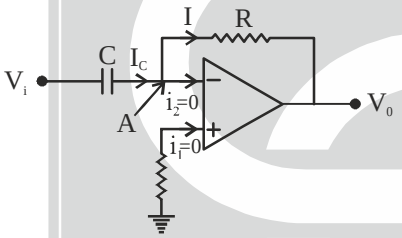
If $F\left[f(t)\right] = f(\omega)$, then $F\left[f(t \pm a)\right] = e^{\mp i\omega a} f(\omega)$

$$F\left[H(t)\right] = -\frac{2i}{\omega};\ F\left\{\frac{1}{2}H\left(t+\frac{1}{2}\right) - \frac{1}{2}H\left(t-\frac{1}{2}\right)\right\}$$

$$= \frac{1}{2}e^{-i\omega/2}\left(-\frac{2i}{\omega}\right) - \frac{1}{2}e^{i\omega/2}\left(-\frac{2i}{\omega}\right) = \frac{i}{\omega}\left[e^{i\omega/2} - e^{-i\omega/2}\right] = \frac{i}{\omega}\left(2i\sin\frac{\omega}{2}\right) = -\frac{\sin\frac{\omega}{2}}{(\omega/2)}$$

Correct option is (a)

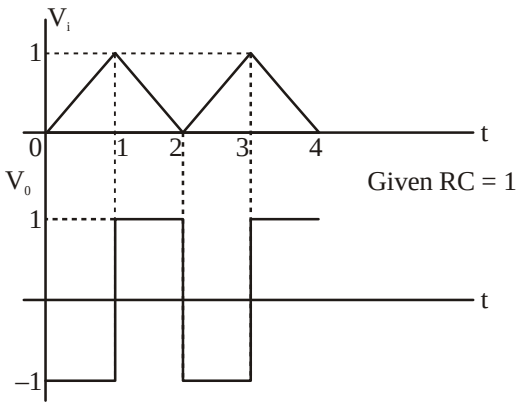
52. It is a differentiator circuit



As ideal op-amp has infinite input resistance. So, there will be no current passing through op-amp. So, by applying KCL at node A.

$$I_C = I;\ I_C = C\frac{dV_i}{dt} \text{ and } V_R = RI_C;\ V_0 = -V_R;\ V_0 = -RC\frac{dV_i}{dt}$$

So, output signal can be obtained by differentiating the input signal.



Note: The circuit given is an inverting differentiator. (Just differentiate the given input signal)

Correct option is (b)

53. $$H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2m}m\omega^2\left(r_1^2 + r_2^2\right) + k\left(\sigma_1 \cdot \sigma_2\right)$$

$$= \frac{1}{2m}p_1^2 + \frac{1}{2}m\omega^2 r_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2 r_2^2 + k\left(\sigma_1 \cdot \sigma_2\right)$$

$$= H_1 + H_2 + k\frac{4}{\hbar^2}\left(\vec{s}_1 \cdot \vec{s}_2\right) = H_1 + H_2 + \frac{2k}{\hbar^2}\left(s^2 - s_1^2 - s_2^2\right)$$

Energy eigenvalues are

$$E = \left(n_1 + \frac{3}{2}\right)\hbar\omega + \left(n_2 + \frac{3}{2}\right)\hbar\omega + 2k\left[s\left(s+1\right) - s_1\left(s_1+1\right) - s_2\left(s_2+1\right)\right]$$

For ground state, ($n_1 = 0$, $n_2 = 0$, $s = 0$)

$$E_{G.S.} = \frac{3}{2}\hbar\omega + \frac{3}{2}\hbar\omega + 2k\left(-\frac{3}{2}\right) = 3\hbar\omega - 3k = -0.3\,eV$$

Correct answer is (–0.2)

54. The wave number of anti-stokes line is given by

$$\nu_{\text{anti Stokes}} - \nu_{\text{Raman}} = \nu_{\text{Raman}} - \nu_{\text{Stokes}}$$

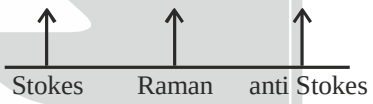
$$\Rightarrow \quad \nu_{\text{anti Stokes}} = 2\nu_{\text{Raman}} - \nu_{\text{Stokes}} \qquad \ldots (1)$$

Given : $\nu_{\text{Raman}} = \frac{1}{\lambda_{\text{Raman}}} = \frac{1}{546\times10^{-9}\,m} = \frac{1}{546\times10^{-7}\,cm}$

$$\nu_{\text{Stokes}} = \frac{1}{\lambda_{\text{Stokes}}} = \frac{1}{552\times10^{-7}\,cm}$$

Substituting values in equation(1),

$$\nu_{\text{anti Stokes}} = 2\times\frac{1}{(546\times10^{-7})} - \frac{1}{(552\times10^{-7})}$$



$$= 0.003663\times10^{7} - 0.0018116\times10^{7}$$

$$= 36630 - 18116 \text{ per cm} = 18514 \text{ per cm}$$

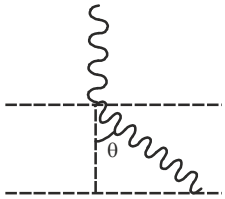
Correct answer is (18514)

55. The condition for n^{th} order maxima is $d\sin\theta = n\lambda$

For first order diffraction $n = 1$

$$d = \left(\frac{\lambda}{\sin\theta}\right)$$

We have, $\vec{k} = |k|\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}\right)$



$$\tan\theta = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^{\circ}\,;\; d = \frac{\lambda}{\sin\theta} = \frac{600\times10^{-9}}{0.5} = 1.2\;\mu m$$

Correct answer is (1.2)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

Q.1 – Q.25 : Carry ONE mark each.

1. We have, $\frac{dy}{dx} = xy \Rightarrow \frac{dy}{y} = xdx \Rightarrow \ln y = \frac{x^2}{2} + c$

Given, $x = 0, y = 2, \therefore c = \ln 2 \Rightarrow \ln y = \frac{x^2}{2} + \ln 2 \Rightarrow \ln\left(\frac{y}{2}\right) = \frac{x^2}{2}$

Putting $x = 2$, we get

$\Rightarrow \quad \ln\left(\frac{y}{2}\right) = 2$

$\Rightarrow \quad y = 2e^2$

Correct option is (d)

2. We known, $\vec{B} = \vec{\nabla} \times \vec{A}$, $\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-B_0 y}{2} & \frac{B_0 x}{2} & 0 \end{vmatrix} = B_0 \hat{k}$

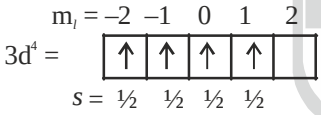
We have, $B = B_0 \hat{k}$, only option (c) satisfy the above condition

Correct option is (c)

3. For Raman active, there must be change in polarizability. Since $^{17}O_2$ is homo diatomic molecule therefore, it will have change in polarizability, thus it will be Raman active. Net dipole moment in $^{17}O_2$ is zero, so IR inactive $^{17}O_2$ has nuclear spin I = 5/2, therefore, it is NMR active.

Correct option is (c)

4. For the 4, 3-d electrons



$L = \sum m_\ell = -2 - 1 + 0 + 1 = -2$

$S = \sum s = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$

Since shell is more than half filled total angular momentum

$J = L + S = -2 + 2 = 0$

The resultant magnetic momentum is given by

$\mu = g\mu_B J = 0 \qquad [J = 0]$

Correct answer is (0)

5. For electric dipole radiation, selection rule is $\Delta\ell = \pm 1$.

Here, in option for transition $2s \rightarrow 1s$, we have $\Delta\ell = 0$, which is forbidden.
Thus, $2s \rightarrow 1s$ is not allowed.

Hence, correct option is (a).

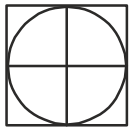
CAREER ENDEAVOUR

6. For triplet of mesons, $2I + 1 = 3 \Rightarrow I = 1$

Mesons are not strange particles. Therefore, S = 0

Correct option is (b)

7. We know magnetic moment, $\vec{m} = Ia^2\hat{n}$



where a is the side of the square and $\hat{n}$ is unit vector normal to the surface

If the square is converted into circle then the radius of the circle will be $r = \frac{a}{2}$

So, magnetic moment, $\vec{m}_c = I\pi r^2\hat{n} = \frac{I\pi a^2}{4}\hat{n} = \frac{m\pi}{4}$

So, $P = \frac{\pi^2}{4} = 2.46$

Correct answer is (2.46)

8. At R, T

Total power emitted = P_1

At $\frac{R}{2}$, $2T$

Total power emitted = P_2

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

$$\text{Total power in space} = \frac{\text{Energy}}{\text{Time}} \times 4\pi R^2$$

$$\Rightarrow \quad \text{Energy} = \sigma T^4 \ (\text{Stefan's Boltzman law})$$

$$\Rightarrow \quad P_1 = \frac{\sigma T^4}{t} \times 4\pi R^2, \ t = \text{time}$$

$$P_2 = \sigma(2T)^4 \times 4\pi\left(\frac{R}{2}\right)^2$$

$$\frac{P_1}{P_2} = \frac{T^4}{16T^4} \times \frac{R^2}{R^2} \times 4 = \frac{1}{4} = 0.25$$

Correct option is (0.25)

9. Given : $S = -k_B N\left[p \ln p + (1-p)\ln(1-p)\right]$

For maxima and minima,

$$\frac{dS}{dp} = 0 \ \Rightarrow \ -k_B N\left[\frac{p}{p} + \ln p + \left(\frac{1-p}{1-p}\right)(-1) - \ln(1-p)\right] = 0$$

$$\Rightarrow \ -k_B N\left[1 + \ln p - 1 - \ln(1-p)\right] = 0$$

$$\Rightarrow \ \ln\left(\frac{p}{1-p}\right) = 0 \quad \Rightarrow \quad p = \frac{1}{2} = 0.5$$

Correct answer is (0.5)

CAREER ENDEAVOUR

10. The Helmholtz free energy (F) is defined as

$$F = U - TS$$

$$\Rightarrow \quad dF = dU - TdS - SdT$$

$$= TdS - PdV - TdS - SdT \qquad [\because \quad dU = TdS - PdV]$$

$$= -PdV - SdT$$

At constant temperature and volume,

$$dF = 0$$

So F is either maximum or minimum for a system at constant volume and temperature. Moreover the second law of thermodynamics sets a limit that in any process at constant volume and temperature, the Helmholtz free energy decreases.

i.e., $dF \le 0$.

where equality holds for reversible and inequality for irreversible processes.

So, Helmholtz free energy decreases and hence minimum at equilibrium.

Correct option is (a)

11. From first law of thermodynamics,

$$dQ = dU + PdV$$

It is given that the total energy is constant.

$$\Rightarrow \quad dU = 0 \quad \Rightarrow \quad dQ = PdV$$

By definition, $\Delta S = \int \frac{dQ}{T} = \int \frac{PdV}{T}$

$$= \int_V^{4V} \frac{Nk_B}{V} dV \qquad [\because \quad \text{for ideal gas, } PV = Nk_BT]$$

$$= Nk_B \left[\ln V\right]_V^{4V} = Nk_B \ln 4 = 2Nk_B \ln 2$$

Correct answer is (2)

12. Let $z = x + iy$

$$f(z) = z^2 = \left(x^2 - y^2\right) + 2xyi = u + iv$$

Here, $u = x^2 - y^2, v = 2xy$

Now, $\frac{\partial u}{\partial x} = 2x, \ \frac{\partial u}{\partial y} = -2y$

$$\frac{\partial v}{\partial x} = 2y, \frac{\partial v}{\partial y} = 2x$$

Therefore, Cauchy Reamann equation is satisfied i.e.

$$\therefore \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \ \& \ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Correct option is (a)

13. **When slit-1 open** **when slit-2 open**

$a^2 = 4I_0$ $b^2 = I_0$

$\Rightarrow a = 2\sqrt{I_0}$ $\Rightarrow b = \sqrt{I_0}$

CAREER ENDEAVOUR

So, when both are open

$$\frac{I_{\max}}{I_{\min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{\left(3\sqrt{I_0}\right)^2}{\left(\sqrt{I_0}\right)^2} = 9:1$$

Correct answer is (9:1)

14. Fermi energy can be given by $E_F = \frac{\hbar^2}{2\pi}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$ ... (i)

where, $\frac{N}{V} = n =$ electron concentration per unit volume.

$\therefore \quad \boxed{E_F \propto n^{2/3}}$ ... (ii)

and Hall coefficient can be given by $R_H = \frac{1}{ne} \propto \frac{1}{n}$

$\Rightarrow \quad R_H \propto \frac{1}{n} \propto \frac{1}{E_F^{3/2}} \quad \Rightarrow \quad \boxed{R_H \propto E_F^{-3/2}}$.

Hence, correct option is (d)

15. $\omega_{\text{acoustic}} = \sqrt{\frac{2K}{m_2}}$ at boundary

$\omega_{\text{optical}} = \sqrt{\frac{2K}{m_1}}$ at boundary

Correct option is (a).

16. $$\int_{-\infty}^{\infty}\left(\psi^* \frac{\partial^2}{\partial x^2}\psi\right)dx = \left[\psi^* \frac{\partial \psi}{\partial x}\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\frac{\partial \psi^*}{\partial x}\cdot\frac{\partial \psi}{\partial x}dx$$

$$= -\left[\frac{\partial \psi^*}{\partial x}\psi\right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty}\frac{\partial^2 \psi^*}{\partial x^2}\psi\, dx = \int_{-\infty}^{\infty}\frac{\partial^2 \psi^*}{\partial x^2}\psi\, dx$$

$\therefore \quad \left(\frac{\partial^2}{\partial x^2}\right)^{\dagger} = \frac{\partial^2}{\partial x^2}$

Correct option is (b)

17. Given $k = m_0c^2$ here, k = kinetic energy

we know that;

$$p = \frac{1}{c}\sqrt{k\left(k + 2\, m_0c^2\right)}$$

$\therefore \quad p = \frac{1}{c}\sqrt{m_0c^2\left(m_0c^2 + 2m_0c^2\right)} = \sqrt{3}\, m_0c = 1.732\, m_0c$

Hence correct answer is (1.73)

CAREER ENDEAVOUR

18. In intrinsic semiconductor, $n_i = 2 \times 10^{19}\ \text{m}^{-3}$

In doped semiconductor, $n = 4 \times 10^{20}\ \text{m}^{-3}$

By doping with donor impurities, the majority carriers will be electrons (n) and minority carriers will be holes (p).

From equation $n_i^2 = np$ ... (i)

$$\Rightarrow \quad p = \frac{n_i^2}{n} = \frac{(2\times10^{19})^2}{4\times10^{20}} \text{ per m}^3 \Rightarrow \quad p = \frac{4\times10^{38}}{4\times10^{20}} = 1\times10^{18}\ \text{m}^{-3}$$

Therefore, ratio of majority to minority charge carriers

$$\frac{n}{p} = \frac{4\times10^{20}}{1\times10^{18}} = 4\times10^2 = 400$$

Hence, correct answer is (400).

19. Given: $m_1 = m,\ m_2 = 2m, \frac{k}{m} = 4\ s^{-2}$

Angular frequency for system shown in figure is given as

$$\omega = \sqrt{\frac{k}{\mu}}, \quad \mu = \frac{m_1 \cdot m_2}{m_1 + m_2} = \text{reduced mass}$$

$$= \frac{m \cdot 2m}{m + 2m} = \frac{2m}{3}$$

$$\therefore \quad \omega = \sqrt{\frac{k}{2m/3}} = \sqrt{\frac{3}{2}\cdot\frac{k}{m}} = \sqrt{\frac{3}{2}\cdot 4} = \sqrt{6} = 2.45\ rad/s$$

Hence correct answer is (2.45)

20. Given, $\vec{F} = -k\vec{r} = -k\left(x\hat{i} + y\hat{j}\right)$, and angular momentum is non zero.

Particle is moving under two perpendicular restoring forces therefore its dynamics can be resolved into two perpendicular simple harmonic motions. We know that superposition of two perpendicular simple harmonic motions leads to three types of path; Straight line through origin, circle about origin, ellipse about origin. Since angular momentum is non zero path cannot be straight line through origin.

Hence correct answer is (b)

21. Here, in this reaction, we have electron on left hand side, which belong to Lepton family. For the conservation of Lepton number in the above equation, we need to add a Lepton member on the right hand side also i.e. either an electron, or electron-neutrino or muon or muon-neutrino.

Since, we have electron at left side, we need to add electron neutrino on right side.

Correct option is (b)

22. First Born approximation state that the energy of the incident particle is large and scattering potential is weak.

Correct option is (a)

CAREER ENDEAVOUR

23. Total scattering cross-section

$$\sigma = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell = \frac{4\pi}{k^2}(2\times 0+1)\sin^2 90^\circ = \frac{4\pi}{\left(\sqrt{2\pi}\right)^2} = 2$$

24. In case of hydrogen atom, a uniform electric field is applied. So, this electric field will be treated as a perturbation.

$$V_p = -qE_0 z = +eE_0 r\cos\theta$$

First order correction to the energy, in the ground state

$$E^1_{G.S.} = \langle\psi_{100}|eE_0 r\cos\theta|\psi_{100}\rangle = 0 \qquad \left[\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}}e^{-\frac{r}{a_0}}\right]$$

In presence of electric field, there is no correction to the first order,
Second order correction to the energy in the ground state

$$= \sum_{n\neq 1}\frac{\left|\langle\psi_{n\ell m}|eE_0 r\cos\theta|\psi_{100}\rangle\right|^2}{E_1^0 - E_m^0} \propto E_0^2$$

This is also well known result in every literature that presence of electric field lifts the degeneracy of energy levels in second order correction which comes out to be proportional to E_0^2

Hence, the value of exponent *n* is (2)

Correct answer is (2)

25. Magnetic susceptibility, $\chi = C$ (independent of *T*) [Property of diamagnets]

For type-I superconductor, the magnetic susceptibility $\chi = -1$, which is independent of temperature.

Hence, correct option is (c).

Q.26 – Q.55 : Carry TWO marks each.

26. The electric field at the upper surface will be $E = \frac{\sigma}{k\varepsilon_0}$

So, polarization, $\vec{P} = \varepsilon_0\chi\vec{E} = \frac{\varepsilon_0(k-1)\sigma}{k\varepsilon_0}\hat{n} = \frac{(k-1)}{k}\sigma\hat{n}$

So, surface bound charge, $\sigma_s = \frac{k-1}{k}\sigma$

Correct option is (d)

27. For 3*p* 3*d* electron, we have

$$s_1 = \frac{1}{2},\ s_2 = \frac{1}{2},\ \ell_1 = 1,\ \ell_2 = 2$$

The possible values of *S* and *L* are:

$$S = |s_1 - s_2|, |s_1 - s_2| + 1, ----, (s_1 + s_2)$$

$$= 0, 1;\ \text{multiplicity}\ (2s+1) = 1, 3$$

CAREER ENDEAVOUR

and $L = |\ell_1 - \ell_2|, |\ell_1 - \ell_2| + 1, ------, (\ell_1 + \ell_2)$

$= 1, 2, 3 \quad (\text{P, D, F states})$

Thus, we have in all six terms, three singlet terms and three triplet terms. All these terms are odd because the configuration 3p3d is odd $(\sum \ell = 1 + 2 = 3)$.

We can write these terms as

^1P, ^1D, ^1F, ^3P, ^3D, ^3F

To take into account spin-orbit interaction, let us combine L and S to form J. Now

$J = |L - S|, -----(L + S)$

Fo singlet terms, we have

$$\begin{cases} \text{S} = 0 \\ \text{L} = 1 \end{cases}; \text{J} = 1;\ ^1\text{P}_1$$

$$\begin{cases} \text{S} = 0 \\ \text{L} = 2 \end{cases}; \text{J} = 2;\ ^1\text{D}_2$$

$$\begin{cases} \text{S} = 0 \\ \text{L} = 3 \end{cases}; \text{J} = 3;\ ^1\text{F}_3$$

For triplet terms, we have

$$\begin{cases} \text{S} = 1 \\ \text{L} = 1 \end{cases}; \text{J} = 0, 1, 2;\ ^3\text{P}_0, ^3\text{P}_1, ^3\text{P}_2$$

$$\begin{cases} \text{S} = 1 \\ \text{L} = 2 \end{cases}; \text{J} = 1, 2, 3;\ ^3\text{D}_1, ^3\text{D}_2, ^3\text{D}_3$$

$$\begin{cases} \text{S} = 1 \\ \text{L} = 3 \end{cases}; \text{J} = 2, 3, 4;\ ^3\text{F}_2, ^3\text{F}_3, ^3\text{F}_4$$

Thus a single degenerate level of configuration 3P 3d is splitted into 12 levels.

Hence, correct answer is (12).

28. Deuteron has binding energy 2.23 MeV. So, can be disintegrated by 4 MeV gamma rays. Deuteron has no excited states. Deuteron ground state is triplet so it cannot have $^1S_0$ state. Its quadrupole moment is positive i.e. prolate.

Correct option is (c)

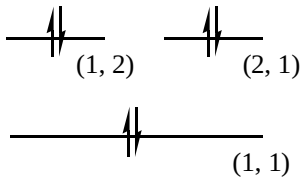
29. $\langle \bar{s}_1 \cdot \bar{s}_2 \rangle = \frac{1}{2}\left[\langle s^2 \rangle - \langle s_1^2 \rangle - \langle s_2^2 \rangle\right]$

$$= \frac{\hbar^2}{2}\left[s(s+1) - s_1(s_1+1) - s_2(s_2+1)\right]$$

$$= \frac{\hbar^2}{2}\left[0 - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right] \qquad (\text{for ground state of He, S = 0})$$

$$= -\frac{3\hbar^2}{4}$$

Correct option is (b)

30. 

So, the ground state energy of the system $E_{n_x n_y} = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2\right)$

$$E_{G.S.} = \frac{2\pi^2\hbar^2}{2mL^2}\times 2 + \frac{5\hbar^2\pi^2}{2mL^2}\times 4 = 24\times\frac{\pi^2\hbar^2}{2mL^2}$$

Correct answer is (24)

31. Maximum kinetic energy, $K.E_{\max} = 2\,NqV$
where N is number of revolutions

$V = 50\times10^3\ V$

$q = 2e^- = 2\times1.6\times10^{-19}\,\text{C}$ of α-particle

$$\therefore \qquad N = \frac{K.E_{\max}}{2qV} = \frac{16\times10^6\times1.6\times10^{-19}}{2\times2\times1.6\times10^{-19}\times50\times10^3} = 80$$

Correct answer is (80)

32. $\in_{ijk} \in_{\ell mk} \partial_j\,\partial_\ell\, V_m$

$= \in_{kij} \in_{k\ell m} \partial_j\,\partial_\ell\, V_m$

$= \left(\delta_{i\ell}\,\delta_{jm} - \delta_{im}\,\delta_{j\ell}\right)\partial_j\,\partial_\ell\, V_m$

$= \partial_j\partial_i V_j - \partial_j\partial_j V_i = \partial_i\left(\partial_j V_j\right) - \partial_j^2 V_i = -\partial_j^2 V_i$

$= -\partial_\ell^2 V_i\left(\text{Given: } \partial_j V_j = 0\right)$

$= -\partial_j^2 V_i$

Correct option is (d)

33. We have, $f\left(x, zy, z\right) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$

$$\therefore \qquad \bar{\nabla} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{1}{2}x^2 - xy + \frac{1}{2}z^2\right) = \left(x - y\right)\hat{i} + \left(-x\right)\hat{j} + z\hat{k}$$

$$\therefore \qquad \bar{\nabla} f\Big|_{(1,1,2)} = \left(-\hat{j} + 2\hat{k}\right)$$

Therefore, direction of $\bar{\nabla} f$ is $\frac{-\hat{j} + 2\hat{k}}{\sqrt{5}}$

Correct option is (b)

34. $2\sigma_x\sigma_y + \sigma_y\sigma_x = 2i\sigma_z - i\sigma_z = i\sigma_z$

Correct option is (c)

CAREER ENDEAVOUR

35. Action is defined as $S = \int_{t_1}^{t_2} L dt$ where $L = T - V$ is Lagrangian.

$$T = \frac{1}{2}mv^2 = \frac{1}{2}\times 0.1(u+at)^2 = \frac{1}{2}\times 0.1(0+10t)^2 = 5t^2$$

$$V = -\int F dx = -\int ma\, dx = -\int 0.1\times 10\, dx = -x = -\left(ut + \frac{1}{2}at^2\right) = -\left(0 + \frac{1}{2}\times 10t^2\right) = -5t^2$$

$$\therefore\ S = \int_0^2 (T-V)\,dt = \int_0^2 \left(5t^2 - \left(-5t^2\right)\right)dt = 10\cdot\left(\frac{t^3}{3}\right)_0^2 = \frac{80}{3} = 26.67$$

$S = 26.67$ J-s

Correct answer is (26.67)

36. $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 > x > \pi \end{cases}$; $f(x) = \frac{a_0}{2} + \sum_n a_n \cos nx + \sum_n b_n \sin nx$

Given $f(x)$ is an odd function :

Therefore, $a_0 = 0,\ a_n = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin nx dx + \int_0^{\pi}\sin nx dx\right]$$

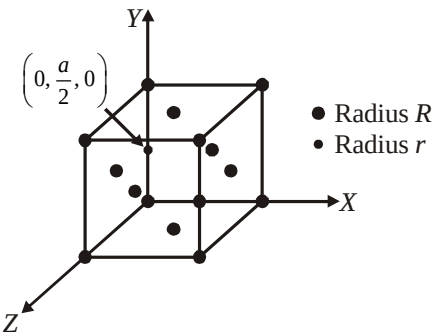
$$= \frac{1}{\pi}\left\{\left[\frac{\cos nx}{n}\right]_{-\pi}^{0} + \left[-\frac{\cos n\pi}{n}\right]_0^{\pi}\right\} = \frac{1}{\pi}\times\frac{1}{n}\left[\left(1-(-1)^n\right)+\left(1-(-1)^n\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{n}\left(1-(-1)^n\right)\right] \Rightarrow \quad b_n = \begin{cases} 0 & \text{for even n} \\ \dfrac{4}{\pi n} & \text{for odd n} \end{cases}$$

$$\therefore\ f(x) = \sum_n b_n \cdot \sin \pi x = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin nx = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + ...\right) \qquad (n \text{ is odd})$$

Correct option is (a)

37. Along the *Y*-axis there are three atoms [two at corners and one at $\left(0, \frac{a}{2}, 0\right)$]. If *a* is the lattice parameters.



Then $a = 2R + 2r$ ... (i)

Diagonal of each face will have 4 atoms (two at corners and one at center).

$\therefore\ \sqrt{2}a = 4R$ ... (ii)

CAREER ENDEAVOUR

$$\Rightarrow \quad R = \frac{\sqrt{2}a}{4}$$

Now putting value of R from equation (ii) to equation (i), we get $a = 2\left(\frac{\sqrt{2}a}{4}\right) + 2r$

$$\Rightarrow \quad 2r = a - \left(\frac{2\sqrt{2}a}{4}\right) = \left[\frac{4-2\sqrt{2}}{4}\right]a$$

$$\Rightarrow \quad r = \left(\frac{4-2\sqrt{2}}{8}\right)a \qquad \text{... (iii)}$$

$$\therefore \quad \frac{r}{R} = \frac{\left(\frac{4-2\sqrt{2}}{8}\right)a}{\left(\frac{\sqrt{2}a}{4}\right)} = \frac{4}{8}\times\frac{\left(4-2\sqrt{2}\right)}{\sqrt{2}} = \frac{2\left(2-\sqrt{2}\right)}{2\sqrt{2}} = \frac{2\sqrt{2}\left(\sqrt{2}-1\right)}{2\sqrt{2}} = \sqrt{2}-1 = 1.414-1 = 0.414.$$

Hence, correct answer is (0.41).

38. Given $\Delta x = x_2 - x_1 = d$

$$\Delta t = t_2 - t_1 = 0$$

From Lorentz transformation

$$\Delta x' = \left(\Delta x - v\Delta t\right)\gamma = d\gamma$$

$$\Delta t' = \left(\Delta t - \frac{\Delta x \cdot v}{c^2}\right)\gamma = -\frac{dv}{c^2}\gamma$$

Hence correct answer is (c)

39. The ratio of effective mass to the free electron mass

$$= \frac{m_e^*}{m_e} = \frac{\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}}{m_e} = \frac{\frac{\hbar^2}{\frac{d}{dk}\left\{2ka^2A + 4Bk^3a^4\right\}}}{m_e} = \frac{\frac{\hbar^2}{\left(2a^2A + 12Bk^2a^4\right)}}{m_e}$$

$$= \frac{\left(\frac{\hbar^2}{2a^2A}\right)}{m_e} \quad \{\text{at the bottom of conduction band } k = 0\}$$

$$= \frac{(1.05\times10^{-34})^2}{2\times(2.1\times10^{-10})^2\times(6.3\times10^{-19})} = \frac{1.05\times1.05\times10^{-68}}{2\times2.1\times2.1\times10^{-20}\times6.3\times10^{-19}\times9.1\times10^{-31}} = \frac{1\times10^2}{8\times6.3\times9.1}$$

$$= \frac{10^2}{458.64} = 0.218 = 0.22$$

Hence, correct answer is (0.22).

40. We have, $E = E_0\cos\left(kz - \omega t\right)\hat{i}$ in free space.

We know, $\vec{k}\times\vec{E} = \omega\mu_0\vec{H}$

CAREER ENDEAVOUR

$$\Rightarrow \qquad \vec{H} = \frac{E_0}{c\mu_0}\cos\left(kz-\omega t\right)\left(\hat{k}\times\hat{i}\right) = \frac{E_0}{c\mu_0}\cos\left(kz-\omega t\right)\hat{j}$$

And $\vec{S} = \vec{E}\times\vec{H} = \dfrac{E_0^2}{c\mu_0}\cos^2\left(kz-\omega t\right)\hat{k}$ $\left(\text{As, } c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$

$$= c\varepsilon_0 E_0^2\cos^2\left(kz-\omega t\right)\hat{k}$$

Correct option is (d)

41. Let n_1, n_2 and n_3 be the number of particles with energy 0, 2ε and 3ε respectively. Then.
According to given,

$$n_1 + n_2 + n_3 = 4 \qquad \text{... (i)}$$

and $n_1(0) + n_2(2\varepsilon) + n_3(3\varepsilon) = 10\varepsilon$

or $2n_2 + 3n_3 = 10$ ... (ii)

Using equation (ii),

n_2	n_3
0	10/3
1	8/3
2	2
3	4/3
4	2/3

Since n_1, n_2 and n_3 all are integers, only possible values of n_2 and n_3 are 2 and 2 respectively.

Now from equation (i), $n_1 = 0$

So possible values of $n_1 = 0$, $n_2 = 2$ and $n_3 = 2$

Also, given that $g_1 = 2$, $g_2 = 2$ and $g_3 = 3$.

The number of microstates for bosons is

$$W = \prod_{i=1}^{3}\frac{(n_i+g_i-1)!}{n_i!(g_i-1)!} = \frac{(n_1+g_1-1)!}{n_1!(g_1-1)!}\times\frac{(n_2+g_2-1)!}{n_2!(g_2-1)!}\times\frac{(n_3+g_3-1)!}{n_3!(g_3-1)!}$$

$$= \frac{1!}{0!1!}\times\frac{3!}{2!1!}\times\frac{4!}{2!2!} = 1\times 3\times 6 = 18$$

Correct answer is (18)

42. $L = \dfrac{1}{2}m\ell^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right) - mg\ell\cos\theta$

φ is cyclic, therefore p_φ is conserved

$$p_\varphi = \frac{\partial L}{\partial\dot{\varphi}} = \text{Conserved}$$

$= m\ell^2\sin^2\theta\dot{\varphi}$ is conserved

Therefore $\sin^2\theta\,\dot{\varphi}$ is conserved.

Correct answer is (a)

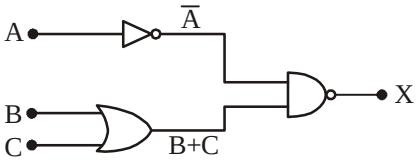
43. Rutherford scattering cross-section

$$\sigma = \frac{NnLZ^2k^2e^2}{4r^2\left(K.E\right)^2\sin^4\left(\frac{\theta}{2}\right)} \qquad \text{(since, momentum is equal)}$$

$$\therefore \qquad \sigma \propto Z^2 \Rightarrow \frac{\sigma_\alpha}{\sigma_p} = \frac{4}{1} = 4$$

Correct answer is (4)

44. To find output X



$$X = \overline{\overline{A}\cdot\left(B+C\right)} \qquad \Rightarrow \qquad X = \overline{\overline{A}} + \overline{B+C} \qquad \Rightarrow \qquad X = A + \overline{B}\cdot\overline{C}$$

(By applying De-Morgan's theorem)

$$\overline{A\cdot B} = \overline{A} + \overline{B}$$

Correct option is (b)

45. The molar specific heat can be given by

$$C_V = \left(\frac{\pi^2 N_A k_B^2}{2E_F}\right)T + \left(\frac{12\pi^4 N_A k_B}{5T_D^3}\right)T^3 \qquad \text{... (i)}$$

$$\Rightarrow \quad C_V = \gamma T + AT^3 \qquad \text{... (ii)}$$

$$\text{where, } \gamma = \frac{\pi^2 N_A k_B^2}{2E_F} \Rightarrow \gamma \propto \frac{1}{E_F} \qquad \text{... (iii)}$$

$$\text{and } A = \frac{12\pi^4 N_A k_B}{5T_D^3} \Rightarrow A \propto \frac{1}{T_D^3} \qquad \text{... (iv)}$$

For X and Y:

$$\frac{\gamma_X}{\gamma_Y} = \frac{\left(\frac{1}{(E_F)_X}\right)}{\left(\frac{1}{(E_F)_Y}\right)} = \frac{(E_F)_Y}{(E_F)_X} = \frac{7\text{ eV}}{5\text{ eV}} = \frac{7}{5} \text{ and } \frac{A_X}{A_Y} = \frac{\left(\frac{1}{(T_D)_X}\right)^3}{\left(\frac{1}{(T_D)_Y}\right)^3} = \frac{(T_D)_Y^3}{(T_D)_X^3} = \left[\frac{(T_D)_Y}{(T_D)_X}\right]^3 = \left(\frac{340}{170}\right)^3 = (2)^3 = 8$$

Correct option is (a).

46. The mean energy is

$$\langle E\rangle = \frac{\sum_i E_i g_i e^{-E_i/k_BT}}{\sum_i g_i e^{-E_i/k_BT}} \quad \text{where } g_i \text{ is the degeneracy of the } E_i \text{ th level.}$$

$$\therefore \quad \langle E\rangle = \frac{0\cdot 1e^{-0/k_BT} + E\cdot 3e^{-E/k_BT}}{1\cdot e^{-0/k_BT} + 3\cdot e^{-E/k_BT}} = \frac{3E\,e^{-E/k_BT}}{1+3e^{-E/k_BT}}$$

Correct option is (d)

CAREER ENDEAVOUR

47. Apply conservation of momentum in x direction to get

$$\frac{Mv}{\sqrt{1-\frac{v^2}{c^2}}} = p_1 \cos 45^o + p_2 \cos 60^o$$

We have energy momentum relation $p = \frac{E}{c}$ for photon

$$\frac{Mv}{\sqrt{1-v^2/c^2}} = \frac{1}{c}\left(\frac{E_1}{\sqrt{2}} + \frac{E_2}{2}\right) \quad \text{... (i)}$$

Apply conservation of energy; given $E_1 = 1GeV$, $E_2 = 0.82\ GeV$

$$\frac{Mc^2}{\sqrt{1-v^2/c^2}} = E_1 + E_2 \quad \text{... (ii)}$$

Divide (i) by (ii) to get

$$\frac{v}{c^2} = \frac{1}{c}\frac{\left(\frac{E_1}{\sqrt{2}} + \frac{E_2}{2}\right)}{(E_1 + E_2)}; \quad v = \frac{c\left(\frac{E_1}{\sqrt{2}} + \frac{E_2}{2}\right)}{(E_1 + E_2)} = \frac{c\left(\frac{1}{\sqrt{2}} + \frac{0.82}{2}\right)}{1+0.82}$$

$v = 0.614\ c$

From equation (ii) we get

$$M = \frac{(E_1 + E_2)}{c^2}\sqrt{1-v^2/c^2} = \frac{(1+0.82)\sqrt{1-(0.614)^2}}{c^2} GeV = 1.44\ GeV/c^2$$

Hence correct answer is (1.44).

48. $\left[x^2, p^2\right] = \left[x^2,\ p \cdot p\right] = \left[x^2, p\right]p + p\left[x^2, p\right] = i\hbar 2xp + pi\hbar 2x = 2i\hbar(xp + px)$

Correct option is (d)

49. According to boundary condition,

$$B_{2n} = B_{1n}; \quad H_{2t} = H_{1t}\ ; \ \frac{1}{\mu_2}B_{2t} = \frac{B_{1t}}{\mu_1}$$

We have, $B_{1n} = B_z$, $B_{1t} = B_x$, $\mu_r = \frac{\mu_2}{\mu_1} = \frac{\mu_2}{\mu_0}$

$$B_{2n} = B_z, \qquad B_{2t} = \mu_r B_x$$

So, magnetic field in the magnetic material will be

$$B = B_{2n} + B_{2t} = \mu_r B_x \hat{i} + B_z \hat{k}$$

Correct option is (d)

50. $\left\langle 1,0\left|L_x + iL_y\right|1,-1\right\rangle = \left\langle 1,0\left|L_+\right|1,-1\right\rangle = \hbar\sqrt{2\times 1}\left\langle 1,0\,|\,1,0\right\rangle = \sqrt{2}\ \hbar$

$\left[L_+\left|\ell, m\right\rangle = \hbar\sqrt{(\ell - m)(\ell + m + 1)}\left|\ell, m+1\right\rangle\right]$

Correct option is (c)

CAREER ENDEAVOUR

51. We know that, parity operator is hermitian i.e. $P^{\dagger} = P$

Parity operator is also unitary operator i.e. $P^{\dagger} = P^{-1}$

$P|\psi\rangle = \pm 1|\psi\rangle \Rightarrow P^2|\psi\rangle = I|\psi\rangle$

$\therefore\ P^2 = I$

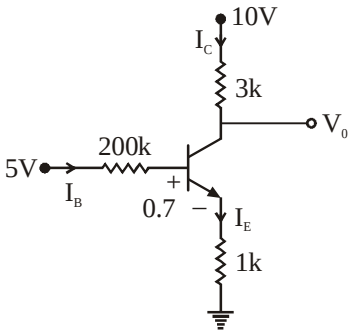
Therefore, $P^2 = -P\,(\text{not correct})$

Correct option is (b)

52. Given $V_{in} = 5V$

$V_{BE} = 0.7\,V$

$\beta_{dc} = 100$



Applying KVL at Base-Emitter Junction

$5 = 200\,I_B + 0.7 + I_E$

$I_E = (\beta + 1)\cdot I_B$

So, $5 = 200\,I_B + 0.7 + 101 \times I_B$

$I_B = 0.0142\ mA$

$I_C = \beta \cdot I_B = 1.42\ mA$

Now, applying KVL at output

$10 = 3I_C + V_0$ (Put value of I_C)

$V_0 = \mathbf{5.714}$ Volt

Correct answer is (5.7)

53. $\psi = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$

$\psi = A\left[|\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle\right]$, where A is a normalization constant

Using the normalization condition, $\langle\psi|\psi\rangle = 1$

$$\Rightarrow \quad |A|^2(1+4+9) = 1 \Rightarrow A = \frac{1}{\sqrt{14}} \Rightarrow |\psi\rangle = \frac{1}{\sqrt{14}}|\phi_1\rangle + \frac{2}{\sqrt{14}}|\phi_2\rangle + \frac{3}{\sqrt{14}}|\phi_3\rangle$$

The probability of finding the system in the state $|\phi_2\rangle$ is $\left|\langle\phi_2|\psi\rangle\right|^2 = \frac{4}{14} = \frac{2}{7} = 0.2857$

Correct answer is (0.29)

54. For $^{15}_{8}O$, $Z = 8\,(\text{even})$

N = 15–8 = 7

$\left(^1s_{1/2}\right)^2 \left(^1p_{3/2}\right)^4 \left(^1p_{1/2}\right)^1$

Therefore, $J = \frac{1}{2}$, $\ell = 1\,(\text{for } p)$ and parity = $(-1)^1 = -1$

CAREER ENDEAVOUR

Thus, spin-parity = $\frac{1^-}{2}$

For $^{17}_{8}O$, Z = 8 (even)

N = 17–8 = 9

$$\left(^1s_{1/2}\right)^2\left(^1p_{3/2}\right)^4\left(^1p_{1/2}\right)^2\left(^1d_{5/2}\right)^1$$

$\therefore \quad J=\frac{5}{2}, \ell=2\left(\text{for } d\right)$

Therefore, parity = $(-1)^2$ = +1 and spin-parity = $\frac{5^+}{2}$

Correct option is (b)

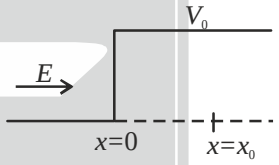
55. Since, $E<V_0$, so according to principle of quantum tunneling there will be a fiinite probability for particle to cross the barrier.

Wevefunction of the particle in the region $\left(x>0\right)$ is

$$\psi\left(x\right)=D\cdot e^{-\alpha x}$$

So, probability of finding the particle in the region $\left(x>0\right)$ is

Given, $P\left(x\right)=\left|\psi\left(x\right)\right|^2=\left|D\right|^2 e^{-2\alpha x}$



$$\Rightarrow \quad P\left(x=x_0\right)=\frac{1}{e}P\left(x=0\right)$$

$$e^{-2\alpha x_0}=e^{-1}$$

$$\Rightarrow \quad 2\alpha x_0=1 \Rightarrow x_0=\frac{1}{2\alpha}$$

Correct option is (c)

CAREER ENDEAVOUR

OBJECTIVE QUESTION

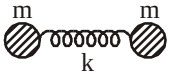
1. $^{13}C_6 + \nu_e \rightarrow {}^{13}N_7 + X$

X should be electron $\left({}_{-1}e^0\right)$

$\therefore \quad {}^{13}C_6 + \nu_e \rightarrow {}^{13}N_7 + {}_{-1}e^0$

Therefore, charge Q will be conserved also electronic lepton number will be conserved.

Correct option is (a)

2. 

$\omega = \sqrt{\frac{k}{\mu}}, \mu = \text{ reduced mass } = \frac{m \cdot m}{m+m} = \frac{m}{2}$

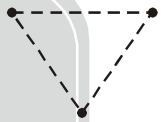
$\therefore \quad \omega = \sqrt{\frac{k}{m/2}} = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times 1}{10\times 10^{-3}}} = \sqrt{200} = 10\sqrt{2} \text{ rad/s} = 10\times 1.414 = 14.14 \text{ rad/sec}.$

Hence correct answer is (14.14)

3. The degree of freedom is

$f = 3N - C$

where N is the number of atoms in a molecule and C is the number of constraints.



Here, $N = 3$ and $C = 3$. So, $f = 6$.

Out of these six degree of freedom, three are due to translational and rest three are due to rotational motion.

At high temperature, all constraints get zero. So total degree of freedom becomes $f = 3N$ at high temperature and hence, nine for the given molecule.

Another three degree of freedom are due to vibrational motion.

According to law of equipartition of energy, each translational and rotational degree of freedom contributes $\frac{1}{2}k_BT$ to the energy and each vibrational degree of freedom contributes k_BT to the energy.

Average energy, $U = 6\times\frac{1}{2}k_BT + 3\times k_BT = 6k_BT$

Heat capacity, $C = \frac{dU}{dT} = 6k_B$

Correct option is (d)

4. $\bar{H} = a_0 I + \bar{b}\cdot\bar{\sigma} = a_0\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + b_x\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + b_y\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + b_z\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}a_0+b_z & b_x - ib_y\\ b_x + ib_y & a_0 - b_z\end{pmatrix}$

If λ is the eigenvalue of Hamiltonian, then $|H - I\lambda| = 0$

$$\begin{vmatrix} a_0 + b_z - \lambda & b_x - ib_y \\ b_x + ib_y & a_0 - b_z - \lambda \end{vmatrix} = 0 \Rightarrow \left(a_0 - \lambda^2\right)^2 - \left(b_z\right)^2 - \left(b_x\right)^2 - \left(b_y\right)^2 = 0 \Rightarrow \lambda = a_0 \pm \sqrt{b_x^2 + b_y^2 + b_z^2}$$

$\Rightarrow \quad \lambda = a_0 \pm |b|$

So, lowest enregy = $a_0 - |b|$

Correct option is (c)

CAREER ENDEAVOUR

5. $\left[x,\ xp_y + yp_x\right] = \left[x,\ xp_y\right] + \left[x,\ yp_x\right]$

$= p_y\left[x, x\right] + x\left[x,\ p_y\right] + p_x\left[x, y\right] + y\left[x,\ p_x\right]$

$= p_y \times 0 + x \times 0 + p_x \times 0 + y \times 1 = y$

Hence correct answer is (b)

6. For **s**-orbitals,

$\ell = 0, m = 0$, wavefunctions are only function of *r*. No angular dependence is there, hence s-orbitals are spherically symmetric

Correct option is (c)

7. The pressure exerted on the mirror

$$P = 2\langle u \rangle = 2 \times \frac{1}{2}\varepsilon_0 E_0^2 = \varepsilon_0 E_0^2 = 8.85 \times 10^{-12}\ Pa$$

Correct answer is (8.85)

8. The energy of the nth state of hydrogen atom

$$E_n \propto \frac{1}{n^2}$$

The highest possible energy state is $n = \infty$

$\therefore \qquad E_{highest} = 0$

Correct option is (a)

9. $U(x) = U_0 \cos\left(\frac{2\pi x}{a}\right) \qquad (1-\text{dimensional lattice})$

Magnitude of energy gap at the edge of Brillouin-zone $\left(k = \frac{\pi}{a}\right)$

$$E_g = \int_0^a dx\, U(x)\left[\left|\psi(+)\right|^2 - \left|\psi(-)\right|^2\right]$$

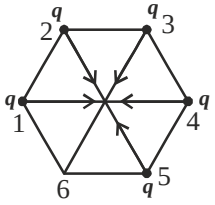
where $\psi(+) = \sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$ and $\psi(-) = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$ are the normalized wavefunctions.

$$E_g = \frac{2}{a}\int_0^a dx U_0 \cos\left(\frac{2\pi x}{a}\right)\left[\cos^2\frac{\pi x}{a} - \sin^2\frac{\pi x}{a}\right]$$

$$= \frac{2U_0}{a}\int_0^a dx \cos^2\left(\frac{2\pi x}{a}\right) = \frac{2U_0}{a}\int_0^a dx\left[\frac{1+\cos\frac{4\pi x}{a}}{2}\right] = \frac{U_0}{a}\left[x + \left(\sin\frac{4\pi x}{a}\right) \times \frac{a}{4\pi}\right]_0^a = U_0$$

Correct option is (a)

CAREER ENDEAVOUR

10. 

At centre : Field due to charge at 1 will be cancelled by field of charge 4. Field due to 2 and 5 are also cancelled at centre.

The resultant electric field will be for only one charge particle which is present at point 3.

So, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$

And potential, $V = V_1 + V_2 + V_3 + V_4 + V_5 = \frac{5}{4\pi\varepsilon_0}\frac{q}{a}$

Correct option is (c)

11. The efficiency of a reversible Carnot engine is independent of any working substance and depends only on the temperature of the source (T_2) and of the sink (T_1). It is given by $\eta = 1 - \frac{T_1}{T_2}$.

Correct option is (b)

12. Given 7 bit A to D converter

Full scale voltage of 5 volt

$$\text{Voltage resolution} = \frac{\text{Full-Scale Voltage}}{2^n - 1} = \frac{5}{2^7 - 1}$$

Resolution = 39.27 mV

13. $L_0 = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2$

$$L = L_0 + \alpha\frac{qdq}{dt} = L_0 + \frac{d}{dt}\left(\frac{1}{2}\alpha q^2\right)$$

The given transformation satisfies gauge transformation of Lagrangian, therefore equation of motion will not change. Since the term added contains $\dot{q}$ therefore generalised momentum will change, as seen below

$$p_{\text{initial}} = \frac{\partial L_0}{\partial \dot{q}} = m\dot{q};\ p_{\text{final}} = \frac{\partial L}{\partial \dot{q}} = \frac{\partial L_0}{\partial \dot{q}} + \frac{\partial}{\partial \dot{q}}(\alpha q\dot{q});\ p_{\text{final}} = m\dot{q} + \alpha q$$

$p_{\text{final}} \neq p_{\text{initial}}$

Hence, correct option is (b)

14. We know, $C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} = \frac{Qd}{A\varepsilon_0 k}$

We have, $Q = 1\ C, d = 10^{-6}\,m, A = 1\,m^2, \varepsilon_0 = 8.854\times10^{-12}$

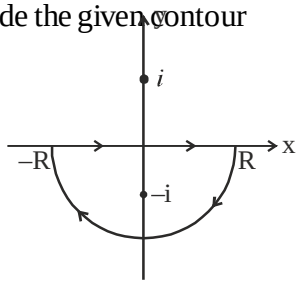
$$\therefore \quad V = \frac{10^{-6}}{8.854\times10^{-12}\times10} = 0.1129\times10^5 = 11.29\ kV$$

Correct answer is (11.29)

CAREER ENDEAVOUR

15. $\oint \frac{dz}{1+z^2} = \oint \frac{dz}{(z+i)(z-i)}$

Therefore, poles are at $z = i, -i$. Only $z = -i$ will lie inside the given contour



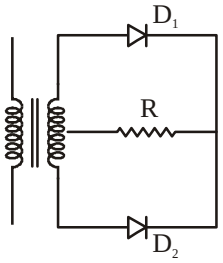
Therefore, Residue at $z = -i$

$$\lim_{z \to -i} \left[(z+i) \times \frac{1}{(z+i)(z-i)} \right] = -\frac{1}{2i}$$

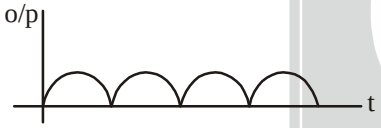
$$\therefore \quad \oint \frac{dz}{1+z^2} = -2\pi i \times (\text{Residual at } z = -i)$$

$$= -2\pi i \times -\frac{1}{2i} = \pi = 3.14$$

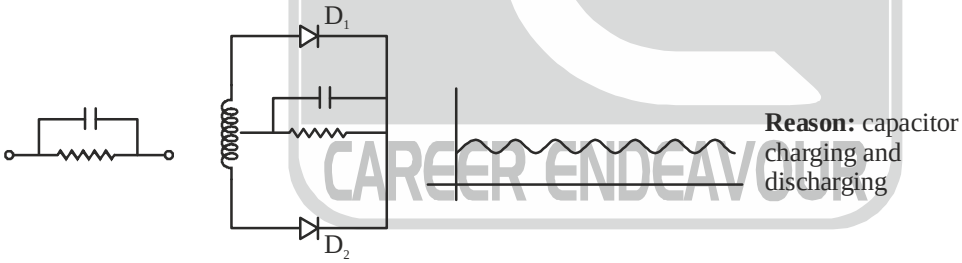
16.

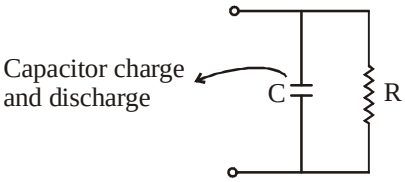


This circuit is a full wave rectifier. So, output wave form



- Output wave form for

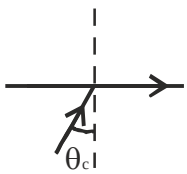




Correct option is (a)

17. We know Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

At critical angle, $n \sin\theta_c = 1$



$$\Rightarrow \quad \sin\theta_c = \frac{1}{1.5}$$

$$\Rightarrow \quad \theta_c = 41.81$$

Correct answer is (41.81)

18. In electromagnetic interactions, all three C, CP and CPT should be conserved.
Therefore, **correct option is (a)**

19. Compton wavelength, $\lambda_c = \dfrac{h}{m_p c} = \dfrac{6.626\times10^{-34}}{1.67\times10^{-27}\times3\times10^{8}} = 1.322\times10^{-15} m = 1.322\,\text{fm}$

Correct answer is (1.32)

20. $\tau^+ \to \mu^+ + \mu^+ + \mu^-;\ \ L_e : -1 \to 0+0+0$

$\therefore \quad \Delta L_\tau \neq 0$ i.e. Tau number is not conserved

Charge is conserved, total lepton number and angular momentum is also conserved.

Correct option is (d)

21. Fourier series in complex form

$$f(x) = C_0 + \sum_{k=1}^{\infty} C_k\, e^{ikx} + \sum_{k=1}^{\infty} C_{-k}\, e^{-ikx}$$

$$u(x) = A\sin^2(ax) = A\left(\frac{e^{iax} - e^{-iax}}{2i}\right)^2 = -\frac{A}{4}\left(e^{2iax} + e^{-2iax} - 2\right)$$

Hence co-efficient of e^{ikx} when $k = -2a$ is $-\dfrac{A}{4}$

Correct option is (b)

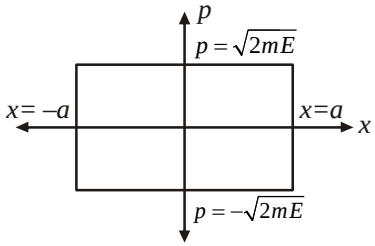
22. The phase-space trajectory of a particle under any potential is the momentum-space diagram.
If the free particle has fixed amount of energy E, then

$$E = \frac{p^2}{2m}$$

where *m* is the mass of the particle.

$\Rightarrow \quad p = \pm(2mE)^{1/2}$

So we have two fixed values of p.
Moreover, the ball is bouncing between two hards walls. So position of it is also restricted, lets say between, $x = -a$ and $x = +a$. So the phase space is



Correct option is (c)

23. The number of density of valence electrons $= \left(\dfrac{N\ \rho_{\text{atomic}}}{M}\right)$

$$= \frac{6.022\times10^{23}}{23}\times0.968\,cm^{-3} = 2.53\times10^{22}\ cm^{-3}$$

24. The number of degeneracy of nth energy level of 3-dimension isotropic harmonic oscillator is given by

$$= \frac{1}{2}(n+1)(n+2) = \frac{1}{2}(4)\times(5)\,(\text{for third energy level } n=3)$$

$$= 10$$

Correct option is (d)

25. The first order correction of the ground state energy

$$E_0^{(I)} = \langle 0|H'|0\rangle = \alpha\langle 0|x|0\rangle + \beta\langle 0|x^3|0\rangle + \gamma\langle 0|x^4|0\rangle = 0+0+\gamma\langle 0|x^4|0\rangle \propto \gamma$$

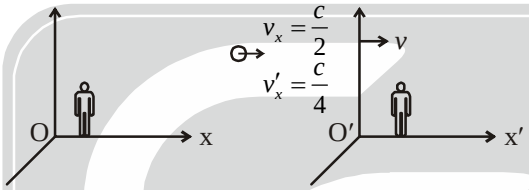
Correct option is (d)

26. Total charge of the system is 2 – 1 – 1 = 0
So, the dipole moment does not depend on the origin
So, the dipole moment w.r.t. the point A is

$$\vec{P} = \sum_i q_i \vec{r}_i = \left(-1+\frac{1}{2}\times 2\right)\hat{i} + \frac{\sqrt{3}}{2}\times 2\,\hat{j} = \sqrt{3}\;\hat{j} = 1.73\hat{j}$$

Correct answer is (1.73)

27. $v =$ velocity of O' w.r.t O



$$v'_x = \frac{v_x - v}{1 - \frac{v_x \cdot v}{c^2}}$$

$$\frac{c}{4} = \frac{\frac{c}{2} - v}{1 - \frac{v}{2c}} \Rightarrow \frac{c}{4} - \frac{v}{8} = \frac{c}{2} - v \;\Rightarrow\; \frac{7v}{8} = \frac{c}{4} \;\Rightarrow\; v = \frac{2c}{7} = 0.28c$$

Hence correct answer is (0.28)

28. Given: (1) $u = \frac{U}{V} = aT^4$ or $U = aT^4V \Rightarrow dU = a\left(4T^3VdT + T^4dV\right)$

(2) $P = \frac{u}{3} = \frac{U}{3V}$

(3) $\frac{S}{V} = \alpha aT^3$ or $S = \alpha aT^3V \Rightarrow dS = \alpha a\left(3T^2VdT + T^3dV\right)$

Using combined I and II law,

$TdS = dU + PdV$, we have

$$\alpha aT\left(3T^2VdT + T^3dV\right) = a\left(4T^3VdT + T^4dV + \frac{U}{3V}dV\right)$$

$$\Rightarrow \quad 3\alpha aT^3VdT + \alpha aT^4dV = 4aT^3VdT + \frac{4}{3}aT^3dV$$

Comparing, we have

$$\alpha = \frac{4}{3} = 1.33$$

Correct answer is (1.33)

29. We know due to rotation of earth, weight of person at latitude λ is given by

$$mg' = mg - m\omega^2 R\cos^2\lambda$$

$$\Rightarrow \quad w_p = mg \text{ (at north pole)}$$

And $\quad w_e = mg - mw^2 R$

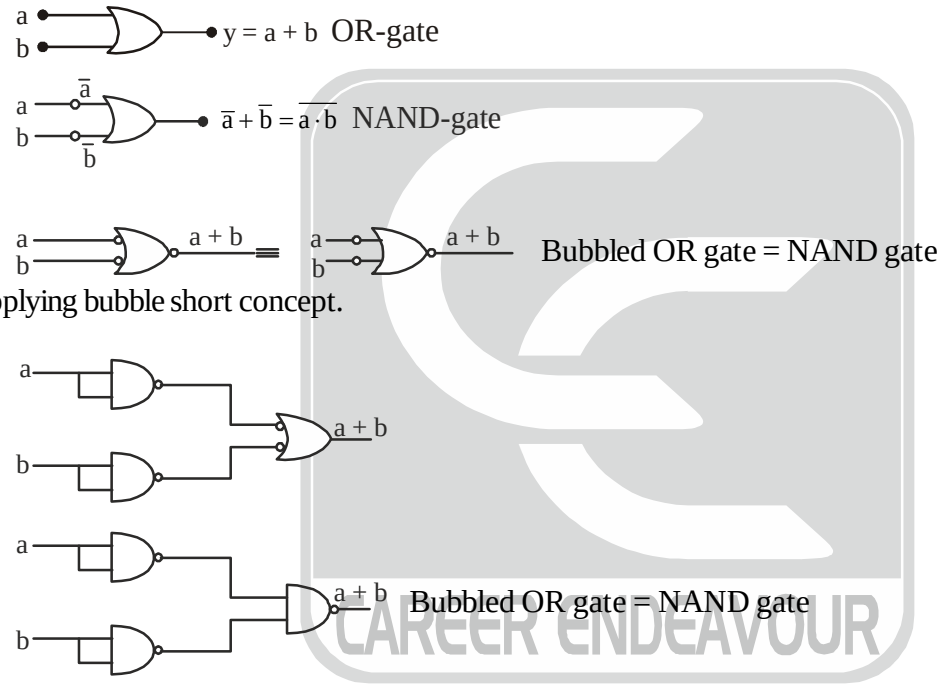
$$\Rightarrow \quad w_e - w_p = -mw^2 R$$

$$\Rightarrow \quad \frac{w_p - w_e}{w_p} = \frac{w^2 R}{g} = \frac{w^2 R}{g}$$

$$\Rightarrow \quad \frac{w_p - w_e}{w_p}\times 100 = \left(\frac{2\pi}{24\times 3600}\right)^2 \times \frac{6400\times 10^3}{10}\times 100 = 0.003337\times 100 = 0.337$$

Correct answer is (0.33)

30. Minimum number of NAND gate required for OR-gate



Applying bubble short concept.

Note: 3 number of NAND are needed.

Correct option is (d)

31. $$E(R) = \frac{0.5}{R^{12}} - \frac{1}{R^6} \Rightarrow \frac{dE}{dR} = -\frac{0.5(12)}{R^{13}} + \frac{6}{R^7}$$

At the equilibrium $\left.\frac{dE}{dR}\right|_{R=R_0} = 0$

Therefore, $-\frac{6}{R_0^{13}} + \frac{6}{R_0^7} = 0 \Rightarrow R_0 = 1$

Correct answer is (1.00)

CAREER ENDEAVOUR

32. $v(x,y) = 2xy + 3y$

According to Cauchy Reamann equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 2x + 3 \Rightarrow u = x^2 + 3x + f(y); \quad \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = -2y \Rightarrow u = -y^2 + g(x)$$

So, $\boxed{u(x,y) = x^2 + 3x - y^2}$

$u(z = 1+i) = u(x = 1,\ y = 1) = 3$

Correct answer is (3)

33. The population of particles in the level with energy ε is

$$\langle N \rangle = \frac{N\, e^{-\varepsilon/k_B T}}{e^{-0/k_B T} + e^{-\varepsilon/k_B T}} = \frac{N\, e^{-\varepsilon/k_B T}}{1 + e^{-\varepsilon/k_B T}} = \frac{N}{e^{\varepsilon/k_B T} + 1}$$

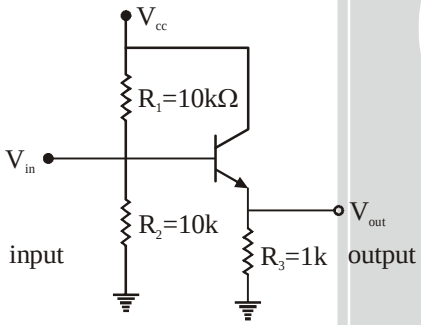
In the high temperature limit,

$$\Rightarrow \quad k_B T \gg \varepsilon \ \Rightarrow \ \frac{\varepsilon}{k_B T} \ll 1$$

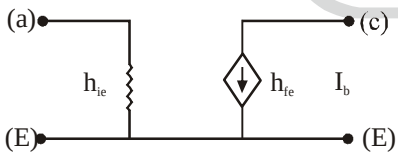
$$\therefore \quad e^{\varepsilon/k_B T} \approx 1 \Rightarrow \quad \langle N \rangle \simeq \frac{N}{1+1} \simeq \frac{N}{2}$$

Correct option is (a)

34.

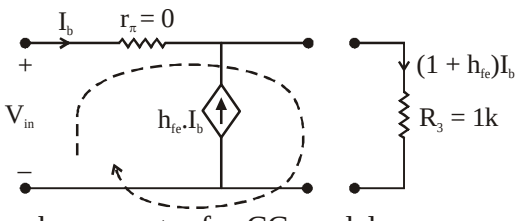


- Its a CC-configuration



$h_{ie} = r_\pi$ (emitter diode —wwww—)

Given: emitter diode resistance is zero; $r_\pi = 0$



h-parameter for CC model

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$$V_{in} = r_\pi \times I_b + \left(1 + h_{fe}\right) I_b \times R_3 \quad r_\pi = 0$$

$$R_i = \frac{V_{in}}{I_b} = \left(1 + h_{fe}\right) R_3 = \left(1 + 99\right) \cdot 1 = 100\ k\Omega$$

Correct answer is (100)

35. For the Raman active, diatomic molecules must have changing polarisability. For IR active molecules must have change in dipole moment. While for NMR, presence of non-zero nuclear spin is compulsory.
Considering given option :

(a) $^{1}H - {}^{1}H$: There is no change in dipole moment due to symmetry, therefore, it is not IR active.

(b) $^{12}C - {}^{16}O$: Net nuclear spin zero, therefore not NMR active.

(c) $^{1}H - {}^{35}Cl$: It has all the condition available for IR, NMR and Raman active.

(d) $^{16}O - {}^{16}O$: There is no change in dipole moment, therefore it is not IR active.

Correct option is (c)

36. **Correct option is (c)**

37. The reflection coefficient,

$$R = \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}}\right)^2 = \left(\frac{1 - \sqrt{0.25}}{1 + \sqrt{0.25}}\right)^2 = \left(\frac{1 - 0.5}{1 + 0.5}\right)^2 = \left(\frac{0.5}{1.5}\right)^2 = \frac{1}{9} = 0.11$$

38. The magnetic field inside the solenoid will be along $\hat{z}$ axis. So, $B_z \neq 0, B_r = B_\theta = 0$

Correct option is (d)

39. **(i) When particles A and B are distinguishable.**
Since particles are distinguishable, there is no limit on the filling of the particles in energy levels. Furthermore since level 1 already contains a particle, we have in total three ways to filled the levels.

1	2
AB	
A	B
B	A

Therefore, P(level 2 also contains a particle) $= \frac{2}{3}$

(ii) When particles A and B are bosons.
Since bosons are indistinguishable particles and there is no restriction on the filling of them in different energy levels, we have in total two ways to fill them.

1	2
..	
.	.

Therefore, P(level 2 also contains a particle) $= \frac{1}{2}$

Correct option is (c)

40. $a_1 = a\hat{x};\ a_2 = \frac{a}{2}\left(\hat{x} + \sqrt{3}\ \hat{y}\right)$

Let $a_3 = \hat{z}$

The reciprocal space vector of this lattice are

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$$\therefore \qquad b_1 = a_1^* = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi}{\frac{\sqrt{3}}{2}a^2}\left(\frac{\sqrt{3}}{2}a\hat{x} - \frac{a}{2}\hat{y}\right) = \frac{2\pi}{a}\left[\hat{x} - \frac{1}{\sqrt{3}}\hat{y}\right]$$

$$\therefore \qquad b_2 = a_2^* = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi\ a\hat{y}}{\frac{\sqrt{3}}{2}a^2} = \frac{4\pi\hat{y}}{\sqrt{3}\,a}$$

Correct option is (a)

41. Consider two colliding protons at very large energies. The effective size of colliding protons is approximately 10^5 times smaller (in radius) than an atom.

Size of an atom is generally considered in Angstroms (Å)

$1Å = 10^{-10}$ m

Effective size of proton will be 10^{-15} m

Geometric cross-section $\propto r^2 = 10^{-30} m = 10^{-2}\ barn \qquad \left(\because\ 1\ barn = 10^{-28} m^2\right)$

$= 10^{-2}$ b = 10 mb

Correct option is (b)

42. We know, $\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon_0}t} \Rightarrow \frac{\rho_0}{e} = \frac{\rho_0}{e^{\frac{\sigma}{\varepsilon_0}t}} \Rightarrow \frac{\sigma}{\varepsilon_0}t = 1$

$$\Rightarrow \qquad t = \frac{\varepsilon_0}{\sigma} = 8.85\times10^{-12}\times10^{-2} = 8.85\times10^{-14} = 88.50\times10^{-15}\ \text{sec} = 88.50\ \text{Femto seconds}$$

Correct answer is (88.50)

43. The charge in the temperature of the melting point of a solid dT due to a change of pressure dP is given by Clapeyron's equtaion

$$\frac{dp}{dT} = \frac{L}{T(v_2 - v_1)}$$

where L is the latent heat of fusion of the solid and $(V_2 - V_1)$ the change in volume, T is melting point

Specific volume of ice $v_{ice} = \frac{1}{\rho_{ice}}$

Specific volume of of water $v_{water} = \frac{1}{\rho_{water}}$

$$\therefore \qquad v_2 = v_{ice} = \frac{1}{\rho_{ice}},\ v_1 = v_{water} = \frac{1}{\rho_{water}}$$

$dT = 10^o C = 10K,\ L = 3.34\times10^5\ J/kg,\ T = 273\ K$

$$dp = \frac{LdT}{273\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)} = \frac{3.34\times10^5\times10}{273\left(\frac{1}{934} - \frac{1}{1000}\right)} = \frac{3.34\times934\times10^9}{273(1000-934)}$$

$$\simeq 0.173\times10^9\ Pa = 0.173\times GPa$$

Correct answer is (0.173)

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44. $\int_0^\infty x^2 e^{-x^2} dx$, [let $x^2 = t \Rightarrow 2xdx = dt$]

$$= \int_0^\infty x \cdot x\, e^{-x^2} \cdot dx = \int_0^\infty t^{1/2} e^{-t} \frac{dt}{2}$$

$$= \frac{1}{2}\int_0^\infty t^{\frac{3}{2}-1} e^{-t} dt = \frac{1}{2}\lceil 3/2 = \frac{1}{2} \times \frac{1}{2}\lceil 1/2 = \frac{1}{2} \times \frac{1}{2}\sqrt{\pi} = \frac{\sqrt{\pi}}{4} \simeq 0.44$$

Correct answer is (0.44)

45. $^{13}C_6$

Proton number $P = 6$ and Neutron number $N = 7$

$P = 6 : 1S_{½}^2\ \ 1P_{3/2}^4$

$N = 7 : 1S_{½}^2\ 1P_{3/2}^4\ 1P_{½}^1$

The total angular moment is due to momentum of last unpaired neutron.

$\therefore \qquad J = \frac{1}{2}$, the parity $= (-1)^\ell = (-1)^1 = -1$

$\therefore \qquad J^P = \frac{1}{2}^{-1}$

Correct option is (d)

46.

Angular momentum about point of contact is conserved

$$mu\,R = mvR + I_c\omega$$

$$mu\,R = mvR + \frac{2}{5}mR^2 \cdot \frac{v}{R}$$

$$u = \frac{7}{5}v$$

$$v = \frac{5}{7}u = \frac{5 \times 5}{7} = \frac{25}{7} = 3.57\ m/s$$

Hence correct answer is (3.57)

47. $(\Delta x)_{\max} = 3\,nm$

$$(\Delta p_x)_{\min} = \frac{\hbar}{2 \times 3 \times 10^{-9}} = \frac{\hbar \times 10^9}{6}$$

$$E_{\min} = \frac{(\Delta p_x)^2}{2m} = \frac{(\hbar \times 10^9 / 6)^2}{2 \times 9.31 \times 10^{-31}} = \frac{\left(\frac{6.626 \times 10^{-25}}{2\pi \times 36}\right)^2}{2 \times 9.31 \times 10^{-31}} = 9.83 \times 10^{-4} \simeq 10 \times 10^{-4}\ eV \simeq 1\ meV$$

So, the minimum depth of the well should be 1 meV.

Correct option is (b)

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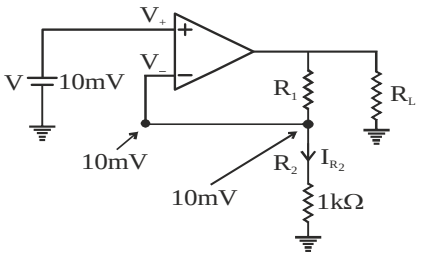
48. Given $R_1 = 5k\Omega$

$R_2 = 1k\Omega$

$R_L = 100\,k\Omega$

$V = 10\,mV$

Current through R_2 ____ μA (micro ampere)



As ideal op-amp has infinite input resistance. So, there will be no current pass through op-amp.

So, $V_+ = V_-$ (Virtual ground concept)

= 10 mV

$$I_{R_2} = \frac{10\,mV}{R_2} = \frac{10\times10^{-3}}{1\times10^{3}}$$

$I_{R_2} = 1\times10^{-6}$ Ampere

$I_{R_2} = \mathbf{1}\,\mu A$

Correct answer is (1)

49. $\underset{\text{Rest}}{\pi^+} \longrightarrow \mu^+ + \nu_\pi$

By conservation of momentum, $P_\mu = P_\nu$ ... (1)

By conservation of energy,

$E_\pi = E_\mu + E_\nu$ ... (2)

If m_π, m_μ and 0 are rest masses of π, ν and ν respectively. Then equation (2) becomes

$$m_\pi c^2 = \sqrt{P_\mu^2 + m_\mu^2 c^4} + P_\nu c \quad \Rightarrow \quad P_\nu c = \frac{\left(m_\pi c^2\right)^2 - \left(m_\mu c^2\right)^2}{2m_\pi c^2}$$

$$\Rightarrow \quad P_\nu c = \frac{(139)^2 - (105)^2}{2\times139} \simeq 29.84 \text{ MeV} \quad \Rightarrow \quad P_\nu = \frac{29.84}{c}\text{ MeV}$$

Correct answer is (29.84)

50. Given differential equation is

$\frac{dy}{dx} + y\tan x = \cos x$ (1st or order linear differential equation)

I.F.= $e^{\int \tan x\,dx} = e^{\log\sec x} = \sec x$

Therefore, solution will be

$\Rightarrow\ y\times\sec x = \int \cos x\cdot\sec x\,dx + c$

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$$y\sec x = x + c$$

$\because \quad y(0) = 0,$

$\Rightarrow \; 0 = 0 + c \;\Rightarrow c = 0$

So, $\quad y\sec x = x \;\Rightarrow\; y = \dfrac{x}{\sec x} = x\cos x$

$\therefore \quad y(\pi/3) = \dfrac{\pi}{3}\cos\dfrac{\pi}{3} = \dfrac{\pi}{3}\times\dfrac{1}{2} = \dfrac{\pi}{6} = 0.5233$

Correct answer is (0.5233)

51. We know that the lowest frequency of electromagnetic wave that can propagate through the metal is proportional to square root of density of free electrons.

$$f \propto \sqrt{n}$$

$$\therefore \qquad \frac{f_1}{f_2} = \sqrt{\frac{n_1}{n_2}}$$

$$\Rightarrow \qquad f_2 = f_1\sqrt{\frac{n_2}{n_1}} = 1.38\times10^{16}\sqrt{\frac{1.8\times10^{23}}{6\times10^{22}}}\ Hz = 1.38\times10^{16}\sqrt{3} = 2.39\times10^{16}\ Hz$$

Correct answer is (2.39)

52. Electronic configuration of the nitrogen ${}_7N = 1s^2\ 2s^2\ 2p^3$

i.e., it has three electrons outside closed shells.

Chart of nitrogen state

m_e	e
1	↑
0	↑
−1	↑
$S = \sum m_s = \frac{3}{2}$	
$L = \sum m_l = 0$	

i.e., total spin $S = \dfrac{3}{2}$, multiplicity $= 2\times\dfrac{3}{2} + 1 = 4$

Total $L = 0$, i.e. *s*-orbital

Total angular momentum $J = |L + S|$.... to $|L - S| = \dfrac{3}{2}$

Therefore, ground state term is ${}^4S_{3/2}$

Correct option is (b)

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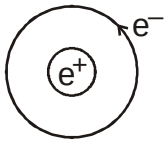
54. $$\left(\frac{i\left(p_x x^2 - x^2 p_x\right)}{2}\right)^{\dagger} = \frac{-i\left(x^2 p_x - p_x x^2\right)}{2} = \frac{i\left(p_x x^2 - x^2 p_x\right)}{2}$$

Correct option is (a)

55. Radius of the Bohr orbit is $r_n = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{Z e^2 \mu}$, *i.e.*, $r_n \propto \frac{1}{\mu}$, where $\mu = \frac{m_1 m_2}{m_1 + m_2}$; reduced mass of the hydrogen like system.

For hydrogen atom : $m_p = 1836\, m_e$

$$\Rightarrow \quad \mu_H = \frac{m_e \cdot m_p}{m_e + m_p} = \frac{1836\, m_e}{1837\, m_e} \simeq m_e$$



For positronium atom : $m_{e^-} = m_{e^+}$

$$\Rightarrow \quad \mu_p = \frac{m_{e^-} \cdot m_{e^+}}{m_{e^-} + m_{e^+}} = \frac{m_e}{2}$$

Ratio of ground state orbit $\frac{R_P}{R_H} = \frac{\mu_H}{\mu_P} \qquad \left[\because\ r_n \propto \frac{1}{\mu}\right]$

$$\Rightarrow \quad R_P = \left(\frac{\mu_H}{\mu_P}\right) R_H = \frac{m_e}{m_e/2} \cdot 0.53\ \text{Å} = 2 \times 0.53\ \text{Å} = 1.060\text{Å} \simeq 1.06\text{Å}$$

Correct answer is (1.06)

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